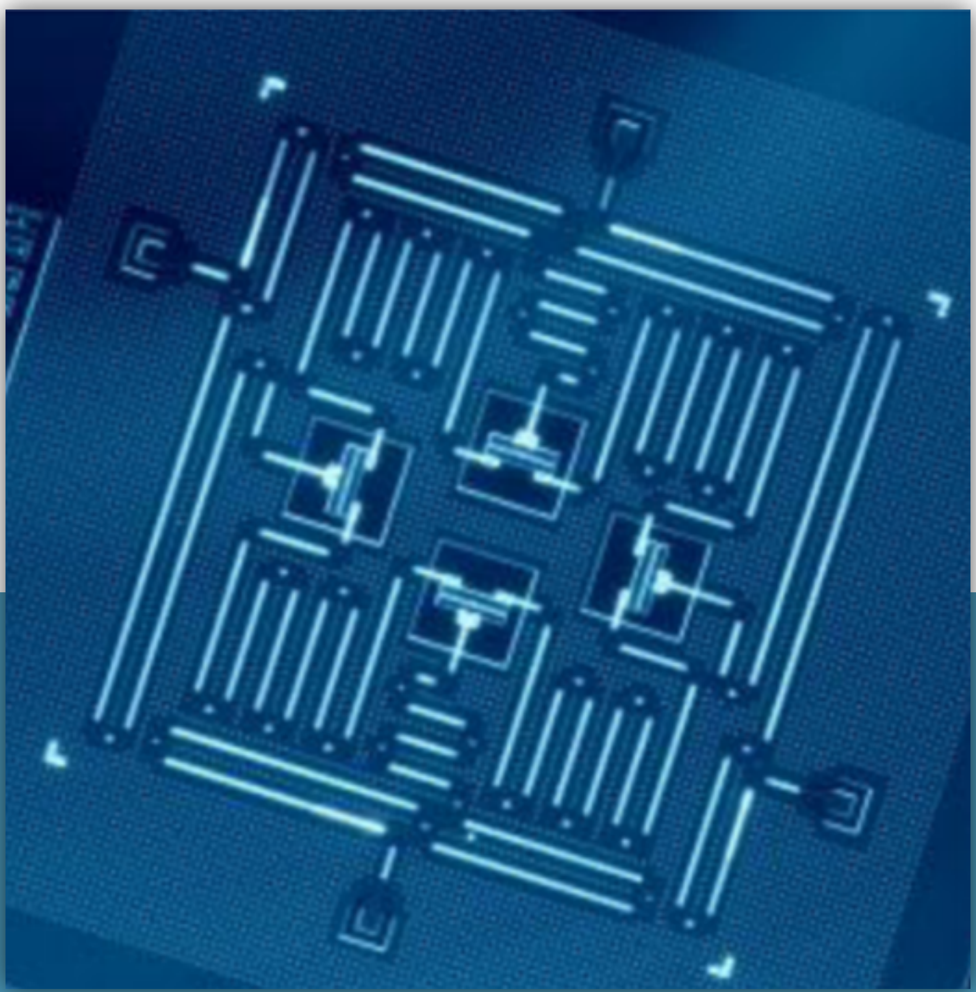
MICHAEL RONEN

MULTI-QUBIT DEVICES

CAPACITIVE AND INDUCTIVE COUPLING



4 Transmon Qubit/4 Bus/4 Readout Chip by IBM

(Fig. 4c, Gambetta et al., Building logical qubits in a superconducting quantum computing system. npj Quantum Inf 3, 2 (2017). https://doi.org/10.1038/s41534-016-0004-0)

Seminar: Superconducting Quantum Hardware for Quantum Computing

CONTENTS

MOTIVATION

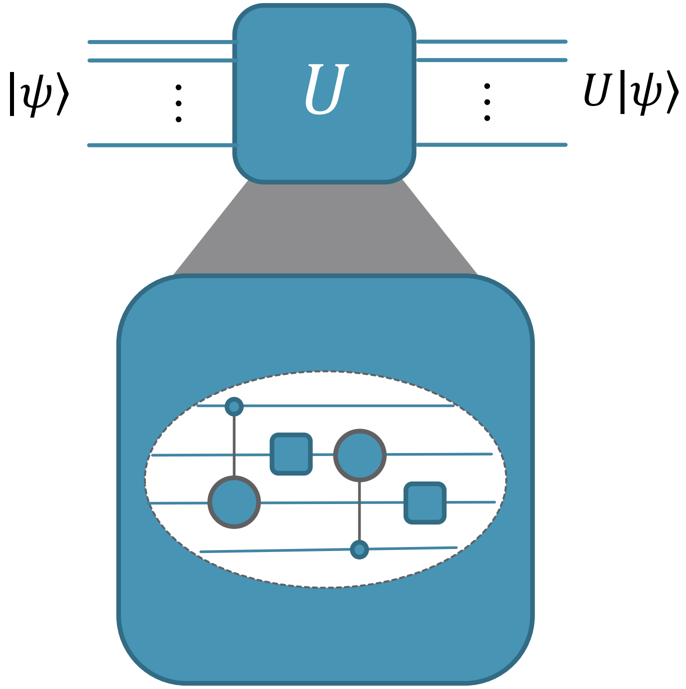
- Qubit Computation:

  Time evolution of state $|\psi(t)\rangle = e^{-iHt/\hbar}|\psi_0\rangle = U|\psi_0\rangle \rightarrow$ unitary transformation

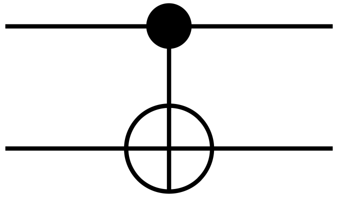
- Quantum Circuit Model:

  $\rightarrow U$ can be constructed in approximation from a finite set of

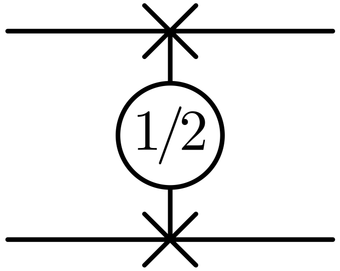
  - 1 qubit operations $U \in SU(2)$ and
  - 2 qubit operations $U \in SU(4)$



- Examples of two qubit gates, necessary for universality:



$CNOT$



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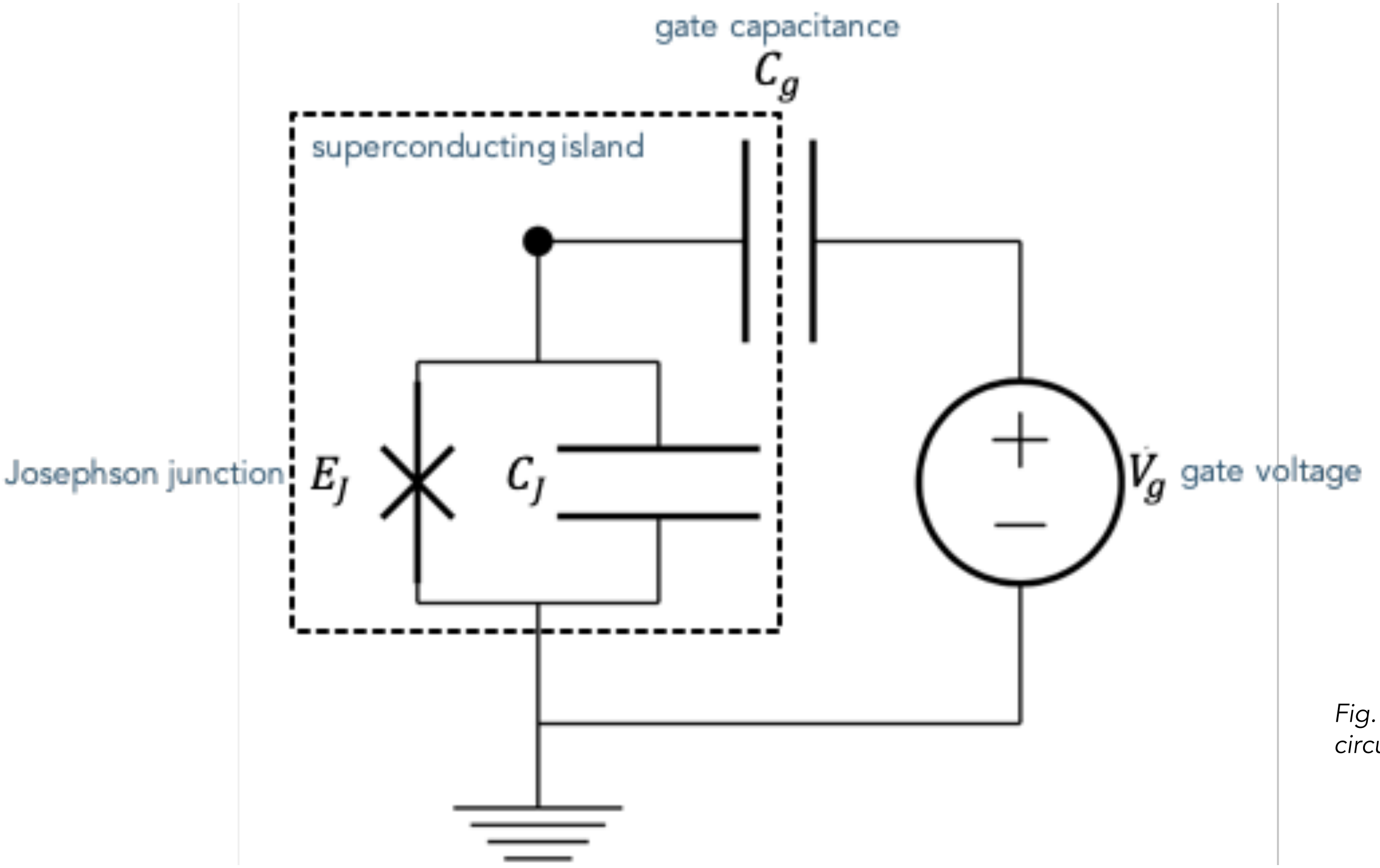
SUPERCONDUCTING QUBIT ARCHETYPES

- The Charge Qubit (Cooper-Pair Box, CPB)



Superconducting Charge Qubit – Cooper-pair box
(Fig. 17.4a, Kockum A.F., Nori F. (2019) Quantum Bits with Josephson Junctions. Springer)

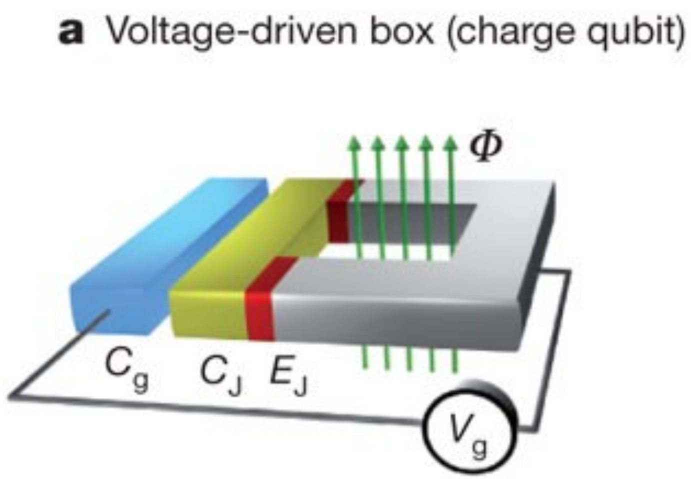


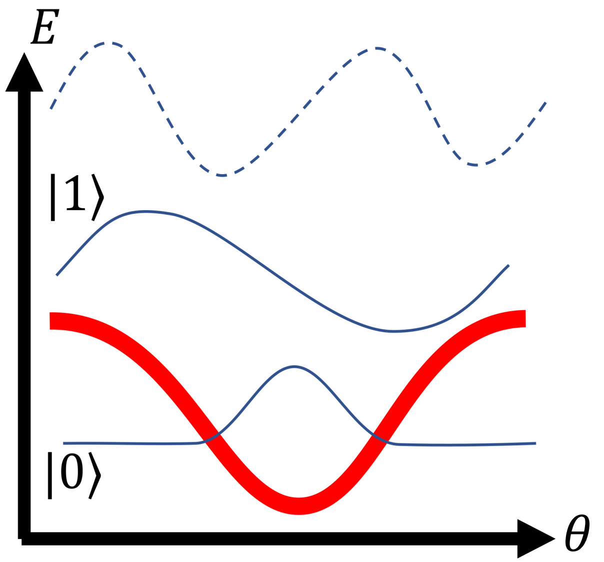
Fig. 1, You, J., Nori, F. Atomic physics and quantum optics using superconducting circuits.Nature 474, 589–597 (2011). https://doi.org/10.1038/nature10122

- Hamiltonian: $$H_{CPB} = \underbrace{E_C\left(N - N_g\right)^2}_{\text{charge energy}} \underbrace{- E_J \cos(\theta)}_{\text{Josephson energy}}$$

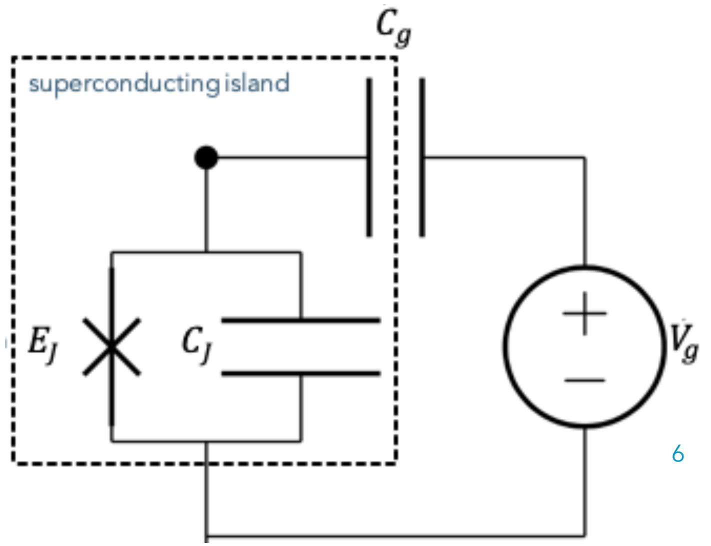
charge energy: $E_C = \frac{(2\,e)^2}{2(C_J+C_g)}$; background charge: $\boldsymbol{N_g} = C_g \frac{\boldsymbol{U}}{2e}$

- Externally adjustable parameters:
  1. Background charge: $\boldsymbol{N_g} = C_g \frac{\boldsymbol{U}}{2e}$ through gate voltage $\boldsymbol{U}$
  2. Josephson energy: $\boldsymbol{E_J}(\boldsymbol{\Phi_{ext}}) = E_J \cos(\frac{\boldsymbol{\Phi_{ext}}}{\boldsymbol{\Phi_0}}\pi)$
- States: number of excess Cooper pairs on island

$$|0\rangle = |N\rangle$$
$$|1\rangle = |N+1\rangle$$

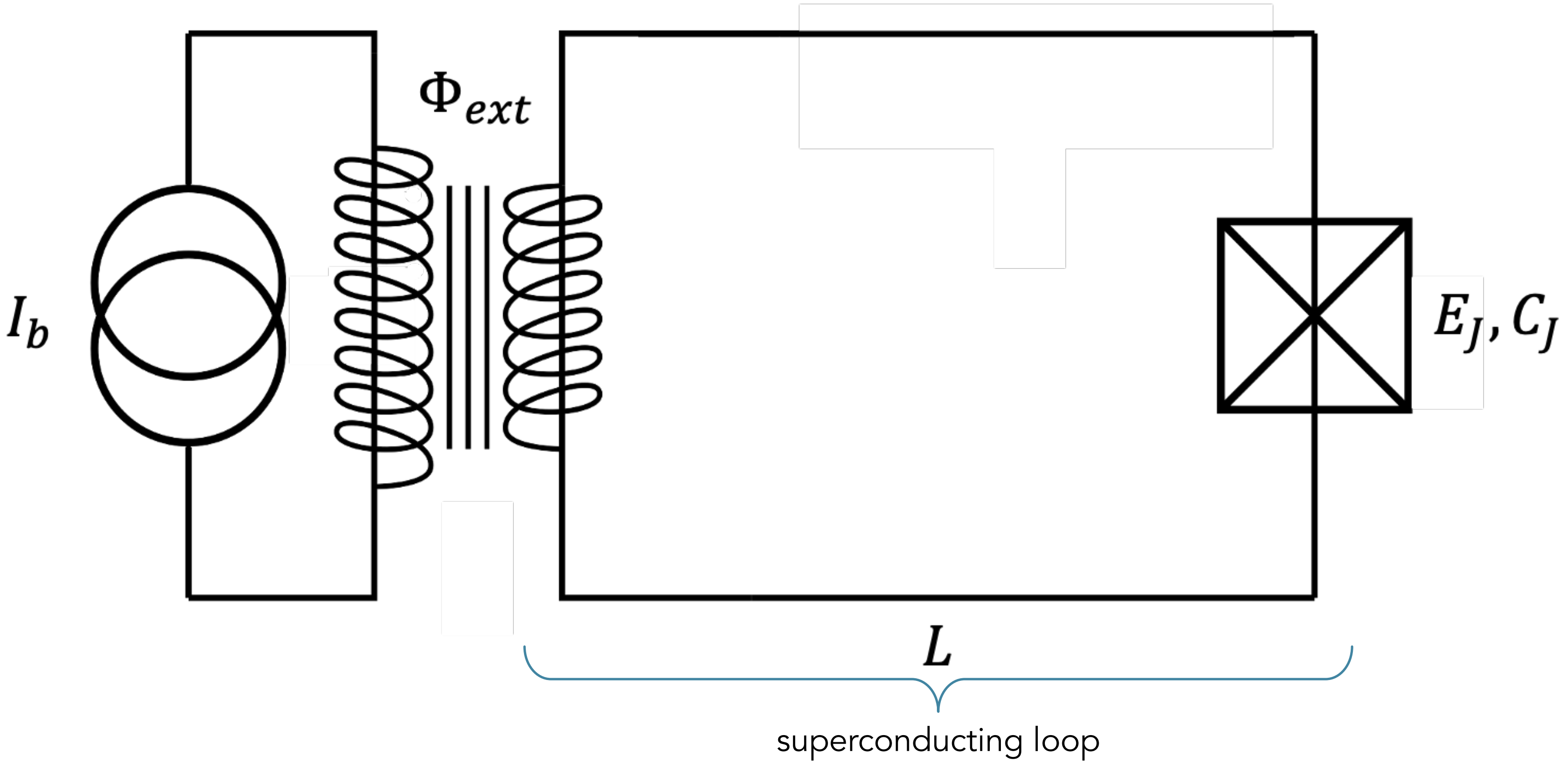


CPB Potential (red) and two lowest states (blue solid)
(Srjmas, „Charge qubit potential", https://commons.wikimedia.org/wiki/File:Charge_qubit_potential.svg)



SUPERCONDUCTING QUBIT ARCHETYPES

- The Flux Qubit (*RF SQUID*)



Superconducting Flux Qubit – RF-SQUID
(Srjmas, „Flux qubit circuit“, https://commons.wikimedia.org/wiki/File:Flux_qubit_circuit.svg)

- Hamiltonian:

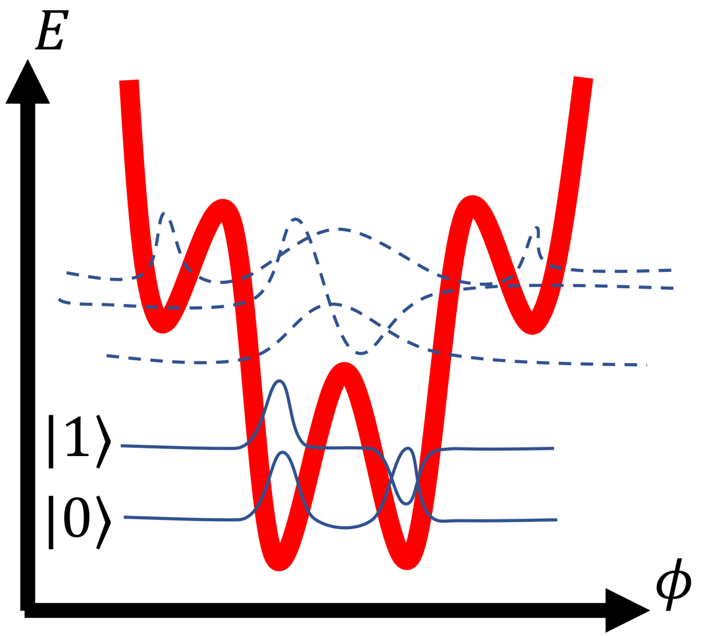
$$H_{Flux} = \frac{\overset{\text{Charge on } C_J}{q^2}}{2C_J} + \underbrace{\left(\frac{\Phi_0}{2\pi}\right)^2 \frac{\overset{\text{Phase difference across J. j.}}{\phi^2}}{2L} - E_J cos\left(\phi - \Phi_{ext}\frac{2\pi}{\Phi_0}\right)}_{\text{Potential}}$$

energy scales: $E_J, \quad E_{C_J} = \frac{(2\,e)^2}{2C_J}, \quad E_L = \frac{\Phi_0^2}{2L}$

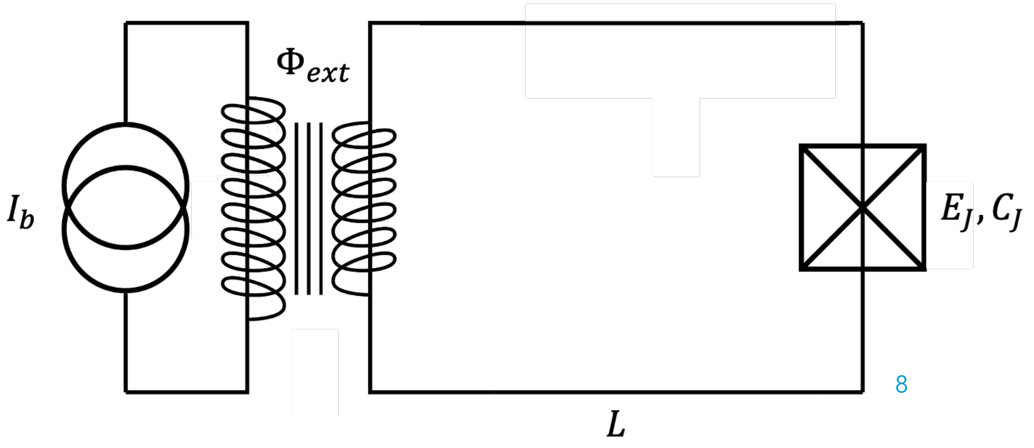
- Externally adjustable parameters:
  1. External bias flux: $\boldsymbol{\Phi_{ext}}$ through bias current $\boldsymbol{I_b}$
  2. Loop inductance: $\boldsymbol{L}$ through coil ℓ and $\boldsymbol{N}$
  3. Josephson energy: $\boldsymbol{E_J(\Phi_J)} = E_J \cos(\frac{\boldsymbol{\Phi_J}}{\boldsymbol{\Phi_0}}\pi)$
- States: symmetrical and antisymmmetrical superposition of flux quanta

$$|0\rangle = \frac{1}{\sqrt{2}}(|↺\rangle + |↻\rangle)$$

$$|1\rangle = \frac{1}{\sqrt{2}}(|↺\rangle - |↻\rangle)$$

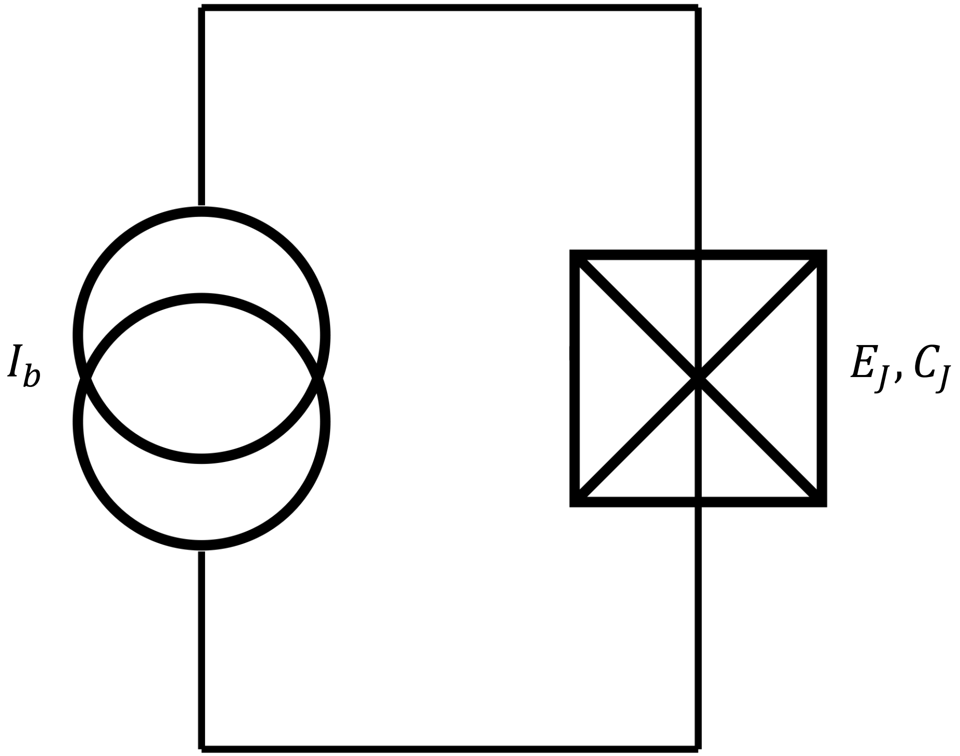


Flux Qubit Potential (red) and two lowest states (blue solid)
(Srjmas, „Flux qubit potential", https://commons.wikimedia.org/wiki/File:Flux_qubit_potential.svg)



SUPERCONDUCTING QUBIT ARCHETYPES

- The Phase Qubit (Current Biased Josephson Junction, *CBJJ*)



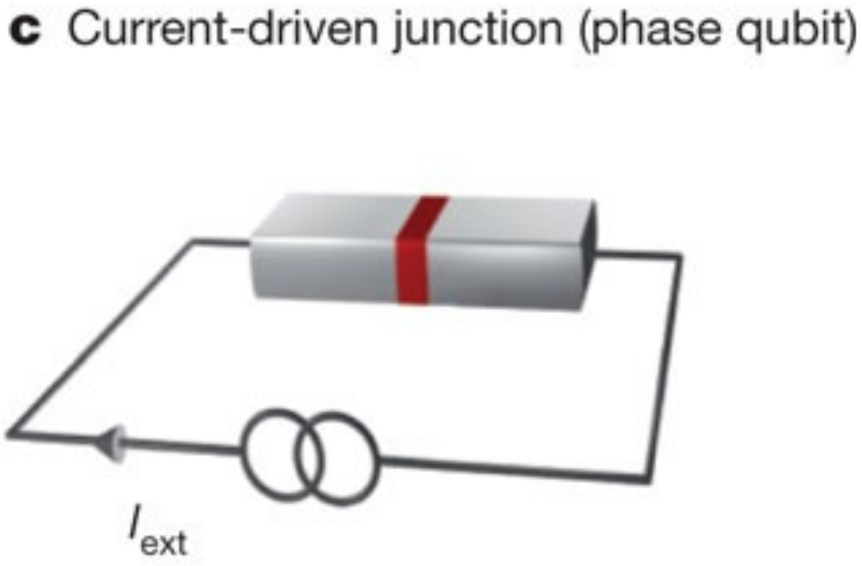
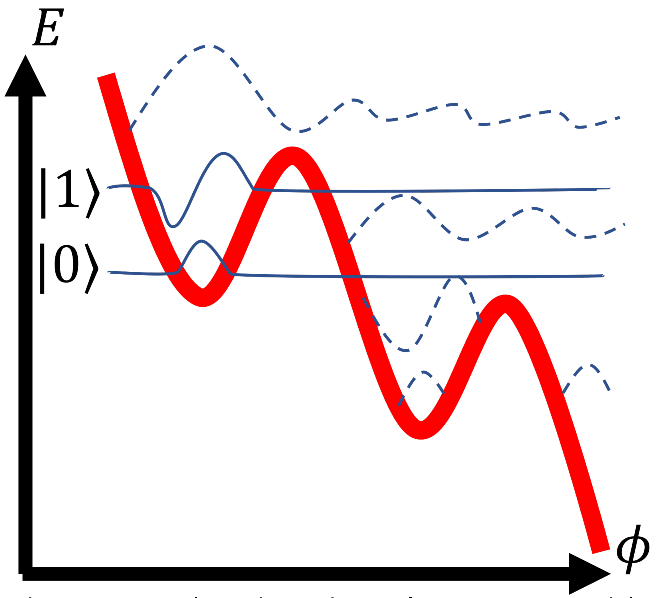
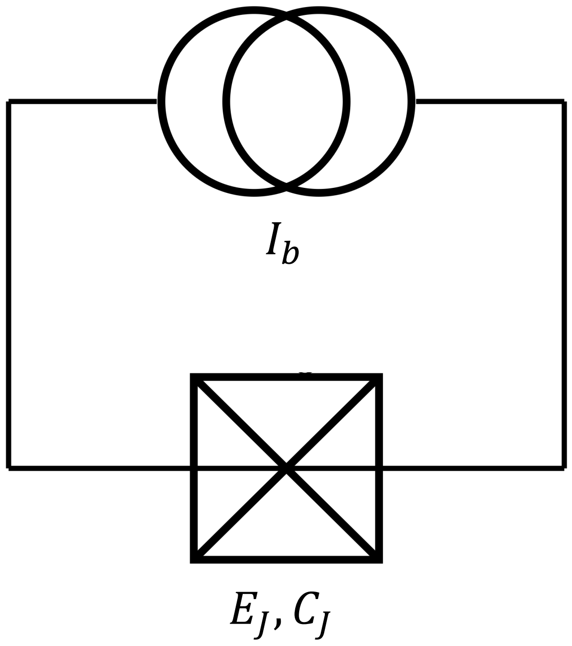


Fig. 1, You, J., Nori, F. Atomic physics and quantum optics using superconducting circuits.Nature 474, 589–597 (2011). https://doi.org/10.1038/nature10122

Superconducting Charge Qubit – Cooper-pair box
(Fig. 17.4b, Kockum A.F., Nori F. (2019) Quantum Bits with Josephson Junctions. Springer)



Phase Qubit Potential (red) and two lowest states (blue solid)
(Srjmas, „Phase qubit potential",
https://commons.wikimedia.org/wiki/File:Phase_qubit_potential.svg)



Charge on C_J Phase difference across J. j.

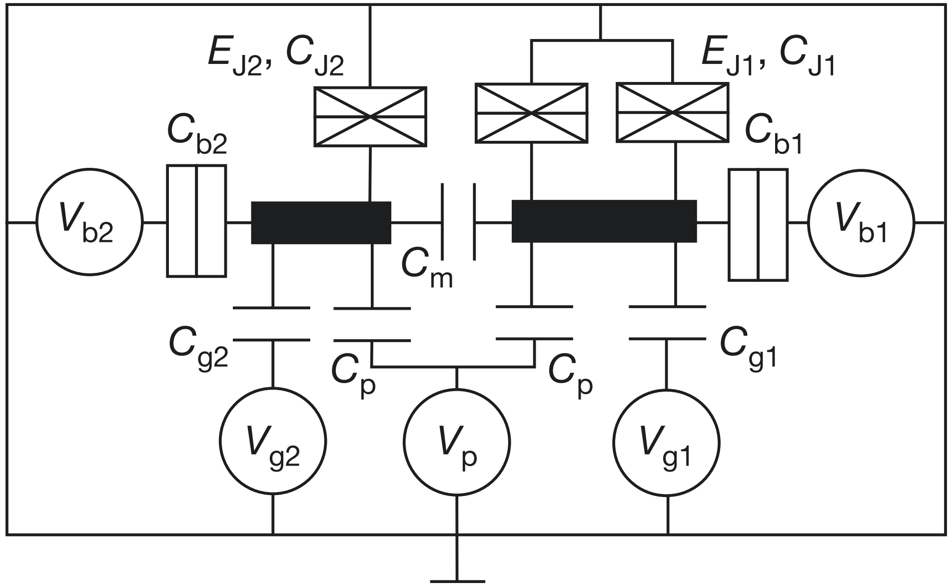
- Hamiltonian:

$$H_{Phase} = \frac{(2e)^2}{2C_J}q^2 - I_b\frac{\Phi_0}{2\pi}\phi - E_J cos\phi$$

- Externally adjustable parameters:
  1. Bias current: $\boldsymbol{I_b}$
  2. Josephson energy: $\boldsymbol{E_J(\Phi_J)} = E_J\cos(\frac{\boldsymbol{\Phi_J}}{\boldsymbol{\Phi_0}}\pi)$
- States: Oscillations modes in superconducting loop

CAPACITIVE COUPLING

- **Two capacitively coupled charge qubits:**
  - Qubit state given by charge on superconducting island
  - Qubit charges on capacitor C_m $\rightarrow$ coupling of charge states



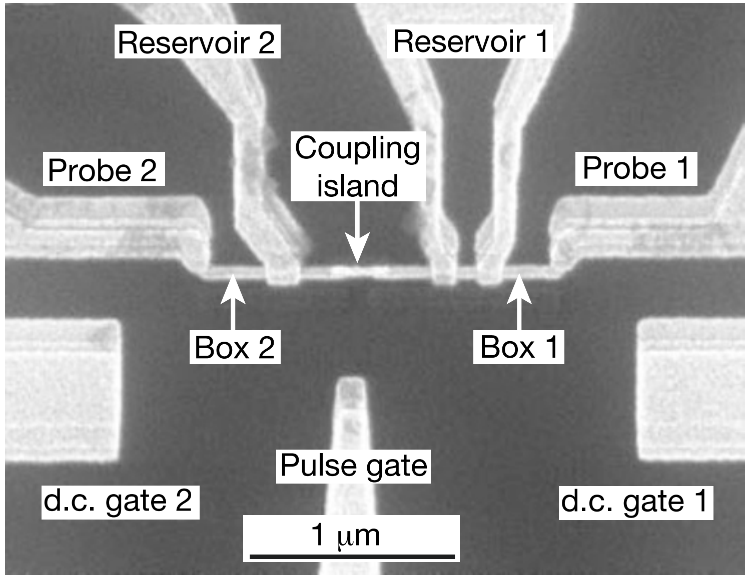


Figure 1: Experiment setup – two coupled charge qubits

Pashkin, Y., Yamamoto, T., Astafiev, O. et al. Quantum oscillations in two coupled charge qubits. Nature 421, 823–826 (2003). https://doi.org/10.1038/nature01365

- Hamiltonian:

normalised charges on qubit (induced by d.c. and pulse gate electrodes)

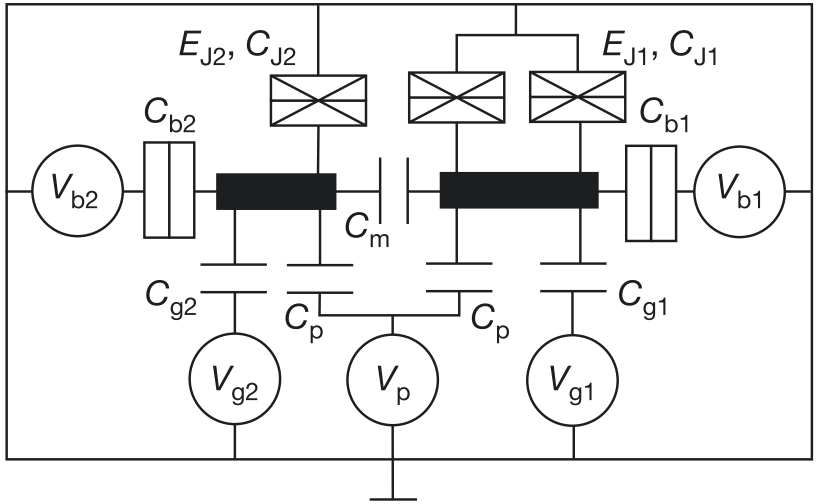
$$n_{g_{1,2}} = \frac{1}{2e}(C_{g_{1,2}} V_{g_{1,2}} + C_P V_P)$$

total electrostatic energy: $E_{n_1 n_2} = E_{C_1}(n_{g_1} - n_1)^2 + E_{C_2}(n_{g_2} - n_2)^2 + E_m(n_{g1} - n_1)(n_{g_2} - n_2)$

effective C. p. charging energies $E_{C_{1,2}} = 4e^2 \frac{C_{\Sigma_{2,1}}}{C_{\Sigma_1} C_{\Sigma_2} - C_m^2}$

coupling energy $E_m = 4e^2 \frac{C_m}{C_{\Sigma_1} C_{\Sigma_2} - C_m^2}$

$$H = \begin{bmatrix} \boxed{E_{00}} & -\frac{1}{2}E_{J1} & -\frac{1}{2}E_{J2} & 0 \\ -\frac{1}{2}E_{J1} & \boxed{E_{10}} & 0 & -\frac{1}{2}E_{J2} \\ -\frac{1}{2}E_{J2} & 0 & \boxed{E_{01}} & -\frac{1}{2}E_{J1} \\ 0 & -\frac{1}{2}E_{J2} & -\frac{1}{2}E_{J1} & \boxed{E_{11}} \end{bmatrix}$$



→ gate voltages allow control over diagonal terms

Josephson coupling $E_{J_{1,2}} \approx E_m < E_{C_{1,2}}$ *→ coherent superpositions of* $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ at $n_{g1,2} = 0.5$

Pashkin, Y., Yamamoto, T., Astafiev, O. et al. Quantum oscillations in two coupled charge qubits. Nature 421, 823–826 (2003). https://doi.org/10.1038/nature01365

- In absence of Josephson coupling:
  - Hexagonal boundaries between states
  - R and L – degeneracy between neighbouring states
    - System will oscillate between neighbouring states
  - $\boldsymbol{n_{g_1}, n_{g_2}}$ inside cell → system will remain in cells' state
  - Pulse gate shifts system along 45° line (black arrows)
- With small Josephson coupling:
  - ➢ States become **superposed on boundaries**!

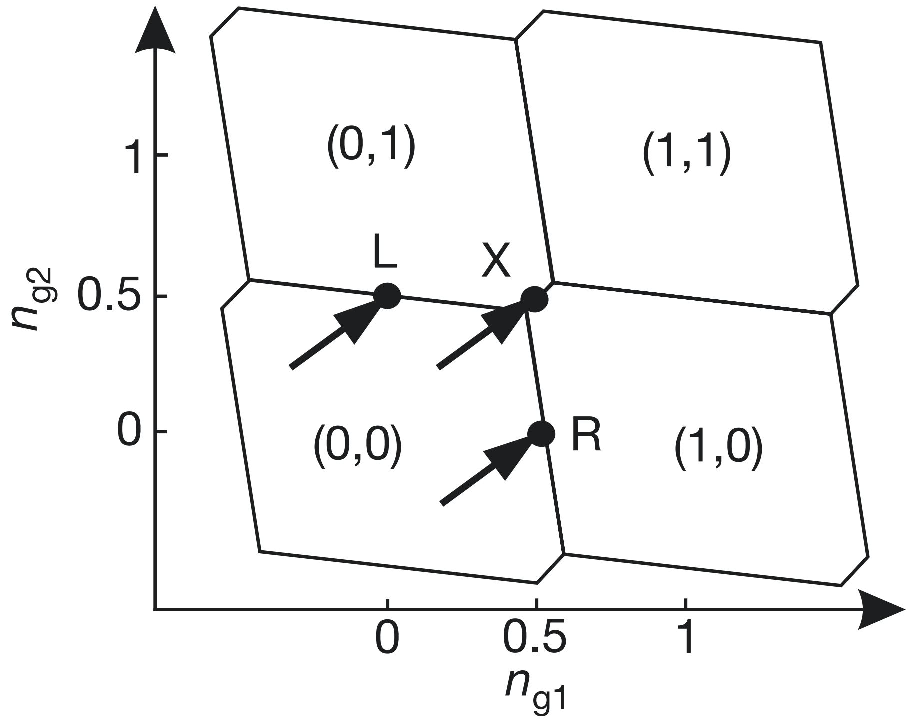


Figure 2a: Ground-state charging diagram of coupled qubits

- Co-resonance point X with $\boldsymbol{n_{g_1} = n_{g_2} = 0.5}$
  - ➢ Double degeneracy
  - ➢ Superposition of all charge states $|\psi(t)\rangle = c_1\,|00\rangle + c_2\,|10\rangle + c_3\,|01\rangle + c_4\,|11\rangle\}$

Pashkin, Y., Yamamoto, T., Astafiev, O. et al. Quantum oscillations in two coupled charge qubits. Nature 421, 823–826 (2003). https://doi.org/10.1038/nature01365

- Idea of the experiment:
  1. Prepare system in $|00\rangle$
  2. Start applying pulse → system at co-resonance point X in $|\psi(t)\rangle = c_1|00\rangle + c_2|10\rangle + c_3|01\rangle + c_4|11\rangle\}$
  3. Stop applying pulse → system 'frozen' in superposition
  4. System decays to $|00\rangle$ emitting quasi particles
- Readout scheme:
  - **Measuring probe currents I_1, I_2** in proportion to probability of each qubit having a C. p. on it:

$$I_1 \propto p_1(1) \equiv |c_2|^2 + |c_4|^2$$
$$I_2 \propto p_2(1) \equiv |c_3|^2 + |c_4|^2$$

  - Time evolution of probabilities:

$$p_{1,2}(1) = \frac{1}{4}(2 - (1-\chi_{1,2})cos((\Omega+\epsilon)\Delta t) - (1+\chi_{1,2})cos((\Omega-\epsilon)\Delta t))$$

$$\chi_{1,2} = \frac{E_{J_{2,1}}^2 - E_{J_{1,2}}^2 + E_m^2/4}{4\hbar^2\Omega\epsilon}$$

$$\Omega = \frac{1}{2\hbar}\sqrt{(E_{J_1}+E_{J_2})^2 + (E_m/2)^2}$$
$$\epsilon = \frac{1}{2\hbar}\sqrt{(E_{J_1}-E_{J_2})^2 + (E_m/2)^2}$$

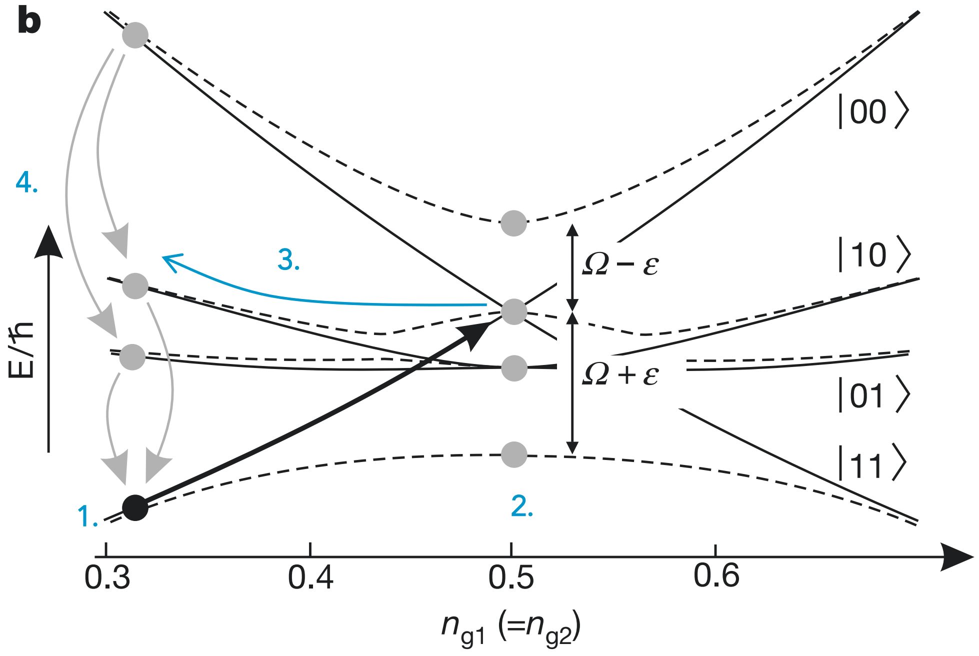
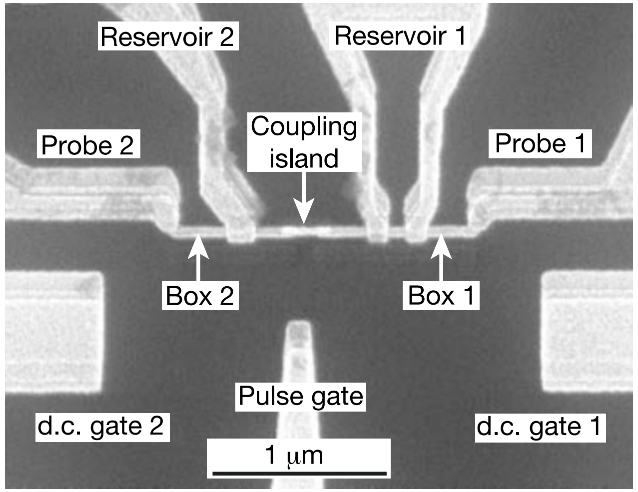


Figure 2b: energy diagram along $n_{g_1} = n_{g_2}$-line



Pashkin, Y., Yamamoto, T., Astafiev, O. et al. Quantum oscillations in two coupled charge qubits. Nature 421, 823–826 (2003). https://doi.org/10.1038/nature01365

(a) Probe current oscillations at R and L

(b) Probe current oscillations at co-resonance X

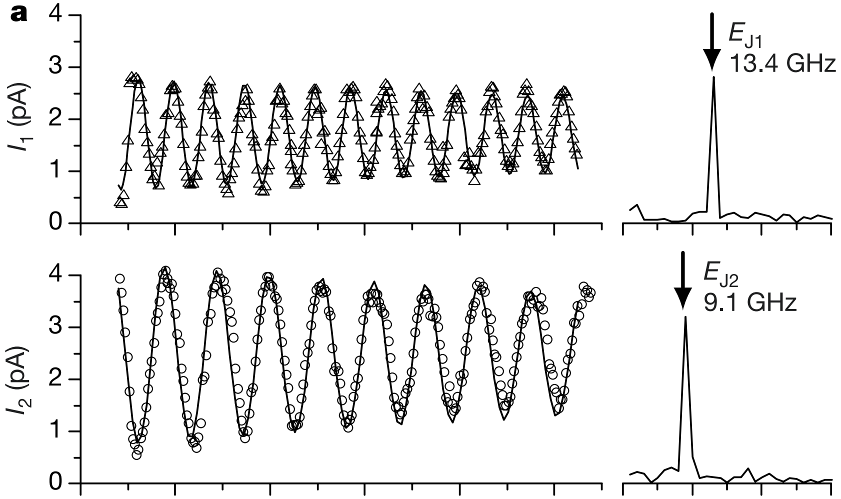


Figure 3a: Oscillations at resonance points

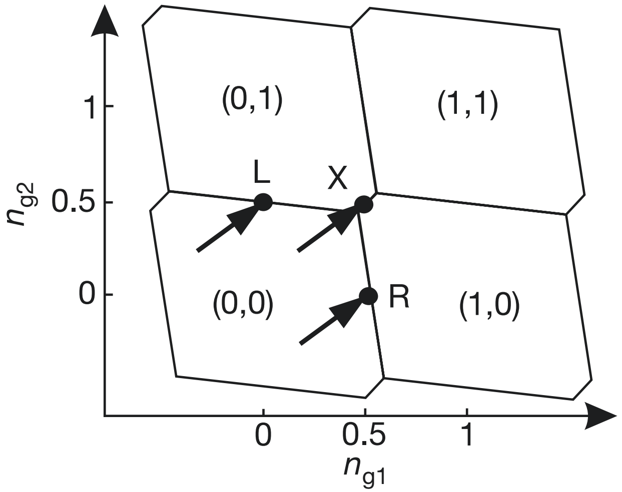


Figure 2a: Ground-state charging diagram of coupled qubits

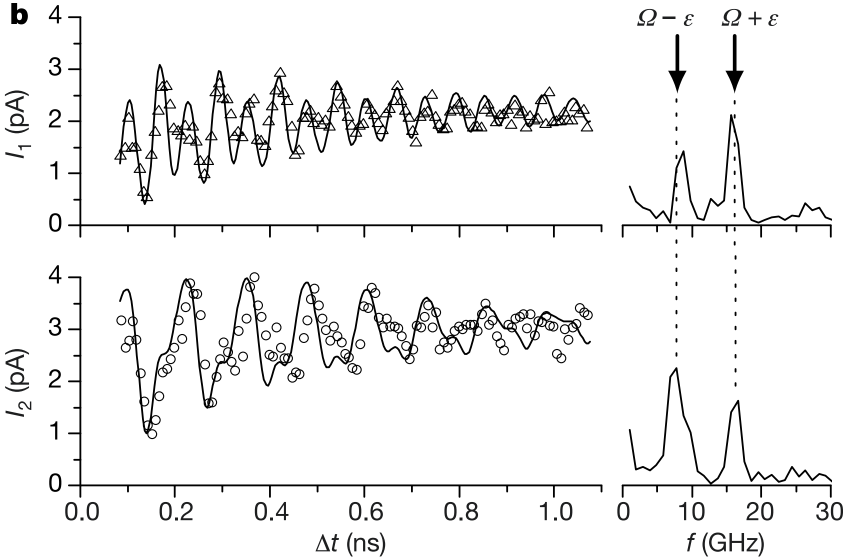


Figure 3b: Oscillations at co-resonance point

- Can be fitted with cosine
- Single peaks at different energies

  → no qubit interaction

- Two peaks in spectrum
- Two peaks at same energies

  → evidence for qubit interaction

Pashkin, Y., Yamamoto, T., Astafiev, O. et al. Quantum oscillations in two coupled charge qubits. Nature 421, 823–826 (2003). https://doi.org/10.1038/nature01365

TUNABLE COUPLING

- **Two capacitively coupled phase qubits:**
  - Hamiltonian:

$$H = \overbrace{\frac{q_q^2}{2\tilde{C}_{jq}} - E_{jq}cos\phi_q - \frac{\Phi_0}{2\pi}I_q\phi_q}^{\text{CBJJ Qubit Hamiltonian}} + \overbrace{\frac{q_b^2}{2\tilde{C}_b} - E_{jb}cos\phi_b - \frac{\Phi_0}{2\pi}I_b\phi_b}^{\text{Tuneable Coupling Bus Hamiltonian}} + \overbrace{\frac{q_q q_b}{\tilde{C}_C}}^{\text{Coupling Term}}$$

effective capacitances: $\tilde{C}_{b(jq)} = C_{jq(b)} + 1/(1/C_{b(jq)} + 1/C_C)$ $\quad\tilde{C}_C = C_{jq}C_{jb}(1/C_{jb} + C_{jq} + 1/C_C)$

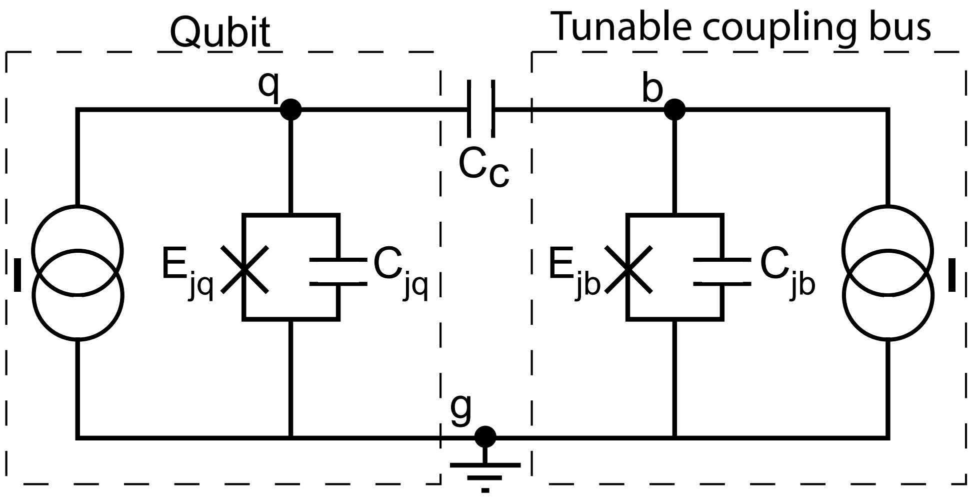


Figure 1: A pair of capacitively coupled CBJJs

- Now: $C_{jq} = C_b \equiv C_j \quad E_{jq} = E_b \equiv E_j$

  $\rightarrow$ approximately cubic potential wells

  $\rightarrow$ junctions treated as anharmonic oscillators

$$m = \tilde{C}_j(\Phi_0/2\pi)^2 \qquad \omega_{qi} = \sqrt{\frac{2\pi I_c}{\tilde{C}_j\Phi_0}\sqrt{1-(\frac{I_b}{I_c})^2}}$$

$$\rightarrow q_i = i\frac{2\pi}{\Phi_0}\sqrt{m\frac{\hbar\omega_q}{2}}(a_i^\dagger - a_i)$$

Blais & v. e. Brink & Zagoskin (2003). Tunable Coupling of Superconducting Qubits. Physical review letters. 90. 127901. 10.1103/PhysRevLett.90.127901.

- Hamiltonian H_2 in $span\{|0_q 1_b\rangle, |1_q 0_b\rangle\}$:

$$H_2 = \begin{pmatrix} E_{q0} + E_{b1} & \gamma/2 \\ \gamma/2 & E_{q1} + E_{b0} \end{pmatrix}$$

coupling coefficient: $\gamma \equiv \hbar\sqrt{\omega_{pq}\omega_{pb}}\frac{\tilde{C}_j}{\tilde{C}_C}$

  - Without coupling: $|0_q 1_b\rangle$ and $|1_q 0_b\rangle$ degenerate for I_b such that $E_{q1} - E_{q0} = E_{b1} - E_{b0}$
  - With coupling: lifts degeneracy and the new eingenstates are

$$|\Psi_\pm\rangle = \frac{1}{\sqrt{2}}(|0_q 1_b\rangle \pm |1_q 0_b\rangle)$$

  - In resonance: H_2 acts like $e^{-i\frac{\sigma_x \gamma \tau}{2\hbar}}$ → prepared in $|1_q 0_b\rangle$: probability for $|1_q\rangle$ oscillates with $T_{Rabi} = h/\gamma$
  - Anharmonicity of qubits supressses leakage out of two level system
  - Coupling term $\frac{q_q q_b}{\tilde{C}_C}$ causes nonresonant leakage to $|2_{q(b)}\rangle$ (close to barrier → large transition rate) → shortens coherence time
  - Bus not tuned to qubit frequency Ω_q → qubit decoupled from bus

Blais & v. e. Brink & Zagoskin (2003). Tunable Coupling of Superconducting Qubits. Physical review letters. 90. 127901. 10.1103/PhysRevLett.90.127901.

- Optimise coupling quality

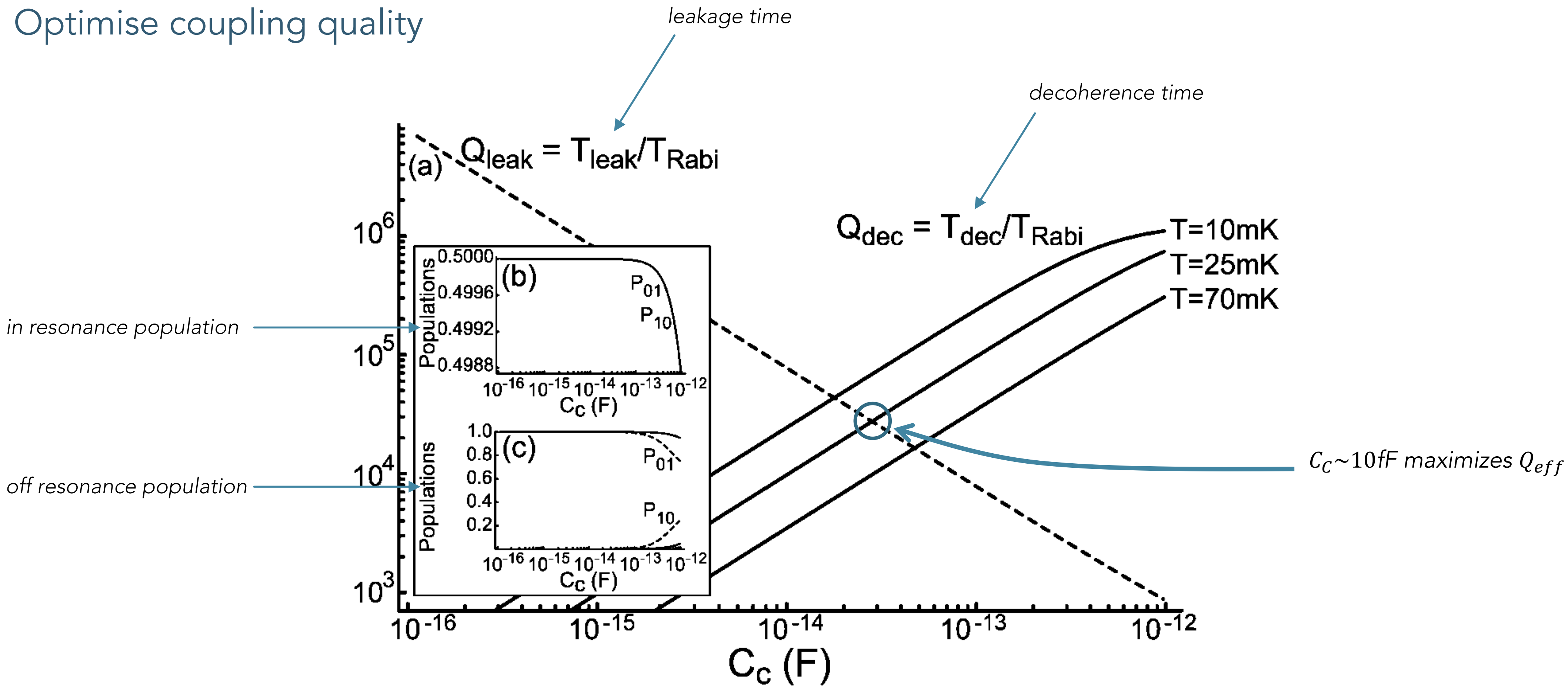


Figure 2: Quality of coupled identical CBJJ qubits

Blais & v. e. Brink & Zagoskin (2003). Tunable Coupling of Superconducting Qubits. Physical review letters. 90. 127901. 10.1103/PhysRevLett.90.127901.

TUNABLE COUPLING

- Pair of charge-phase qubits coupled through CBJJ
  - For qubits: consider only two levels
  - $\Omega_1 \neq \Omega_2$ → bus can be coupled to only one qubit (by tuning bus to Ω_i)

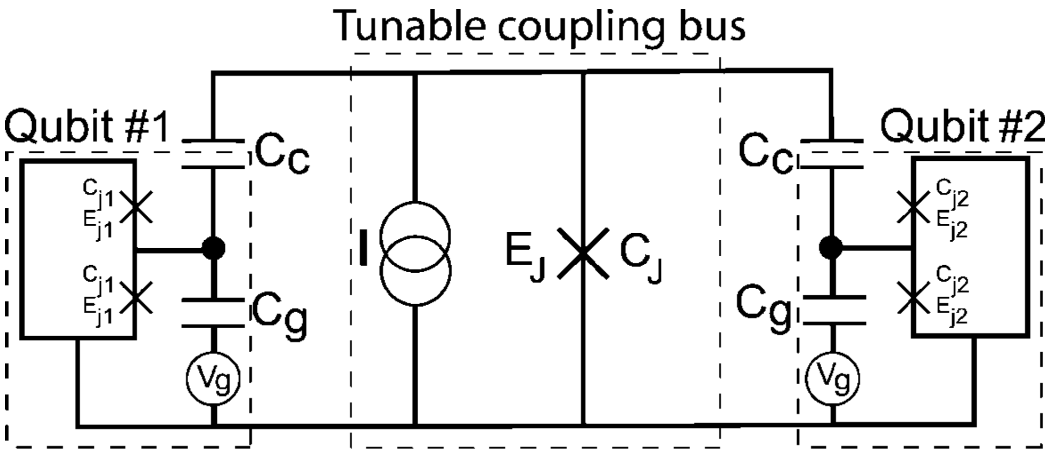


Figure 3: A pair of charge-phase qubits capacitively coupled to a CBJJ

- Hamiltonian:
  - Couples bus charge to island charge
  - Takes form $\sigma_{ix} q_b / \tilde{C}_C$
  - Qubit-bus coupling coefficient $\gamma' \equiv \beta 2e(\frac{2\pi}{\Phi_0})^2 \frac{\sqrt{2m\hbar\omega_{pb}}}{\tilde{C}_C}$

$$\omega_{qi} = \sqrt{\frac{2\pi I_c}{\tilde{C}_j \Phi_0}} \sqrt{1-(\frac{I_b}{I_c})^2}$$

depends on ratio between E_C and E_J

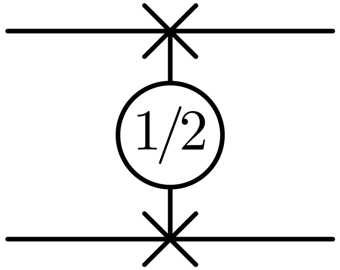
effective coupling capacitances depend on $C_{\Sigma,i} = C_{gi} + 2C_{ji}$

Blais & v. e. Brink & Zagoskin (2003). Tunable Coupling of Superconducting Qubits. Physical review letters. 90. 127901. 10.1103/PhysRevLett.90.127901.

- Two qubit operation:

  Assume qubits are in arbitrary state, bus in ground state.

  1. Tune bus to Ω_1 for t_1 such that $\frac{\gamma' t_1}{2\hbar} = \frac{\pi}{2}$
  2. Tune bus to Ω_2 for t_2 such that $\frac{\gamma' t_2}{2\hbar} = \frac{\pi}{4}$
  3. Tune bus to Ω_1 again for t_1
  4. Disentangle bus from the qubits



$\sqrt{SWAP}$

- Problems:
  - Phase factors accumulating → can be calculated numerically from known parameters
  - Leakage to higher bus states
    - Charge-phase qubits at least as anharmonic as CBJJ → leakage not larger than with two phase qubits.

Blais & v. e. Brink & Zagoskin (2003). *Tunable Coupling of Superconducting Qubits. Physical review letters. 90. 127901. 10.1103/PhysRevLett.90.127901.*

INDUCTIVE COUPLING

- **Passive inductive coupling of flux qubits**
  - Flux in one qubit induces current in second qubit

$\rightarrow$ inductance matrix L_{ik}: connects flux in i-th loop with current in k-th loop:

$$\Phi_i = \sum_k L_{ik} I_k$$

L_{12} - mutual inductance

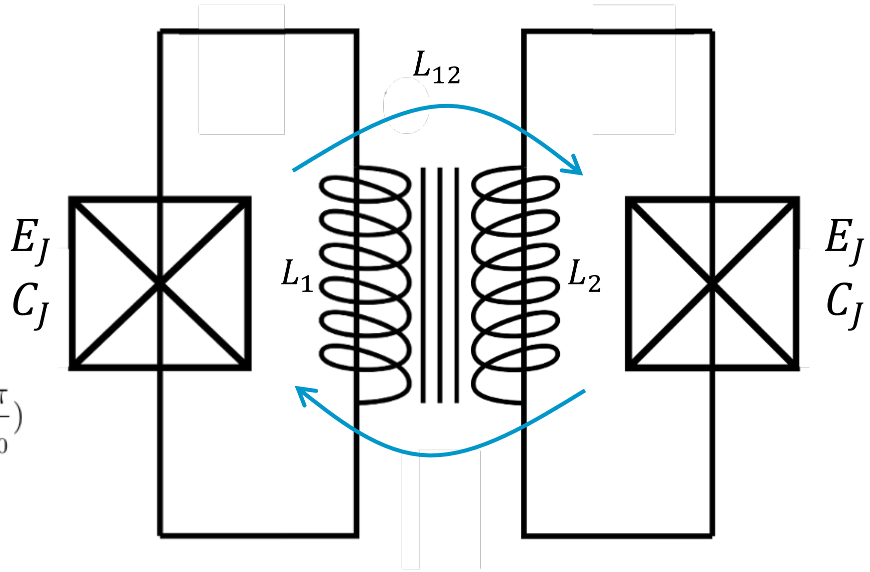
  - Generalized magnetic potential energy $H_{Flux} = \frac{q^2}{2C_J} + \underbrace{(\frac{\Phi_0}{2\pi})^2 \frac{\phi^2}{2L}} - E_J cos(\phi - \Phi_{ext}\frac{2\pi}{\Phi_0})$

$$\frac{1}{2}\left(\frac{\hbar}{2e}\right)^2 \sum_{ik} (L^{-1})_{ik}(\phi_i - \phi_{ei})(\phi_k - \phi_{ek})$$

  - Interaction term

$$\hat{H}_{int} = \lambda \sigma_{z1} \sigma_{z2}$$

$$\lambda = \frac{1}{8}\left(\frac{\hbar}{2e}\right)^2 (L^{-1})_{12} (\phi_l - \phi_r)_1 (\phi_l - \phi_r)_2$$



Wendin, G., & Shumeiko, V. S. (2005). Superconducting quantum circuits, qubits and computing. arXiv preprint cond-mat/0508729.

- Flux qubit Hamiltonian $$\mathcal{H} = -E_J \cos\left(2\pi \frac{\Phi}{\Phi_0}\right) + \frac{(\Phi - \Phi_x)^2}{2L} + \frac{Q^2}{2C_J}$$

- Currents and fluxes in lower loop coupled (dashed line)
  - Fluxes control barriers between potential wells $\rightarrow \propto \sigma_x^1 \sigma_x^2$ interaction
  - Placing loop differently $\rightarrow \propto \sigma_z^1 \sigma_z^2$ interaction
  - Interaction energy in order of MI_c^2 *(M- mutual inductance)*
    typically $0.01E_J$
  - *For typical RF-SQUID:*
    coupling stronger than tunnelling rate between flux states

  - Turn coupling of by switch controlled by high-frequency pulses
    - *Trade-off:* coupling to external circuit leads to decoherence
    - ***Alternative:*** **use ac driving pulses to induce state transitions two-qubit system**

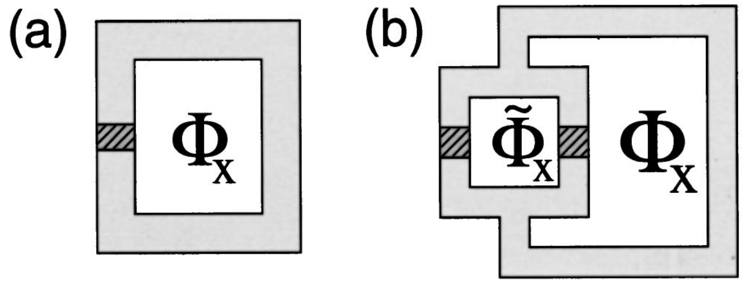


Figure 9: (a) Flux qubit and (b) improved design for flux qubit

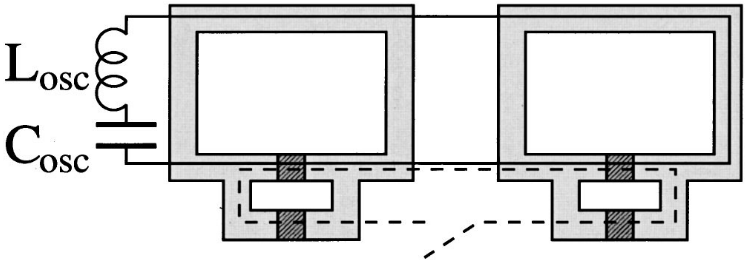


Figure 11: Direct inductive coupling (dashed line) vs. coupling by LC-circuit (solid line).

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- Flux qubit Hamiltonian $$\mathcal{H} = -E_J \cos\left(2\pi \frac{\Phi}{\Phi_0}\right) + \frac{(\Phi - \Phi_x)^2}{2L} + \frac{Q^2}{2C_J}$$

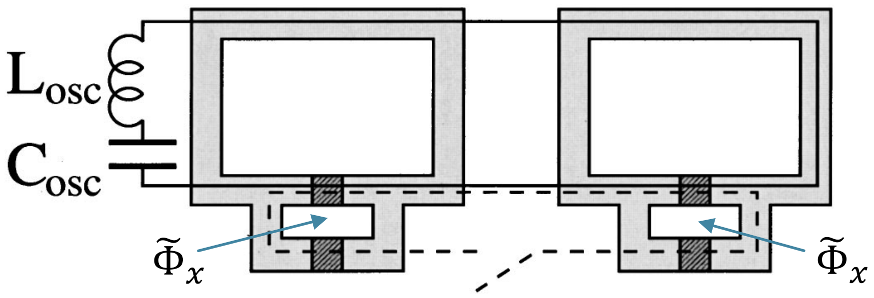
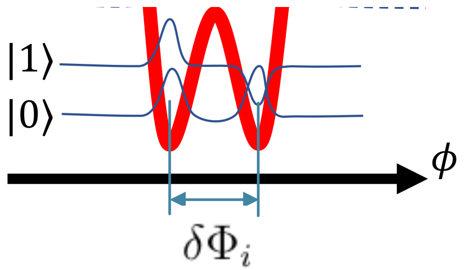


Figure 11: Direct inductive coupling (dashed line) vs. coupling by LC-circuit (solid line).

- Coupling by *LC* circuit (solid line)
  - Without additional switches
  - Coupling controlled by qubit parameters
  - Oscillator Hamiltonian: $$H_{osc} = \frac{\Phi^2}{2L_{osc}} + \frac{Q^2}{2C_{osc}} - VQ$$
  - Weak coupling to *LC* circuit $$\Phi_i = \frac{i}{\hbar}[H_i, \Phi_i] = \delta\Phi_i B_x^i \sigma_y^i \sigma_y^j$$ *Distance between two minima in potential*; *Tunnelling amplitude*; *(Heisenberg equation)*



  - Interaction Hamiltonian: $$H_{int} = -\left(\frac{\pi M}{L}\right)^2 \sum_{i<j} \frac{\delta\Phi_i \delta\Phi_j}{\Phi_0} \frac{B_x^i B_x^j}{e^2/C_{osc}} \sigma_y^i \sigma_y^j$$ *Mutual inductance*; *Qubit inductance*
  - Turn off coupling: Supress $B_x^i \leftrightarrow$ increase potential barrier via $\tilde{\Phi}_x$

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