### MICHAEL RONEN

## MULTI-QUBIT DEVICES CAPACITIVE AND INDUCTIVE COUPLING



4 Transmon Qubit/4 Bus/4 Readout Chip by IBM (Fig. 4c, Gambetta et al., Building logical qubits in a superconducting quantum computing system. npj Quantum Inf 3, 2 (2017). https://doi.org/10.1038/s41534-016-0004-0)

Seminar: Superconducting Quantum Hardware for Quantum Computing

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# MOTIVATION

Qubit Computation:

Time evolution of state  $|\psi(t)
angle=e^{-iHt/\hbar}|\psi_0
angle=U|\psi_0
angle
ightarrow$  unitary transformation

- Quantum Circuit Model:
  - $\rightarrow$  U can be constructed in approximation from a finite set of
  - 1 qubit operations  $U \in SU(2)$  and
  - 2 qubit operations  $U \in SU(4)$
- Examples of two qubit gates, necessary for universality:





SUPERCONDUCTING QUBIT ARCHETYPES

The Charge Qubit (Cooper-Pair Box, CPB)



Superconducting Charge Qubit – Cooper-pair box (Fig. 17.4a, Kockum A.F., Nori F. (2019) Quantum Bits with Josephson Junctions. Springer)

• Hamiltonian: 
$$H_{CPB} = E_C (N - N_g)^2 - E_J \cos(\theta)$$

charge energy:  $E_C = \frac{(2 e)^2}{2(C_J + C_g)}$ ; background charge:  $N_g = C_g \frac{U}{2e}$ 

charge energy

Josephson energy

through gate voltage **U** 

- Externally adjustable parameters:
  - 1. Background charge:  $N_g = C_g \frac{v}{2e}$
  - 2. Josephson energy:  $E_J(\Phi_{ext}) = E_J \cos(\frac{\Phi_{ext}}{\Phi_0}\pi)$
- States: number of excess Cooper pairs on island

 $|0\rangle = |N\rangle$  $|1\rangle = |N+1\rangle$ 



CPB Potential (red) and two lowest states (blue solid) (Srjmas, "Charge qubit potential",

https://commons.wikimedia.org/wiki/File:Charge\_qubit\_potential.svg)



### Capacitive Coupling

### Tunable Coupling

nductive Coupling

### SUPERCONDUCTING QUBIT ARCHETYPES

The Flux Qubit (*RF SQUID*)



Superconducting Flux Qubit – RF-SQUID (Srjmas, "Flux qubit circuit", https://commons.wikimedia.org/wiki/File:Flux\_qubit\_circuit.svg)

#### Capacitive Coupling

#### Tunable Coupling

### • Hamiltonian:

energy scales:

3.

Charge on 
$$C_J$$
  
 $H_{Flux} = \frac{q^2}{2C_J} + (\frac{\Phi_0}{2\pi})^2 \frac{\phi^2}{2L} - E_J cos(\phi - \Phi_{ext}\frac{2\pi}{\Phi_0})$   
 $E_J, \quad E_{C_J} = \frac{(2e)^2}{2C_J}, \quad E_L = \frac{\Phi_0^2}{2L}$ 
Potential

- Externally adjustable parameters:
  - 1. External bias flux:  $\Phi_{ext}$
  - 2. Loop inductance:
    - Josephson energy:  $E_J(\Phi_J) = E_J \cos(\frac{\Phi_J}{\Phi_0}\pi)$

L

States: symmetrical and antisymmmetrical superposition of flux quanta

$$|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |0\rangle)$$
$$|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |0\rangle)$$

through bias current  $I_b$ through coil  $\ell$  and N



Flux Qubit Potential (red) and two lowest states (blue solid) (Srjmas, "Flux qubit potential",

https://commons.wikimedia.org/wiki/File:Flux\_qubit\_potential.svg)



#### Iunable Coupling

### SUPERCONDUCTING QUBIT ARCHETYPES

The Phase Qubit (Current Biased Josephson Junction, CBJJ)



c Current-driven junction (phase qubit)



Fig. 1, You, J., Nori, F. Atomic physics and quantum optics using superconducting circuits.Nature 474, 589–597 (2011). https://doi.org/10.1038/nature10122

Superconducting Charge Qubit – Cooper-pair box (Fig. 17.4b, Kockum A.F., Nori F. (2019) Quantum Bits with Josephson Junctions. Springer)

Iunable Coupling



States: Oscillations modes in superconducting loop

# CAPACITIVE COUPLING

## • Two capacitively coupled charge qubits:

- Qubit state given by charge on superconducting island
- Qubit charges on capacitor  $C_m \rightarrow$  coupling of charge states







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Pashkin, Y., Yamamoto, T., Astafiev, O. et al. Quantum oscillations in two coupled charge qubits. Nature 421, 823–826 (2003). https://doi.org/10.1038/nature01365

Tunable Couplir



 $\rightarrow$  gate voltages allow control over diagonal terms

Josephson coupling  $E_{J_{1,2}} \approx E_m < E_{C_{1,2}} \rightarrow \text{coherent superpositions of } \{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$  at  $n_{g_{1,2}} = 0.5$ 

Pashkin, Y., Yamamoto, T., Astafiev, O. et al. Quantum oscillations in two coupled charge qubits. Nature 421, 823–826 (2003). https://doi.org/10.1038/nature01365

- In absence of Josephson coupling:
  - Hexagonal boundaries between states
  - R and L degeneracy between neighbouring states
    - System will oscillate between neighbouring states
  - **n** $_{g_1}$ ,  $n_{g_2}$ inside cell ightarrow system will remain in cells' state
  - Pulse gate shifts system along 45° line (black arrows)
- With small Josephson coupling:
  - > States become superposed on boundaries!



- Double degeneracy
- > Superposition of all charge states  $|\psi(t)\rangle = c_1 |00\rangle + c_2 |10\rangle + c_3 |01\rangle + c_4 |11\rangle$



**Figure 2a:** Ground-state charging diagram of coupled qubits

Pashkin, Y., Yamamoto, T., Astafiev, O. et al. Quantum oscillations in two coupled charge qubits. Nature 421, 823–826 (2003). https://doi.org/10.1038/nature01365

Tunable Couplin

- Idea of the experiment:
  - Prepare system in  $|00\rangle$ 1.
  - 2.

Start applying pulse  $\rightarrow$  system at co-resonance point X in  $|\psi(t)\rangle = c_1 |00\rangle + c_2 |10\rangle + c_3 |01\rangle + c_4 |11\rangle$ 

- Stop applying pulse  $\rightarrow$  system 'frozen' in superposition
- System decays to  $|00\rangle$  emitting quasi particles 4.
- Readout scheme:

3.

- Measuring probe currents  $I_1$ ,  $I_2$  in proportion to probability of each qubit having a C. p. on it:  $I_1 \propto p_1(1) \equiv |c_2|^2 + |c_4|^2$
- Time evolution of probabilities:



b

4.

3.

Pashkin, Y., Yamamoto, T., Astafiev, O. et al. Quantum oscillations in two coupled charge qubits. Nature 421, 823–826 (2003). https://doi.org/10.1038/nature01365

 $|00\rangle$ 

|10 >

### (a) Probe current oscillations at R and L

(b) Probe current oscillations at co-resonance X



Figure 3a: Oscillations at resonance points

- Can be fitted with cosine
- Single peaks at different energies ~
  - $\rightarrow$  no qubit interaction

Figure 2a: Ground-state charging diagram of coupled qubits

 $\left( \begin{array}{c} \mathbf{A} \\ \mathbf{A} \\$ 

Figure 3b: Oscillations at co-resonance point

- Two peaks in spectrum
- Two peaks at same energies

 $\rightarrow$  evidence for qubit interaction

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 $\Omega - \varepsilon \quad \Omega + \varepsilon$ 

Pashkin, Y., Yamamoto, T., Astafiev, O. et al. Quantum oscillations in two coupled charge qubits. Nature 421, 823–826 (2003). https://doi.org/10.1038/nature01365

Tunable Couplin

## TUNABLE COUPLING

- Two capacitively coupled phase qubits:
  - Hamiltonian:



effective capacitances:  $\tilde{C}_{b(jq)} = C_{jq(b)} + 1/(1/C_{b(jq)} + 1/C_C)$   $\tilde{C}_C = C_{jq}C_{jb}(1/C_{jb} + C_{jq} + 1/C_C)$ 

- Now:  $C_{ig} = C_b \equiv C_i$   $E_{ig} = E_b \equiv E_i$ 
  - $\rightarrow$  approximately cubic potential wells
  - $\rightarrow$  junctions treated as anharmonic oscillators

Qubit

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Blais & v. e. Brink & Zagoskin (2003). Tunable Coupling of Superconducting Qubits. Physical review letters. 90. 127901. 10.1103/PhysRevLett.90.127901.

**Tunable Coupling** 

Tunable coupling bus

 $\mathsf{C}_{\mathsf{jb}}$ 

C<sub>C</sub>

۲E<sub>ib</sub>

• Hamiltonian  $H_2$  in  $span\{|0_q1_b\rangle, |1_q0_b\rangle\}$ :

$$H_{2} = \begin{pmatrix} E_{q0} + E_{b1} & \gamma/2 \\ \gamma/2 & E_{q1} + E_{b0} \end{pmatrix}$$

$$\sum_{\text{coupling coefficient: } \gamma \equiv \hbar \sqrt{\omega_{pq} \omega_{pb}} \frac{\tilde{C}_{j}}{\tilde{C}_{C}}$$

- Without coupling:  $|0_q 1_b\rangle$  and  $|1_q 0_b\rangle$  degenerate for  $I_b$  such that  $E_{q1} E_{q0} = E_{b1} E_{b0}$
- With coupling: lifts degeneracy and the new eingenstates are

$$|\Psi_{\pm}\rangle = \frac{1}{\sqrt{2}}(|0_q 1_b\rangle \pm |1_q 0_b\rangle)$$

- In resonance:  $H_2$  acts like  $e^{-i\frac{\sigma_X\gamma\tau}{2\hbar}} \rightarrow$  prepared in  $|1_q0_b\rangle$ : probability for  $|1_q\rangle$  oscillates with  $T_{Rabi} = h/\gamma$
- Anharmonicity of qubits supresses leakage out of two level system
- Coupling term  $\frac{q_q q_b}{\tilde{c}_c}$  causes nonresonant leakage to  $|2_{q(b)}\rangle$  (close to barrier  $\rightarrow$  large transition rate)  $\rightarrow$  shortens coherence time

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• Bus not tuned to qubit frequency  $\Omega_q \longrightarrow$  qubit decoupled from bus

Blais & v. e. Brink & Zagoskin (2003). Tunable Coupling of Superconducting Qubits. Physical review letters. 90. 127901. 10.1103/PhysRevLett.90.127901.

		Tunable Coupling	
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Figure 2: Quality of coupled identical CBJJ qubits

Blais & v. e. Brink & Zagoskin (2003). Tunable Coupling of Superconducting Qubits. Physical review letters. 90. 127901. 10.1103/PhysRevLett.90.127901.

Tunable Coupling

# **TUNABLE COUPLING**

#### Pair of charge-phase qubits coupled through CBJJ

- consider only two levels For qubits:
- $\rightarrow$  bus can be coupled to only one qubit  $\Omega_1 \neq \Omega_2$ (by tuning bus to  $\Omega_i$ )



- Hamiltonian:
  - Couples bus charge to island charge
  - Takes form  $\sigma_{ix} a_{h} / \tilde{C}_{c}$

Figure 3: A pair of charge-phase qubits capacitively coupled to a CBJJ

Qubit-bus coupling coefficient 
$$\gamma' \equiv \beta 2e(\frac{2\pi}{\Phi_0})^2 \frac{\sqrt{2m\hbar\omega_{pb}}}{\tilde{C}_C} \qquad \omega_{qi} = \sqrt{\frac{2\pi I_c}{\tilde{C}_j \Phi_0}} \sqrt{1 - (\frac{I_b}{I_c})^2}$$
  
depends on ratio  
between  $E_c$  and  $E_J$  effective coupling capacitances depend on  $C_{\Sigma,i} = C_{qi} + 2C_{ji}$ 

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Blais & v. e. Brink & Zagoskin (2003). Tunable Coupling of Superconducting Qubits. Physical review letters. 90. 127901. 10.1103/PhysRevLett.90.127901.

**Tunable Coupling** 

• Two qubit operation:

Assume qubits are in arbitrary state, bus in ground state.

- 1. Tune bus to  $\Omega_1$  for  $t_1$  such that  $\frac{\gamma' t_1}{2\hbar} = \frac{\pi}{2}$
- 2. Tune bus to  $\Omega_2$  for  $t_2$  such that  $\frac{\gamma' t_2}{2\hbar} = \frac{\pi}{4}$
- 3. Tune bus to  $\Omega_1$  again for  $t_1$
- 4. Disentangle bus from the qubits



- Problems:
  - Phase factors accumulating
  - Leakage to higher bus states
    - Charge-phase qubits at least as anharmonic as CBJJ
- $\rightarrow$  can be calculated numerically from known parameters
- $\rightarrow$  leakage not larger than with two phase qubits.

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Blais & v. e. Brink & Zagoskin (2003). Tunable Coupling of Superconducting Qubits. Physical review letters. 90. 127901. 10.1103/PhysRevLett.90.127901.

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	UU	

# INDUCTIVE COUPLING

- Passive inductive coupling of flux qubits
  - Flux in one qubit induces current in second qubit
  - $\rightarrow$  inductance matrix  $L_{ik}$ : connects flux in *i*-th loop with current in *k*-th loop:

$$\Phi_{i} = \sum_{k} L_{ik} I_{k}$$

$$L_{12} - \text{mutual inductance}$$
Generalized magnetic potential energy  $H_{Flux} = \frac{q^{2}}{2C_{J}} + (\frac{\Phi_{0}}{2\pi})^{2} \frac{\phi^{2}}{2L} - E_{J} cos(\phi - \Phi_{ext} \frac{2\pi}{\Phi_{0}})$ 

$$\frac{1}{2} \left(\frac{\hbar}{2e}\right)^{2} \sum_{ik} (L^{-1})_{ik} (\phi_{i} - \phi_{ei})(\phi_{k} - \phi_{ek}) \leftarrow \mathbf{T}$$

 $E_{J}$   $C_{J}$   $L_{1}$   $L_{2}$   $L_{2}$ 

 $L_{12}$ 

Interaction term

$$\hat{H}_{int} = \lambda \sigma_{z1} \sigma_{z2}$$

$$\lambda = \frac{1}{8} \left(\frac{\hbar}{2e}\right)^2 (L^{-1})_{12} (\phi_l - \phi_r)_1 (\phi_l - \phi_r)_2$$

Wendin, G., & Shumeiko, V. S. (2005). Superconducting quantum circuits, qubits and computing. arXiv preprint cond-mat/0508729.

Tunable Coupling

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 $E_J$ 

 $C_I$ 

Flux qubit Hamiltonian

$$\mathcal{H} = -E_{\rm J} \cos \left( 2 \pi \frac{\Phi}{\Phi_0} \right) + \frac{(\Phi - \Phi_{\rm x})^2}{2L} + \frac{Q^2}{2C_{\rm J}}$$

- Currents and fluxes in lower loop coupled (dashed line)
  - Fluxes control barriers between potential wells  $\rightarrow \propto \sigma_x^1 \sigma_x^2$  interaction
  - Placing loop differently  $\rightarrow \propto \sigma_z^1 \sigma_z^2$  interaction
  - Interaction energy in order of  $MI_c^2$  (*M* mutual inductance)
  - For typical RF-SQUID: coupling stronger than tunnelling rate between flux states
  - Turn coupling of by switch controlled by high-frequency pulses
    - Trade-off: coupling to external circuit leads to decoherence
    - Alternative: use ac driving pulses to induce state transitions two-qubit system



**Figure 9:** (a) Flux qubit and (b) improved design for flux qubit



**Figure 11:** Direct inductive coupling (dashed line) vs. coupling by LC-circuit (solid line).

Makhlin & Schön & Shnirman. Quantum-state engineering with Josephson-junction devices. Rev. Mod. Phys. 73, 357 (2001). https://link.aps.org/doi/10.1103/RevModPhys.73.357

Introduction

Capacitive Coupli

**Tunable Couplir** 

typically  $0.01E_{I}$ 

Flux qubit Hamiltonian

$$\mathcal{H} = -E_{\mathrm{J}} \cos\left(2\pi \frac{\Phi}{\Phi_{0}}\right) + \frac{(\Phi - \Phi_{\mathrm{x}})^{2}}{2L} + \frac{Q^{2}}{2C_{\mathrm{J}}}$$

- Coupling by LC circuit (solid line)
  - Without additional switches

Interaction Hamiltonian:

- Coupling controlled by qubit parameters
- Oscillator Hamiltonian:

$$H_{osc} = \frac{\Phi^2}{2L_{osc}} + \frac{Q^2}{2C_{osc}} - VQ$$
istance between two minima in potential

/Mutual inductance

Qubit inductance

00

• Weak coupling to *LC* circuit 
$$\Phi_i = \frac{i}{\hbar}[H_i, \Phi_i] = \delta \Phi_i B_x^i \sigma_y^i \sigma_y^j$$

 $H_{int} = -\left(\frac{\pi M}{L}\right)$ 



 $\frac{\sigma_j}{e^2/C_{osc}}\sigma_y^i\sigma_y^j$ 

(Heisenberg equation)



**Figure 11:** Direct inductive coupling (dashed line) vs. coupling by LC-circuit (solid line).



• Turn off coupling: Supress  $B^i_x \leftrightarrow$  increase potential barrier via  $\widetilde{\Phi}_x$ 

Makhlin & Schön & Shnirman. Quantum-state engineering with Josephson-junction devices. Rev. Mod. Phys. 73, 357 (2001). https://link.aps.org/doi/10.1103/RevModPhys.73.357

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	active galaxies such as NGC4261 using the Habble Space Tele- scope <sup>10</sup> . We have applied an axisymmetric term model with a circulate cross-section to the WVM data, in coder so determine the	Vagd, S.J. & Hein, M.A. 2012. 2013 (2017) representation 2017. Automotival factory of the Pacific, San Frontier, 2011. 17. Taples, G. K., Ulymand, J. & K. Callle, C. J. The authors: In the sound hiling-ance of Hardward E.S. Souries, T. H. M. (2017) (2017).	
	structural boundaries in the nuclear region. The compact molecular structure surrounding the nucleus is missingned with the large-scale molecular dals of the galaxy by 54°. The radius of the inner cavity of the teens would be about 50 m, as suggested by the location of the	<ol> <li>Brynn, T.M. &amp; Eurello, Y. Z. High resolution CO-thraneosisms of the abustance induced galaxy Mathematical XI, Assophys. J. 495, 191–1911 (1994).</li> <li>Drenne, D. &amp; Malamon, T.M. Estadog machaterings and nationare mathematic is abushamisma galaxin. Astrophys. J. 305, 102–1041 (1996).</li> <li>Collin, C.L., World, D.M. &amp; Fillerond, D.A. Andelfannense, doi: to Mathematical Zd. Astron. J. 105, 102–1051.</li> </ol>	
	nerthwotern interaction region. The conter edge of the torus spans a diameter of 200 pc, which is based on the 4-only contrours of the diffuse radio emission? and on far-infrared blackbody estimato?". "This backwood is the statement of the backwood of the ACI modifier	<ul> <li>SCI-OPC (2001)</li> <li>SCI Main, N. A. Moini, C. &amp; Orston, R. Norationally ratioal OPC in meganner galaxies. Action. Actiophys. Biol. 14: 2012 (2012)</li> <li>Bandel, L. Hold, D. Sano, K. S., Stan, J. A. &amp; Sray, M. D. The 'DF sensite OP meganner galaxies. International Action Science Sci</li></ul>	
	tion process. These parameters result in a tarus structure with a radius of 63 pc and a thickness of 70 pc, which accounts for an obscuration angle of 60° centred on the plane of the tarus and which	<ol> <li>Bann, N. B., Pisod P. A. D. &amp; Tanchila, S. D. Broudlipshoul aminimis in E. (2014). Arraphys. J. 200, IEEE/CVF (2014).</li> <li>Bankill, W. K. Komman, U. A. Obcoming metatiod around Section model with numbersts. Astrophys. J. 200, 1211–1214 (2014).</li> </ol>	
	is representative of values found for other gatacons <sup>11</sup> . The otherstation and the shape of the extended radio contours and the spennetry axis of the velocity field indicate that the terena is tilled upward from the line of sight by 56°, and is retarned articlockwise by 10°. This inferred	<ol> <li>Campandar, S. et al. String der antikel meide eine in Vol. arXiv arXiv arXiv property Physics and Differ.</li> <li>Inhesis, H. &amp; et al. String der antikel meide with a inherent scherel arXiv physics. J. Berl Ampping, J. Berl, M. A. El (2014).</li> <li>Yilliam, J., Korna, J. A. &amp; Kor, A.J. Permation of the value pix in DOC at 106 formareability and items.</li> </ol>	
	model for the torus is shown as a wire-structure representation in Fig. 5c. The nuclear indication course with opening angles of 67, assumed to be similar to those seen?' in M87, represent 67, directions with an involvement size of the nuclear. The radio	the semantificials look. Money <b>60</b> , 600 400 (1999). <b>BKDOWEDGENETE</b> We doub C. Cardili for providing a map of the diffuse seminous structure in MACC M. R.J. Bank O. Millow for advice on programming in OpenEL software. The function VM Threads is a single-toffer of function (chance Works Works) and other advices in the set of	
	coeffices follows a twisted path within these cones, starting at the molecus (at $PA = 60^{\circ}$ ) and ending up as a double source aligned with the symmetry axis of the outer disk (at $PA = 8^{\circ})^{A-1}$ . As a	antisomy instrums landed by their national research search, the Wararboth Synthesis Rade Talscope is sported by ISTRON (Performing Research or Antisomy) with support from the Nethenlands Feedballon for Scientific Research (SNPC).	
	result, this parameterization indicates that the machine accretion disk, which collimates the jet outflow inside the central cavity, is actually minuligned by 30° with the plane of the molecolar torus. In reality, the proposed torus structure does not need to have a	Emopologistmosts statement: The authors ducker that they have an computing financial inductor. Emogeneouslessor and requests for materials should be addressed to 15.4.X.	
	circular cross section. In addition, it will most probably be wrapped inside a cocoor-blar surface region with a higher temperature, and will have the same outer extent of 400 pi as the diffuse radio emission and the 101 shorehow out? However, the retrementation	press management que	
	of the inner part of this sources, dusty and molecular terms in Mrk231 is consistent with (all) current models of galactic molei and theoretical investigations', and supports the unification schemes for	Quantum oscillations in two	VOLUME 90, NUMBER 12 PHYSICAL REVIEW LETTERS 28 MARCH 2009
	Access which a different eventuation of the focus and the nacional accession disk represents a special case or is a general daracteristic of accise madels needs to be investigated with VLBI observations of other megamaser galaxies hosting a similar nuclear power	Coupled charge qubits	Tunable Coupling of Superconducting Qubits
	Beneroe.     Beneroel 32 My, accepted 17 December 2005, doi:10.1019/sciented1301.     Tenes/M 33 Weak hole models for anti-application models. Immed Rev. Annuel.	<ul> <li><sup>1</sup> The business of Hypital and Chemical Research (HKRN), Hole, Salama 331 (2006, Reprint 1970) The Internet al Property Laboratory Technics, Bandri 105,4701 Science</li> </ul>	Alexandre Bais, <sup>1,4</sup> Alexandre Massen van den Brink, <sup>2,4</sup> and Alexandre M. Zapokin <sup>2,5,4</sup> <sup>1</sup> Dipartenent de Pipsique and Centre de Recherche au les Propriétés Électroniques de Matérica Irancés, Université de Statevoola, Barbroola, Dubler, Casada IIX 287
	<ol> <li>Kinik, J. H. &amp; Rughma, M. C. Melender and in-feylor galaxies Facility to ensure multiliding in Amought, J. Md. Vol. 211 (1986).</li> <li>Kinik, J. &amp; Antor Galaxie Olade (Privation Review Amoughta), Neuron-Nels Pena, Peterson, Neurophys. Rev. 1999.</li> </ol>	S Coperander of Physics and Accounty, SUNT Stary Book, New York S Coperander of Physics and Accounty, SUNT Stary Book, New York 11794-3000, USA	<sup>1</sup> D:Wire Synows Lee, 335–485. Wei Roudway, Vaccurer, British Cohonki, Conada 109 477 <sup>3</sup> Physics and Autonomy Department, The Utivitienty of Pathic Cohonkis, Care 24 Agricultural Road, However, British Cohombis, Caredo 1871 121 (Revenuer, British Cohombis, Caredo 1871 121)
	<ol> <li>Gendell, S. J. et al. Detection of unitgance alument alid insite nucleus arXiV:22.0216 Amplys, J. MI, U.S. &amp; Group, J. P. Bans, S. J. &amp; O'Des, G. P. J. datase image of the obscuring data networking an article patient molecus. March 2014, 1971 (2014).</li> <li>Karrison, J. J. Bans, M. M. &amp; M. M. (2014).</li> <li>Karrison, M. M. M. M. M. (2014). (2014).</li> </ol>	coupled two-level quantum systems (qubits). Among the variety of qubits implemented', solid-state qubits are of particular interest because of their potential suitability for integrated	We study an LC circuit implemented using a current-based asseption junction (CBU) as a turable coupler for superconducting qubits. By modulating the bias current, the junction can be turned in and out of remanance on the intervelse with the path coupled in CD from on the implement framewhile meetings.
	<ol> <li>Hidaney, J. K. Pomili near many incoins galaxie multi. PML Annes. An Paril: 14, 481–481 (2015).</li> <li>Hermith, J. B. et al. Palatinesis: discriminan of the many in NGC 404 on upper limit on the large-mult suspects field https://www.fice.inteo.org/in.jpp. Amplicit. 2906, 305–307 (2008).</li> </ol>	have been implemented <sup>1,1</sup> ; these exploit the coherence of Cooper-pair transiting in the superconducting state <sup>1,10</sup> . Doughts apparent progress in the implementation of individual solid-	by mediating entanglement. We consider the examples of CBIJ and charge-phase qubits. A simple recoupling scheme leads to a gamenilization to arbitrary qubit designs.
	<ol> <li>Otsmend, P. J., Lanshie, G. L. J. andeli, C. S. J. McMill, N. E. Galdel VM intervalues of the comparability suppresses variation from 10 200 fill and Eldor Global Acadepity. J. 66, Ch. 106 (2008).</li> <li>Emandali, G. S. J., Lanshiel, C. S. J. Mannadi, P. J. &amp; Anthi, H. E. Jymmetric process and CM1 (magnetized effective). In No. 201 Academic J. 201 Media. (2014).</li> </ol>	state qubits, there have been no experimental reports of multiple qubit gates—a basic requirement for building a real quantum computer. Here we demonstrate a Josephana circuit consisting of two complet charge qubits. Using a pulse technique, we coher-	DOI: 10.1189/PhysRevLet00122701 19523 semiersi 10.912A, 73.2386, 74.50 or Sanifoant successes in manirolating the quantum states. However, for anharmonic oscillators, transitions
	<ol> <li>Hildman, Y. M., Ganog J. E., Bords, B. S., Diannal, P. J. &amp; Palarida, A. G. 1976 and MRUOS shown in a 2012 for 30. Assess. Assess for apply 1074 (2014) (2014).</li> <li>Bans, W. A., Bhoda graphical of 400 galaxies. Anaplity, J. 50, 504–513 (1996).</li> <li>Bans, W. A., Bhoda galaxy and the statistication of magnetizer galaxies. Amplyin, J. 500, 103 Bans, W. A. Borto, J. &amp; Golfman, D. Optical disalification of magnetizer galaxies. Amplyin, J. 500, 103 Bans, W. A. Borto, J. &amp; Golfman, D. Optical disalification of magnetizer galaxies. Amplyin, J. 500, 103 Bans, W. A. Borto, J. &amp; Golfman, D. Optical disalification of magnetizer galaxies. Amplyin, J. 500, 103 Bans, W. B., Martin, J. &amp; Golfman, D. Optical disalification of magnetizer galaxies. Amplyin, J. 500, 103 Bans, W. B., Martin, M. B. 1990, 2014 (2014), 2</li></ol>	ently raix quartum states and observe quantum oscillations, the spectrum of which reflects interaction between the qubits. Our results demonstrate the feasibility of coupling multiple solid- tates achieved and the feasibility of coupling multiple solid- tates achieved achi	state of superconducting qubits [1–4] once more make them prime candidates for a solid-state quantum com- puter [5]. Since the experiments yield single-quark older- puter [5]. Since the experiments yield single-quark older- for subtate yield mid-quark older- f
	<ol> <li>Chennell, S. &amp; et al. Solvalativitis radie jates and panes-solvalwappion in two Society galaxies. Anappior. J. 1994; 101–104 (2008).</li> <li>Chennell, S. S., Woold, S. M. &amp; Walk, B. &amp; Walk, and S. S. Woold, S. M. &amp; Walk, B. &amp; Walk, S. M. &amp; Walk, B. &amp; Walk, S. W. &amp; Walk, S. W. &amp; Walk, S. W. &amp; Walk, W.</li></ol>	atable. One of the physical realizations of a solid-state qubit is provided	ence times close to the accepted limits [6], one can focus coupled by a capacitance C, (Fig.1 et al. Ref. [17]b. One on other steps towards realizing the potential of quantum information processing [7] in those systems: The critical Controlled coupling of charge-phase qubits follows:
-	<ol> <li>M. Kidash, A. et al. Indianae and the Constraints of Cliphol Apple Residence and Schlims, R. S. SUKUREJ, VOL. 421 (20 FEREDARY 2010) www.sukers.com/autors</li> </ol>	r Primero delas Colas Plana Indian, Nero CERTO, Ress. Aliableg Group 823	next step is controlled coupling of, at least, two ophils. Several coupling mechanisms are possible, e.g., capaci- live coupling [8] for charge, charge-phase [1], and A fooephoon junction biased by a dc current has the
Э	Superconducting Quantum	Circuits. Oubits and Computing	current-based Josephson-parcines (CBJ) qubits [2,3]. well-known washboard potential [19] Close to the criti- Importantly. It is simple to implement and recently on- abled entangling two charge sphire [4]. Also, this type consider a large junction with the simulation of the simulati
	G. Wendin a Department of Microtecia	and V.S. Shumetko nology and Nanoveience - MCE	or coupling can be written to any only of any of the second se
	10 00 00 00 00 00 00 00 00 00 00 00 00 0	terning of Jeonology, iotheniung, Sunden whenaury 2, 2008)	tuning the qubits themselves may cause extra decoher- ence. Meterover, not all qubits are thus tunkley, or have off-diarent interactions. For you'd this reoblem, the con- ence off-diarent interactions. For you'd this reoblem, the con-
	50 conducting electrical circuits for quantum info	remation processing.	pling can be controlled using reforensing pulses—similar to liquid-state NMR, where the J coupling must be re- foreased P(). In this case, numerical quantum computing is $-\frac{\Phi_0}{2L_{col}} + \frac{\mu_0}{2P_0}$ . (1)
	C L Introduction	Variable Josephane coupling Variable Josephane coupling Variable phase coupling Variable canacitize coupling	still possible, but imperfect reflecusing introduces errors $2\pi^{-6/3}$ $\tilde{C}_{c}$ and the thrashold for fault islemance is not yet known. We prepose to capacitively couple sepreconducting qu-
	II. Nanotechnology, computers and qubits III. Basics of quantum computation (a) Conditions for quantum information processing (b) Online and extraordimentation	(h) Two qubits coupled via a resonator X. Dynamics of multi-qubit systems (a) General N-qubit formulation	bits to a CBD, implementing an A.C. creat and acting as a <u>Qubit</u> . Tunble coupling bus, numble to a CBD, implementing and acting as a <u>Qubit</u> . Tunble coupling bus, include net desired that the cavity and atoms, respec-
	(c) Operations and gates (c) Operations and gates (d) Resident and state preparation IV. Dynamics of two-level systems	(b) 1 we optical, imaging the framework is coupling Blasing far away from the degeneracy point Blasing at the degeneracy point (c) Two qubits, transverse xc coupling	to find with (inductively) [12] and other superconducting devices [13-13]. In another scheme to entangle qubits through an $C_{cl}$ (rad) [13] the large scheme to entangle qubits
	(a) The two-level state (b) State evolution on the Bloch sphere (c) do pulses, sublice avoidening and precession (c) do pulses in avoid the second state of the sec	<ul> <li>(d) Two qubits, sy coupling</li> <li>(e) Effects of the environment: noise and decoherence</li> <li>XI. Experiments with single qubits and readout</li> </ul>	the an effective qubit qubit interaction. The CBJ7's kitetic inductance (depends on the bias and modifies the circuit's overall inductance (16). It thas acts
	(e) flarmonic perturbation and Rabi oscillation (g) Decoherence of qubit systems V. Classical superconducting circuits	(a) Readout detectors (b) Operation and measurement procedures (c) NIST current-biased Josenhoon inaction qubit	as a tranship arkarwasoic LC circuit, which guarantees a nonuniform level spacing, reducing leakage to higher FIG. 1. A pair of capacitively coupled CR03 qubits.
	(a) Current biased Josephson junction (b) rf-SQUID (c) do SQUID (c) do SQUID	(d) Flux qubits (e) Charge-phase qubit XII. Experiments with qubits coupled to quan	123901-1 0031-9007/03/90(12)/127908(4)520.00 0 2003 The American Physical Society 127908-1
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	VIII. Qubit read-out and measurement of quan tum information (a) Readout: why, when and how? (b) Direct and it resonancest	<ul> <li>(c) Capacitively coupled JJ phase qubits XIV. Quantum state engineering with multi qubit JJ systems</li> <li>(c) Ball systems</li> </ul>	- Germany and Landau Institute for Theoretical Physics, Kosygin et. 2, 117840 Moscow, Russia Gave Schön
	(c) Measurement of charge qubit with SET (d) Measurement via coupled oscillator (e) Threshold detection	(b) Teleportation (c) Qubit bases and estanglement transfer (d) Qubit encoding and quantum error correction	hultur Kir Theoretische Fissklicpophysik, Universität Kahlsche, D-78128 Kahlsche, Gennany and Foschanssenteum Kelsche Jostitut Kir Neceschnalasie, D-70021 Kahlsche
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	<ul> <li>(d) JJ coupling of charge qubits</li> <li>(f) Coupling via oscillators</li> <li>Coupling of charge qubits</li> </ul>		Institut AX: Theoretiston Functiongraphysis, Universität Kanforuhe, D-7872 Kanforuhe, Germany (Published 8 May 2001)
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