Superconducting circuit protected by two-Cooper-pair tunneling[1]

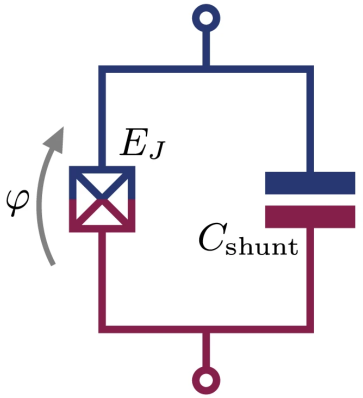
Margaretha Sandor

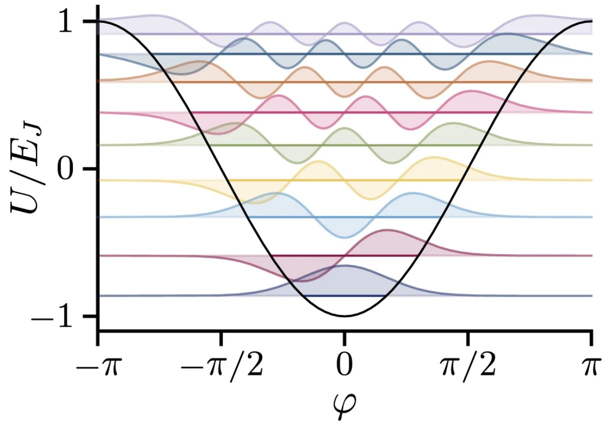
03.07.2020

[1] Smith, W. C. et al. Superconducting circuit protected by two-Cooper-pair tunneling. npj Quantum Inf 6, 8 (2020). https://doi.org/10.1038/s41534-019-0231-2

Idea

Transmon qubit:





Topologically protected qubits:

Circuit elements with degenerate phase states:
-> Only tunneling of pairs of Cooper pairs
Potential Energy: $U = -E_{\mathrm{J}} \cos 2\,\varphi$

➡ Combination of both!

Content

1) Protection in the cos2φ qubit
2) Superconducting circuit
   - Hamiltonian
   - Energy spectrum
3) Qubit States
   - Static properties
   - Decoherence estimates

Protected qubit

- $|g\rangle, |e\rangle$: two lowest energy eigenstates
- $\mathcal{O}$ any operator coupling to fluctuations with mean $\mathcal{O}_0$
- $l \gg 1$ phase space distance between $|g\rangle$ and $|e\rangle$

$$\langle\mu|\mathcal{O} - \mathcal{O}_0|\vartheta\rangle \sim \mathrm{e}^{-l} \qquad \forall\mu, \vartheta \in \{g, e\}$$

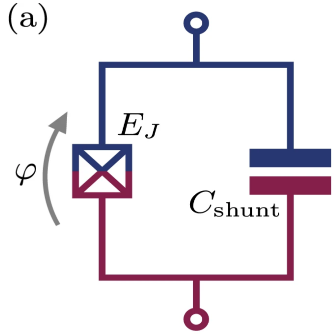
-> Qubit exponentially insensitive to variations in $\mathcal{O}$

- $\mathcal{O}$ should not be able to map:
  - $|g\rangle \rightarrow |e\rangle$ (relaxation)
  - $\frac{1}{\sqrt{2}}(|g\rangle + |e\rangle) \rightarrow \frac{1}{\sqrt{2}}(|g\rangle - |e\rangle)$ (dephasing)

Cos 2φ qubit

Transmon qubit:

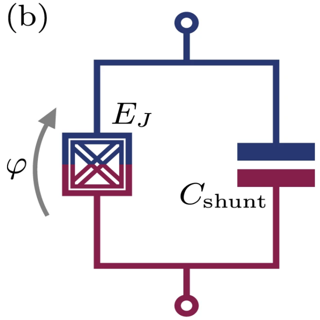
$H = 4E_C(N - N_g)^2 - E_J \cos\varphi$



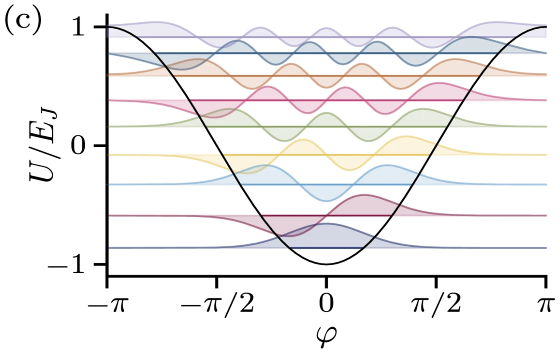
$-E_J \cos\varphi$
$= -\frac{1}{2} E_J \sum_{N=-\infty}^{\infty} (|N\rangle\langle N+1| + |N+1\rangle\langle N|)$

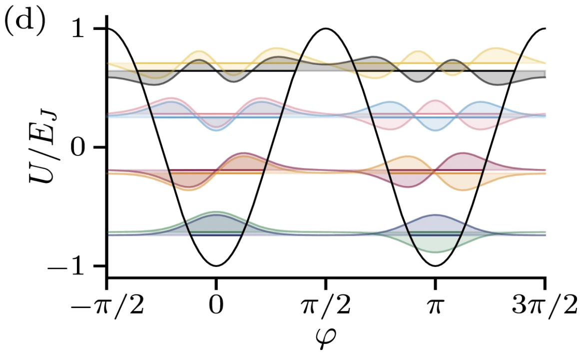
Cos 2φ qubit:

$H = 4E_C(N - N_g)^2 - E_J \cos 2\varphi$



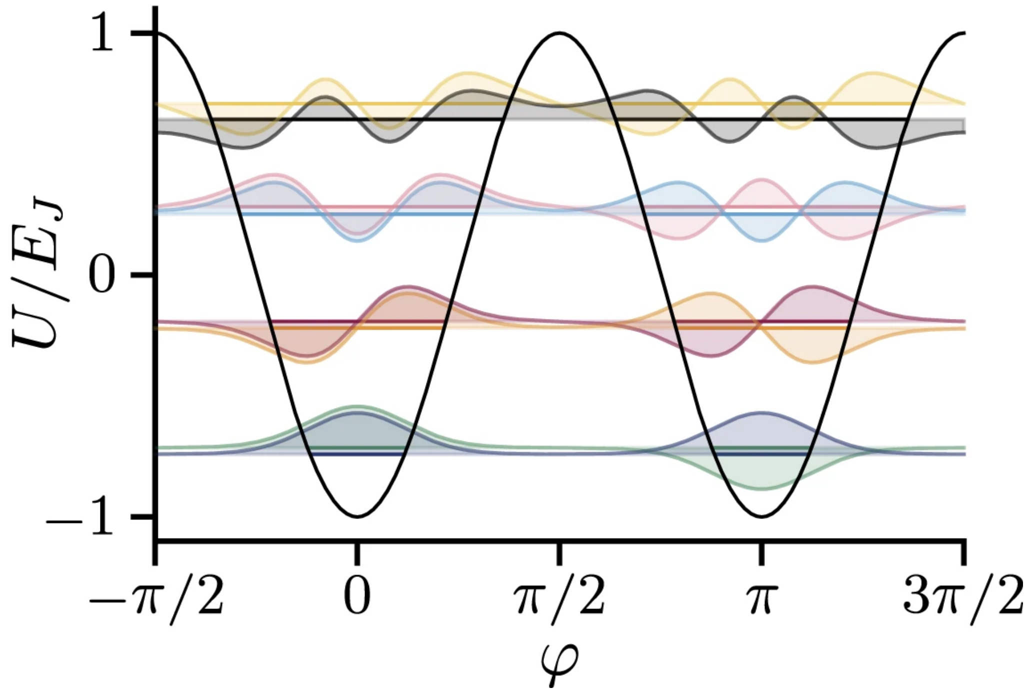
$-E_J \cos 2\varphi$
$= -\frac{1}{2} E_J \sum_{N=-\infty}^{\infty} (|N\rangle\langle N+2| + |N+2\rangle\langle N|)$





N: number of tunnelled Cooper pairs
N_g: off- set charge
E_J: tunneling energy
E_C: charging energy
φ: superconducting phase

Cos 2φ qubit



- π-periodicity
- Two nearly degenerate ground states $|+\rangle, |-\rangle$
- No overlap in charge space, opposite periodicity in phase space => $\langle -|\mathcal{O}|+\rangle \approx 0$
- $|\circlearrowright/\circlearrowleft\rangle = \frac{1}{\sqrt{2}}(|+\rangle \pm |-\rangle)$ localized near $\varphi = 0$ and π
- Supressed overlap in phase space for large E_J / E_C roughly inversely periodic in charge space => $\langle \circlearrowleft |\mathcal{O}| \circlearrowright\rangle \approx 0$
- Groundstate splitting: $\Delta E \approx 16 E_C \sqrt{\frac{2}{\pi}} \left(\frac{2E_J}{E_C}\right)^{\frac{3}{4}} e^{-\sqrt{(2E_J/E_C)}} \cos(\pi N_g)$

->splitting and charge dispersion suppressed exponentially in E_J / E_C

Constructing a quantum Hamiltonian for a circuit

1. Reduce circuit
2. Sketch Lagrangian
3. Impose Kirchoff´s laws
4. Formulate Hamiltonian
5. Promote variables to operators

Superconducting circuit

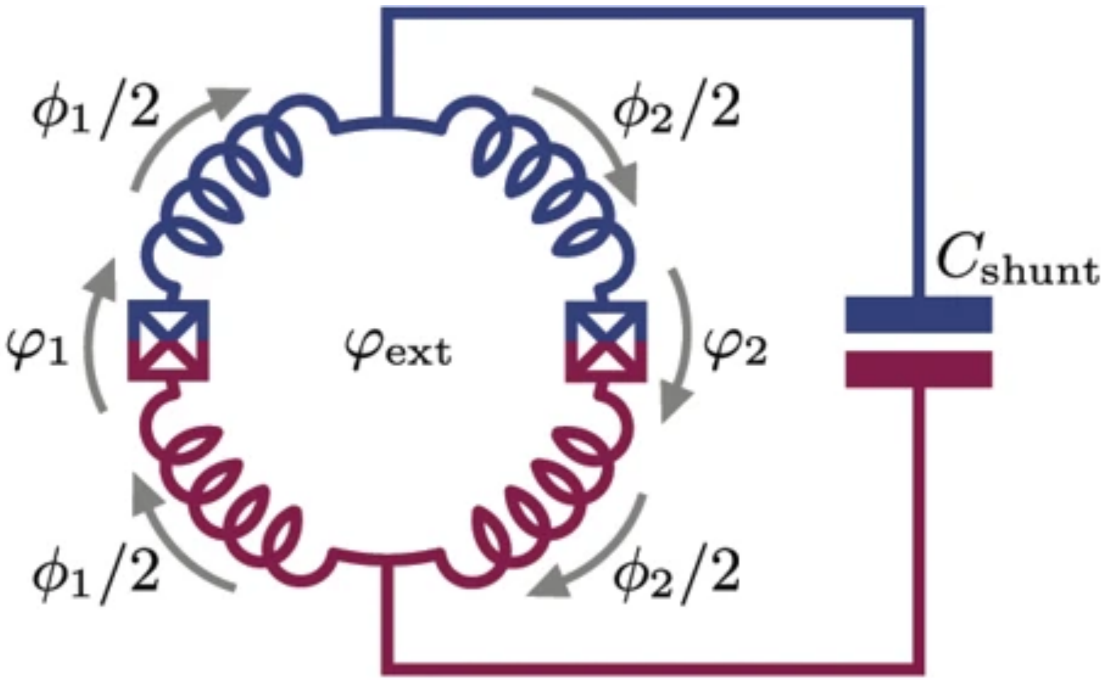
- New phase coordinates:

$$\phi = \varphi_1 + \varphi_2;\ \varphi = \frac{1}{2}(\varphi_1 - \varphi_2);\ \theta = \frac{1}{2}(\phi_1 - \phi_2)$$

n N η

conjugate charges

- $H = 4\epsilon_C\left[2n^2 + \frac{1}{2}(N - N_g - \eta)^2 + x\eta^2\right]$
  $+ \epsilon_L\left[\frac{1}{4}(\phi - \varphi_{ext})^2 + \theta^2\right] + 2\epsilon_J \cos\varphi \cos\frac{\phi}{2}$
- => three strongly coupled modes:
  - ϕ: flux dependent, coupled to φ via Josephson junctions
  - φ: off-set charge dependent, capacitiely coupled to θ



$x \equiv C_J/C_{shunt}$

φ_{ext}: external flux

$2\epsilon_L$: inductive energy of each superinductance

ϵ_C:single junction charging energy

ϵ_J: tunneling energy

Effective Hamiltonian

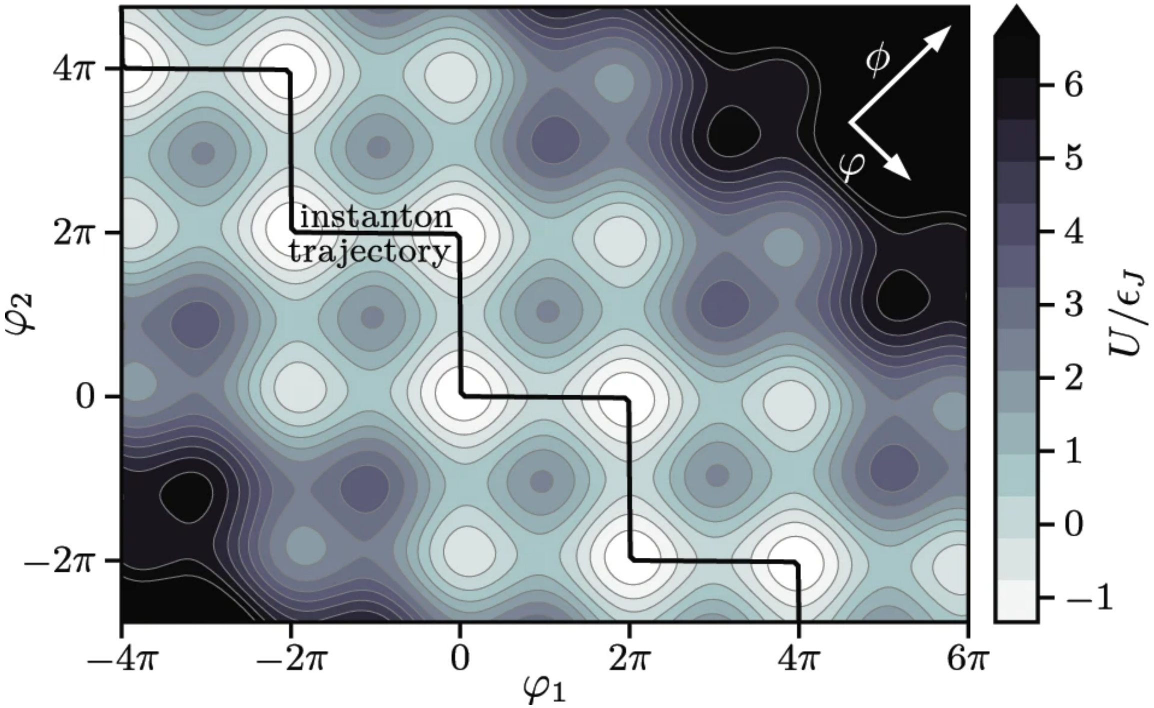
Min at $\varphi_1 + \varphi_2 = \varphi_{ext}$ $\theta = 0$

$$U = \epsilon_L \left[\frac{1}{4}(\varphi_1 + \varphi_2 - \varphi_{ext})^2 + \theta^2\right]$$
$$-2\epsilon_J \cos\frac{\varphi_1 - \varphi_2}{2} \cos\frac{\varphi_1 + \varphi_2}{2}$$

For $\varphi_{ext} = \pi$ -> degenerate ridges

1D potential: minimizing energy, eliminating coordinate

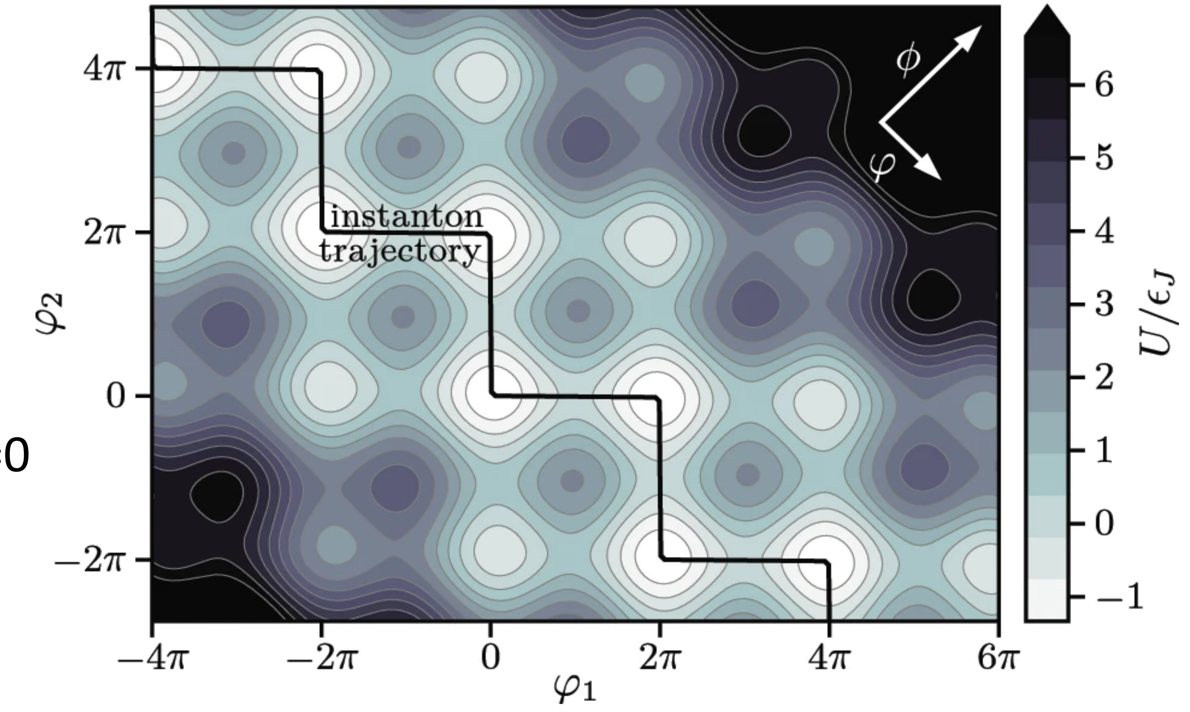
-> Instanton trajectory



Semiclassical theory

- U-> -U
- => Lagrangian
- => Euler Lagrangian equation of motion
- $\phi = \frac{1}{1+z}\left(2\left|\varphi - 2\pi \,\mathrm{round}\frac{\varphi}{2\pi}\right| + z\varphi_{\mathrm{ext}}\right)$
- Plugging in Hamiltonian and approximating with truncated Fourier series and Taylor expansion about z=0
- $H_{\mathrm{eff}} = 4\epsilon_{\mathrm{C}}\left[\frac{1}{4(1-z)}\left(N - N_{\mathrm{g}} - \eta\right)^2 + x\eta^2\right] + \epsilon_{\mathrm{L}}\theta^2$

  $- \frac{16}{3\pi}\epsilon_{\mathrm{L}}(\pi - \phi_{\mathrm{ext}}) \cos\varphi - \epsilon_{\mathrm{J}}\,(1 - \frac{5}{4}z)\cos 2\varphi$



$z = \epsilon_L / \epsilon_J$

$\phi_{\mathrm{ext}} = |\varphi_{\mathrm{ext}} - 4\pi \,\mathrm{round}\frac{\varphi_{\mathrm{ext}}}{4\pi}|$

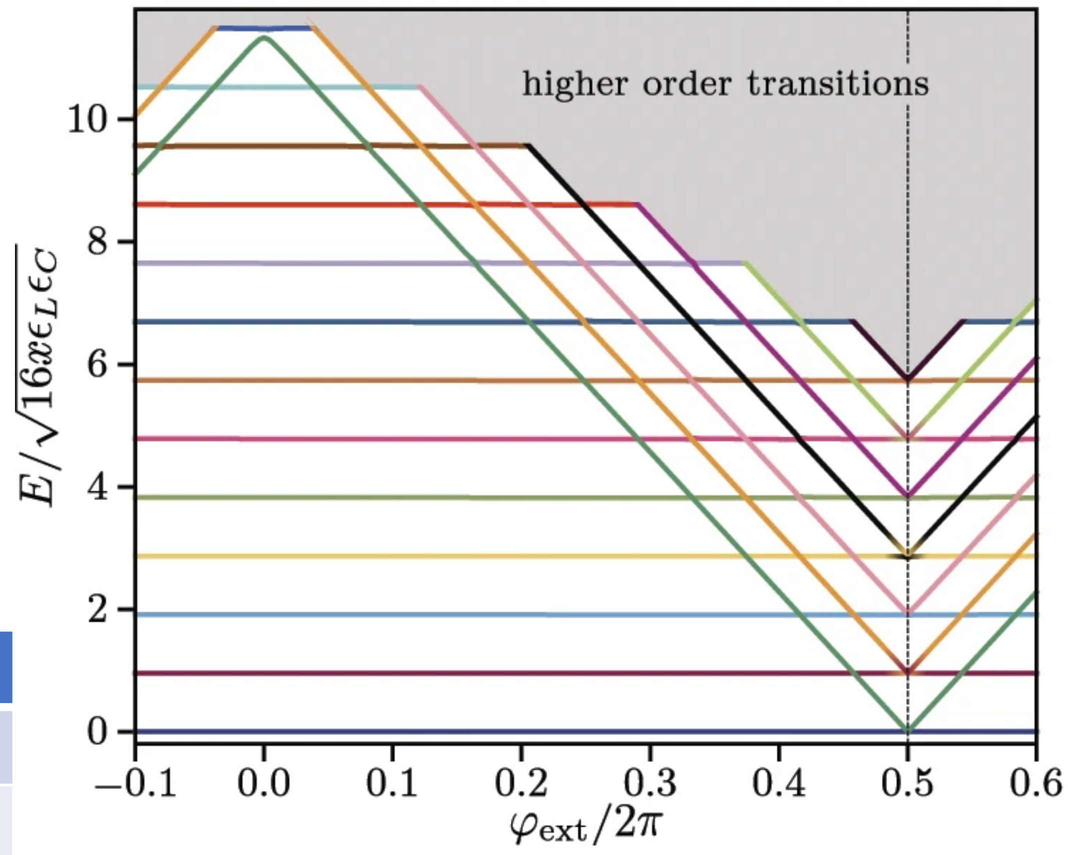
Energy spectrum

Dependence of energy levels on external flux:

At $\varphi_{ext} = \pi$: harmonic oscillator, $\Delta E = \sqrt{16x\epsilon_L\epsilon_C}$

Otherwise:

	Fluxon mode:	Plasmon mode:
Flux	Dependent	Independent
Energy	Linear increase; slope $\frac{32}{2}\epsilon_L$	Harmonic ladder
Excitations	Fluxons enclosed by loop -> magnitude and chirality of persistent current	Quantized charge density oscillations Capacitance and superinductances



$$●/○ = \begin{cases} ↺/↻ \text{ for } \varphi_{ext} \bmod 2\pi < \pi \\ -/+ \text{ for } \varphi_{ext} \bmod 2\pi = \pi \\ ↻/↺ \text{ for } \varphi_{ext} \bmod 2\pi > \pi \end{cases}$$

Label: m: number of plasmons

●/○ : presence/absence of fluxon excitation

Matrix elements

- Properties of logical qubit formed by: $\{|0-\rangle, |0+\rangle\}$ at $\varphi_{\mathrm{ext}} = \pi$

  $\{|0\bullet\rangle, |0\circ\rangle\}$
- Operators inducing transitions ? -> matrix elements

Capacitive coupling:

Voltage V coupling to superconducting island via gate capacitance C_{g}

$\Rightarrow H_{\mathrm{int}} = \frac{C_{\mathrm{g}}}{C_{\mathrm{shunt}} + C_{\mathrm{g}}}(2e\eta)V$

$\Rightarrow$ Transition directly related to $\langle\psi|\eta|0\circ\rangle$

Inductive coupling:

Current I coupling to circuit via inductance L_{s}

$\Rightarrow H_{\mathrm{int}} = \frac{L_s}{2L}(\phi_0\phi)I$

$\Rightarrow \langle\psi|\phi|0\circ\rangle$

$\phi_0 = \hbar/2e$ reduced magnetic flux

L :Superinductance in each arm

Matrix elements

- Normalized matrix elements:

$$\left|\mathcal{O}_\psi\right|^2 \equiv \frac{|\langle\psi|\mathcal{O}|0\circ\rangle|^2}{|\langle 0\circ|\mathcal{O}^\dagger\mathcal{O}|0\circ\rangle|};\ \ \sum_\psi\left|\mathcal{O}_\psi\right|^2 = 1; \left|\mathcal{O}_\psi\right|^2 > 0$$

- For $\mathcal{O}$= η:
- Transitions only form $|0\circ\rangle$ to $|1\circ\rangle$; no transition between qubit states
- Resulting from decoupling from even and odd Cooper pair number parity manifolds

=> Measurement and control

Matrix elements

- Normalized matrix elements:

$$\left|\mathcal{O}_{\psi}\right|^2 \equiv \frac{|\langle\psi|\mathcal{O}|0\circ\rangle|^2}{|\langle 0\circ|\mathcal{O}^{\dagger}\mathcal{O}|0\circ\rangle|}; \;\; \sum_{\psi}\left|\mathcal{O}_{\psi}\right|^2 = 1; \left|\mathcal{O}_{\psi}\right|^2 > 0$$

- For $\mathcal{O}$= ϕ:
- Transitions form $|0\circ\rangle$ to $|0\bullet\rangle$
- ϕ induces transition between Cooper pair parity manifolds

=> Relaxation mainly due to inductive loss

Disorder

- Influence of imperfections in supercoducting circuit?
- Symmertry breaking possible in junctions, capacitances and superinductances
- Numerical diagonalization of H
  -> energy splitting ΔE at N_{g}=0
  -> charge dispersion $\epsilon = \max_{N_g} \Delta E - \min_{N_g} \Delta E$ at $\varphi_{\text{ext}} = \pi$ of $\{|0+\rangle, |0-\rangle\}$ manifold
- $\delta \in [0,1)$ as parameter of asymmetry

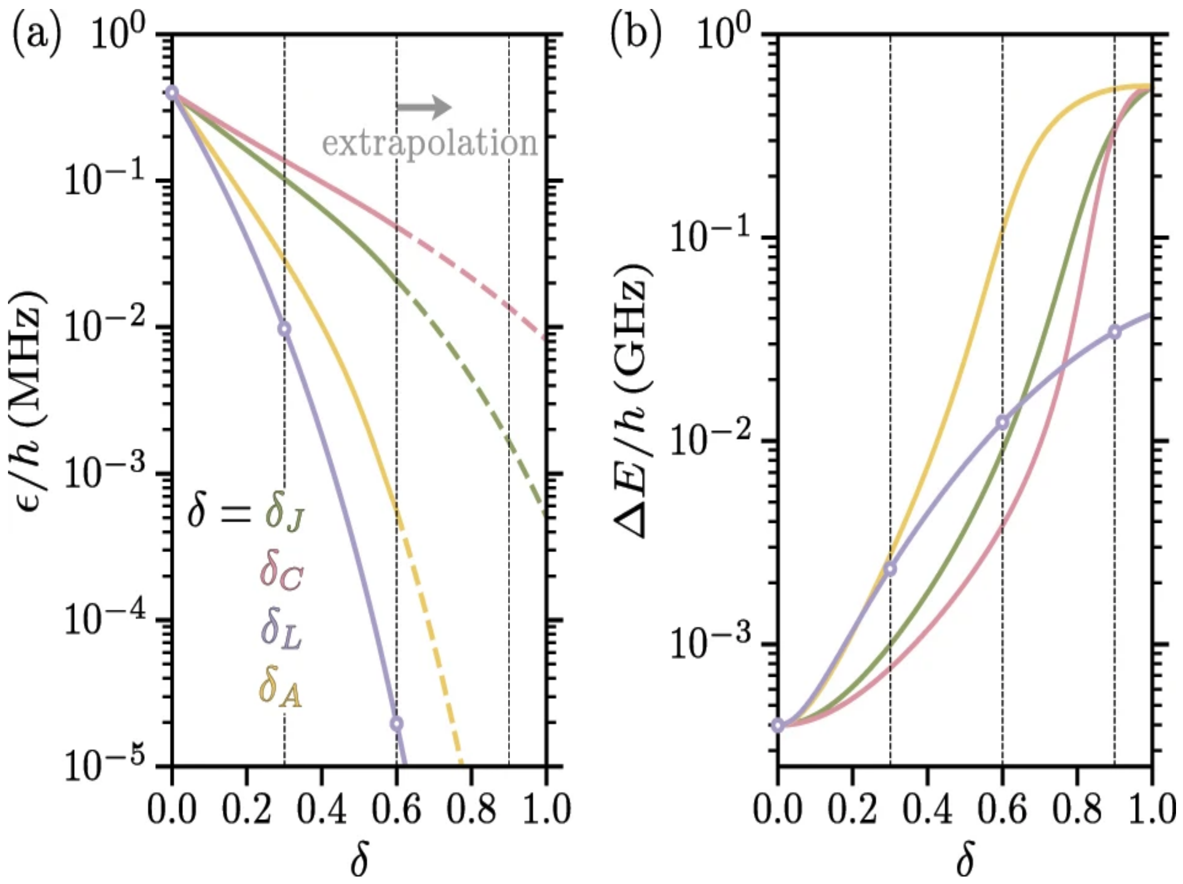
Asymetry

- $H = 4\epsilon_C\left[2n^2 + \frac{1}{2}(N - N_g - \eta)^2 + x\eta^2\right] + \epsilon_L\left[\frac{1}{4}(\phi - \varphi_{ext})^2 + \theta^2\right] - 2\,\epsilon_J \cos\varphi \cos\frac{\phi}{2}$

Josephson junctions:

- $(1 \pm \delta_J)\epsilon_J$
- $H' = 2\,\epsilon_J \delta_J \sin\varphi \sin\frac{\phi}{2}$
- $H'_{eff} = -\frac{16}{3\pi}\epsilon_J\delta_J(\sin\varphi - \frac{1}{5}\sin 3\varphi)$

=> tunnelling of single cooper pairs, symmetric and asymmetric circuit characteristics

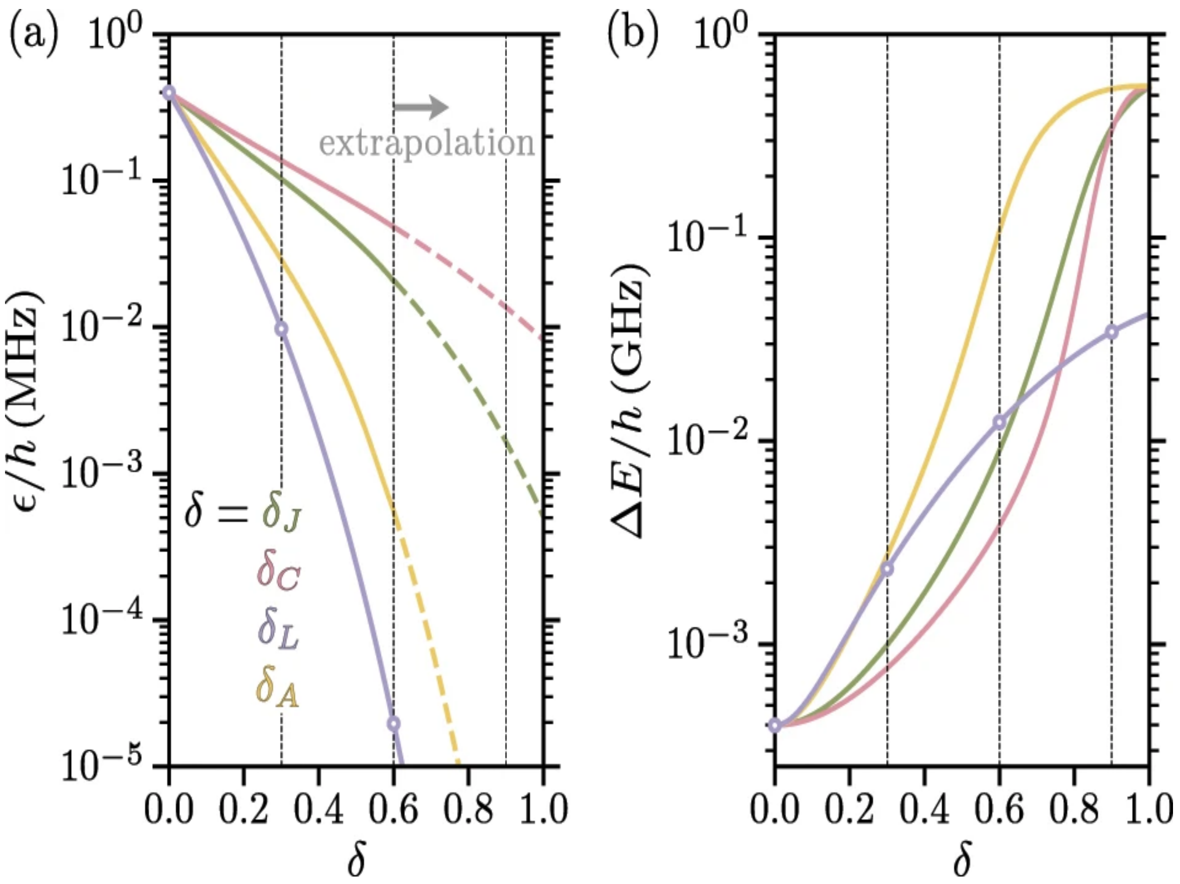


Asymetry

- $H = 4\epsilon_C\left[2n^2 + \frac{1}{2}(N - N_g - \eta)^2 + x\eta^2\right] + \epsilon_L\left[\frac{1}{4}(\phi - \varphi_{ext})^2 + \theta^2\right] - 2\,\epsilon_J \cos\varphi \cos\frac{\phi}{2}$

Capacitances:

- $\epsilon_C / (1 \pm \delta_C)$
- $H' = -8\epsilon_C \frac{\delta_C}{1-\delta_C^2} n(N - N_g - \eta)$
- $\delta_J = \delta_C \equiv \delta_A$
- $\epsilon_J \epsilon_C$=const.; plasma frequencies fixed, area imperfections: $(1+\delta_A)A,\ A \propto \sqrt{\epsilon_J/\epsilon_C}$



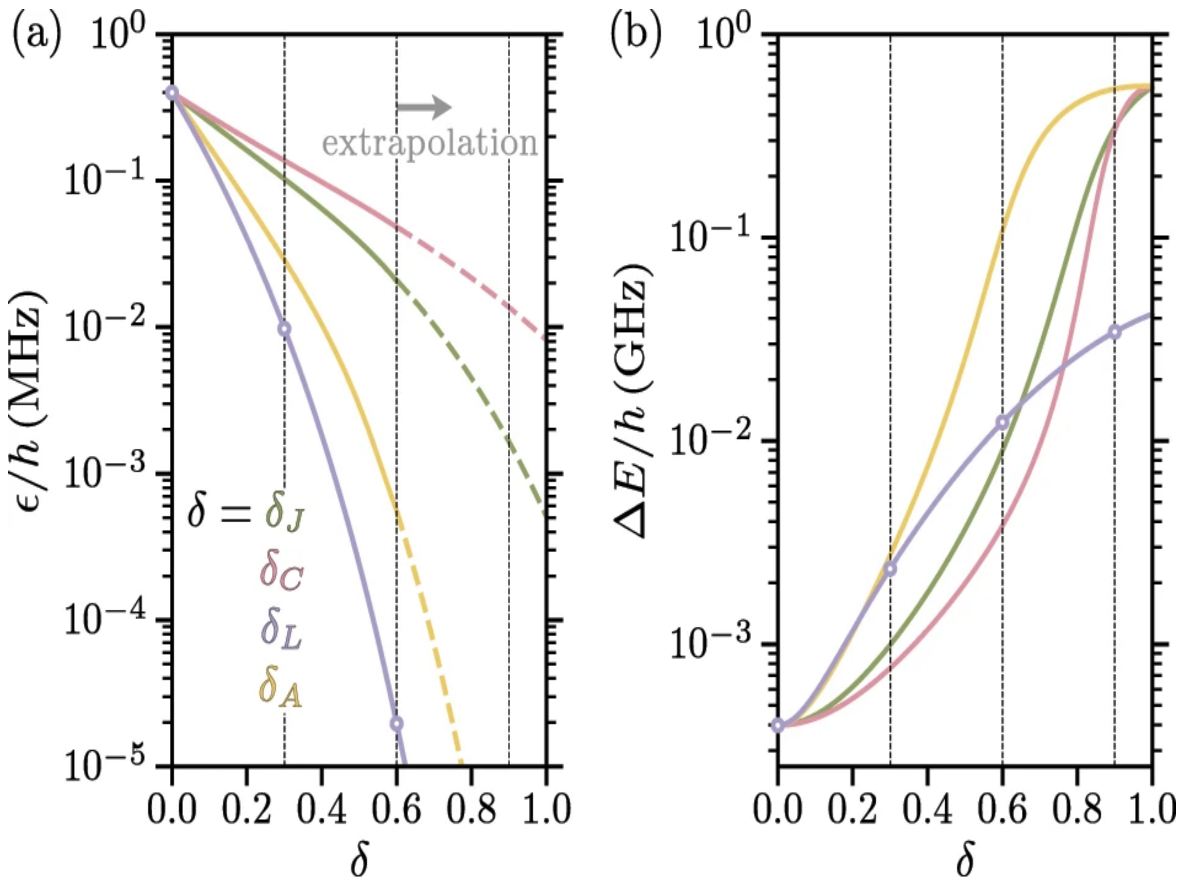
Asymetry

- $H = 4\epsilon_C\left[2n^2 + \frac{1}{2}(N - N_g - \eta)^2 + x\eta^2\right] + \epsilon_L\left[\frac{1}{4}(\phi - \varphi_{ext})^2 + \theta^2\right] - 2\,\epsilon_J \cos\varphi \cos\frac{\phi}{2}$

Superinductances:

- $\epsilon_L / (1 \pm \delta_L)$
- $H' = \epsilon_L \frac{\delta_L}{1-\delta_L^2}(\phi - \varphi_{ext})\theta$

=> sufficiently non degenerate ground states and largely supressed charge dispersion



Relaxation

- Fermi´s Golden rule to model loss -> relaxation rate:

$$\frac{1}{T_1} = \frac{1}{\hbar^2} |\langle 0+|\mathcal{O}|0-\rangle|^2 [S_{\mathcal{E}\mathcal{E}}(\Delta\omega) + S_{\mathcal{E}\mathcal{E}}(-\Delta\omega)]$$

$\Delta\omega = \Delta E / \hbar$

$$\frac{2\hbar}{\lambda Q(\omega)} \coth \frac{\hbar|\omega|}{2k_{\mathrm{B}}T}$$

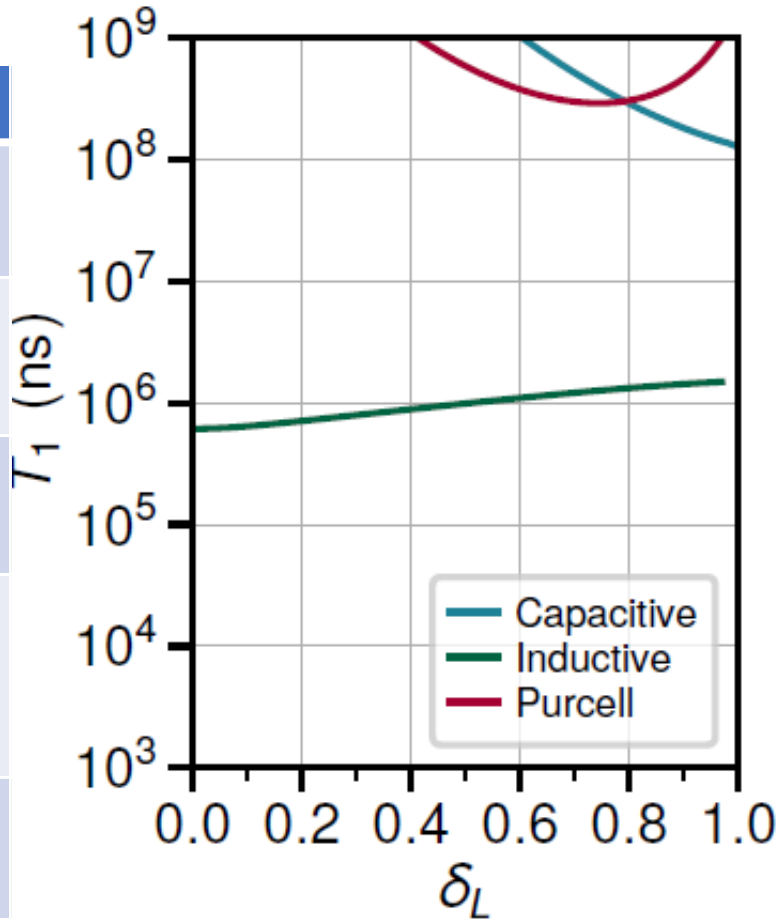
$\mathcal{O}$ Operator coupling to noisy bath $\mathcal{E}(t)$

spectral noise density $S_{\mathcal{E}\mathcal{E}}(\omega)$

- Four main loss mechanism:
  - capacitive loss
  - inductive loss
  - Purcell loss
  - Quasiparticle tunneling

Properties of relaxation mechanisms

Channel	$\mathcal{O}$	$\mathcal{E}$	λ	Quality factor
Capacitive	$2eN_i = 2e[n \pm \frac{1}{2}(N - \eta)]$	V	C_J	$Q_{cap} \sim 1\times10^6$
Inductive	$\phi_0 \phi_i$	I	L_l	$Q_{ind} \sim 500\times10^6$
Quasiparticle	$2\,\phi_0 \sin\frac{\phi_i}{2}$		L_J	$\frac{1}{x_{qp}} \sim 0.3\times10^6$
Purcell	$2e\eta$	V	C_{shunt}	$Q_{cap} \sim 1\times10^6$



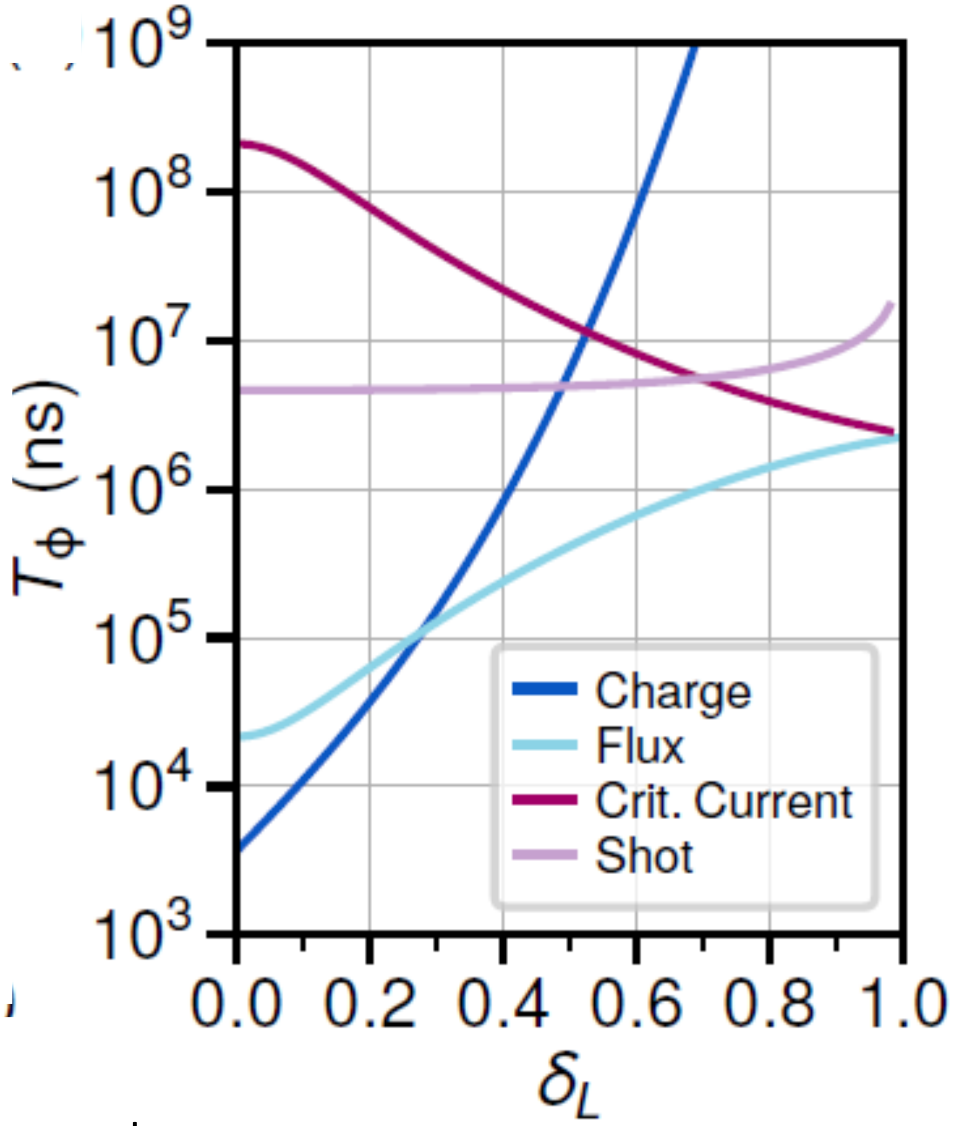
$$[S_{\mathcal{E}\mathcal{E}}(\Delta\omega) + S_{\mathcal{E}\mathcal{E}}(-\Delta\omega)] = \frac{2\hbar}{\lambda Q(\omega)} \coth\frac{\hbar|\omega|}{2k_B T}$$

Pure dephasing

- Dependence of $\Delta \mathrm{E}$ on λ -> dephasing mechanisms
- Noise spectral densities $\frac{1}{f}$ -> $S_{\lambda\lambda}(\omega) = 2\pi A_{\lambda}/|\omega|$
- $\sqrt{A_{\lambda}}$: noise spectral amplitude

Pure dephasing

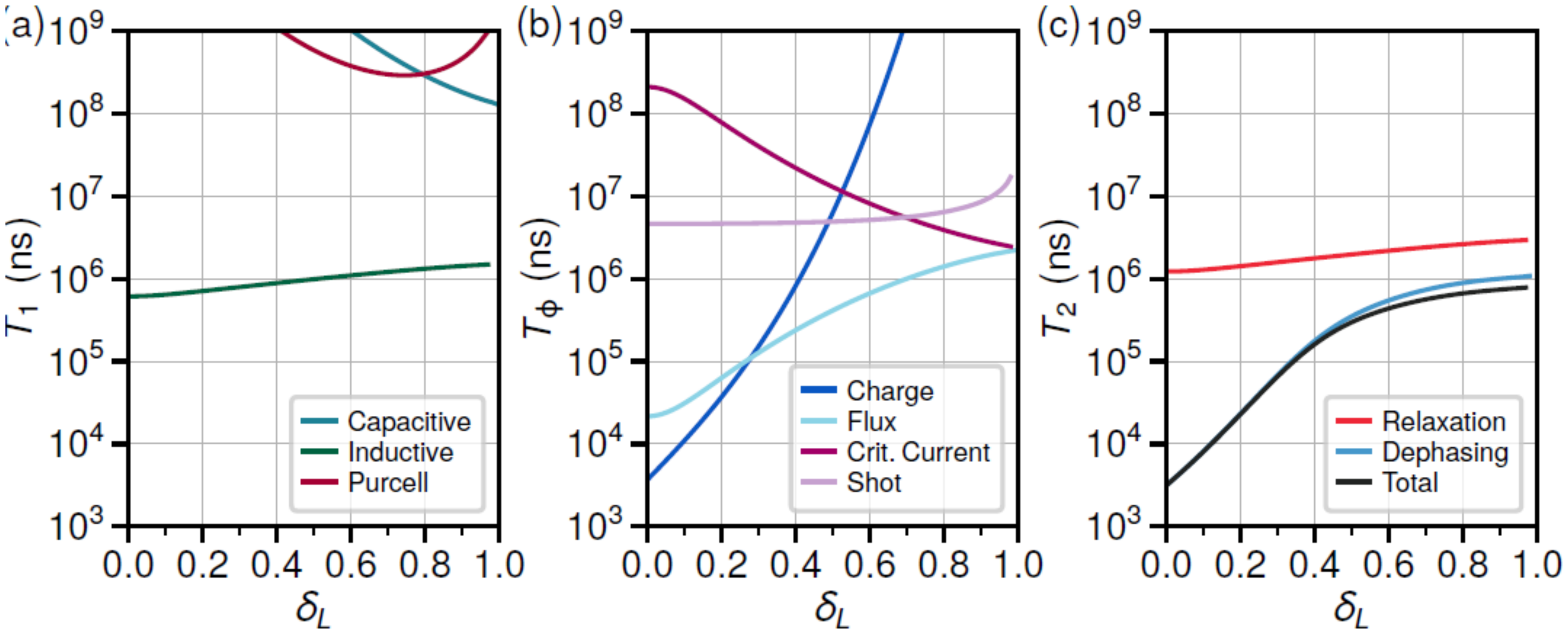
Dephasing channel	λ	$\frac{1}{T_\phi}$	Spectral density amplitude
Charge	N_g	$=\frac{\pi}{(2e)^2}\epsilon/\hbar$	$\sqrt{A_{N_g}} \sim 1\times10^{-4}$
Flux	φ_{ext}	$= A_{\varphi_{\text{ext}}}\left\lvert\frac{\partial^2 \Delta \text{E}}{\partial \varphi_{\text{ext}}^2}\right\rvert$	$\sqrt{A_{\varphi_{\text{ext}}}}/2\pi$ $\sim 3\times10^{-6}$
Critical current	ϵ_{J}	$= \sqrt{A_{\epsilon_{\text{J}}}}\left\lvert\frac{\partial \Delta \text{E}}{\partial \epsilon_{\text{J}}}\right\rvert$	$\sqrt{A_{\epsilon_{\text{J}}}}/\epsilon_{\text{J}}$ $\sim 5\times10^{-7}$
Photon Shot	n_p	$=n_{th}\kappa\frac{\chi^2}{\chi^2+\kappa^2}$	n_{th}/Q_{cap} $\sim 1\times10^{-7}$



χ: dispersive shift of plasmon mode

κ: $\omega_p/Q_{\text{cap}(\omega_p)}$, linewidth of plasmon mode

Decoherence estimates



Control and readout

- Problem in general: staying isolated to preserve coherence
- Non local encoding: $|g\rangle \rightarrow |e\rangle$ control, $|g\rangle, |e\rangle$ read out
- Large degree of insensitivity of frequency -> no dispersive measurement
- Cos 2φ qubit: qubit transition via inductive coupling
- -> capacitive coupling to higher levels
- Readout problem: no native dispersive shift between qubit and external electromagnetic mode
- ->dispersive coupling plasmon mode 20MHz, small anharmonicity -> ancillary anharmonic mode to measure plasmon, two readout tones

Conclusion

- Few body superconducting circuit
- Charge carriers: pairs of Cooper pairs at paticular bias
- Josephson tunneling element: characterized by $\cos 2\varphi$ term in Hamiltonian
- Numerical simulations: protection against relaxation and dephasing sources
- Enhanced in the presence of disorder

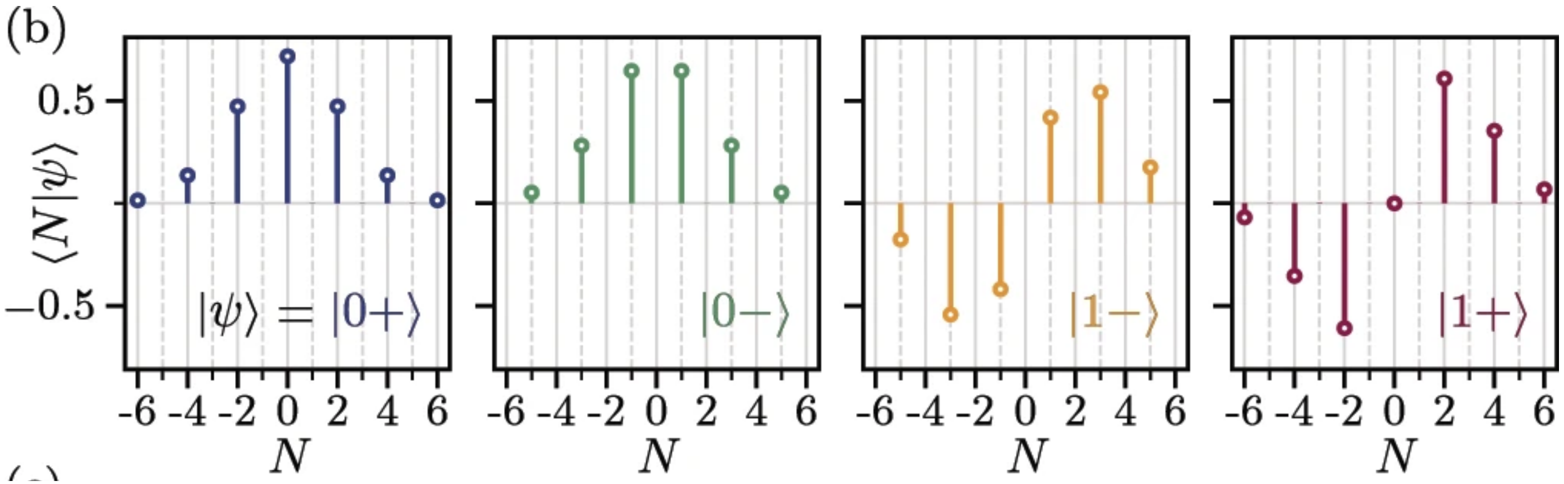
Thank you for your attention!

Literature

[1] Smith, W.C., Kou, A., Xiao, X. *et al.* Superconducting circuit protected by two-Cooper-pair tunneling. *npj Quantum Inf* **6,** 8 (2020). https://doi.org/10.1038/s41534-019-0231-2

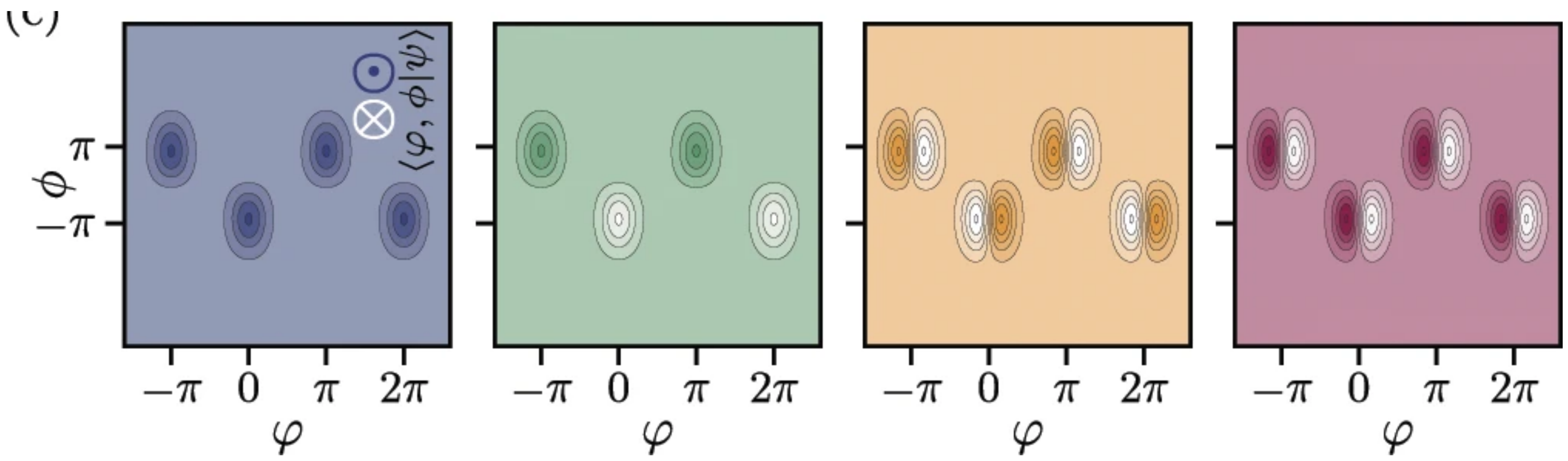
[2] Smith, W.C., Design of Protected Superconducting Qubits, Yale University (2019)

Wavefunctions



Numerical diagonalisation: -> Four lowest energy eigenstates, $\varphi_{ext} = \pi$
Charge wave functions: $\langle N|\psi\rangle$ Projection of θ and constrain to the trajectory
Grid states with Fock state envelopes
+/ -: superpostitions of even/odd number states
0,1: order of Fock state envelope
$|0+\rangle$, $|0-\rangle$ protected from spurious transitions, except operator flipping parity

Wavefunctions



Phase wavefunctions: $\langle \varphi, \phi \,|\psi\rangle$: Projection of θ and FT of $\varphi\phi$ plane
Fock states localized within the potential energy wells
+/-: symmetric, antisymmetric states localized in within opposite ridges of potential wells
ridges correspond to persistent currents of opposite chirality, and hence also to the absence/presence of a fluxon in the inductive loop of the circuit
0,1: Fock order
flip Cooper pair parity odd functions of φ and ϕ period an odd division of 2π