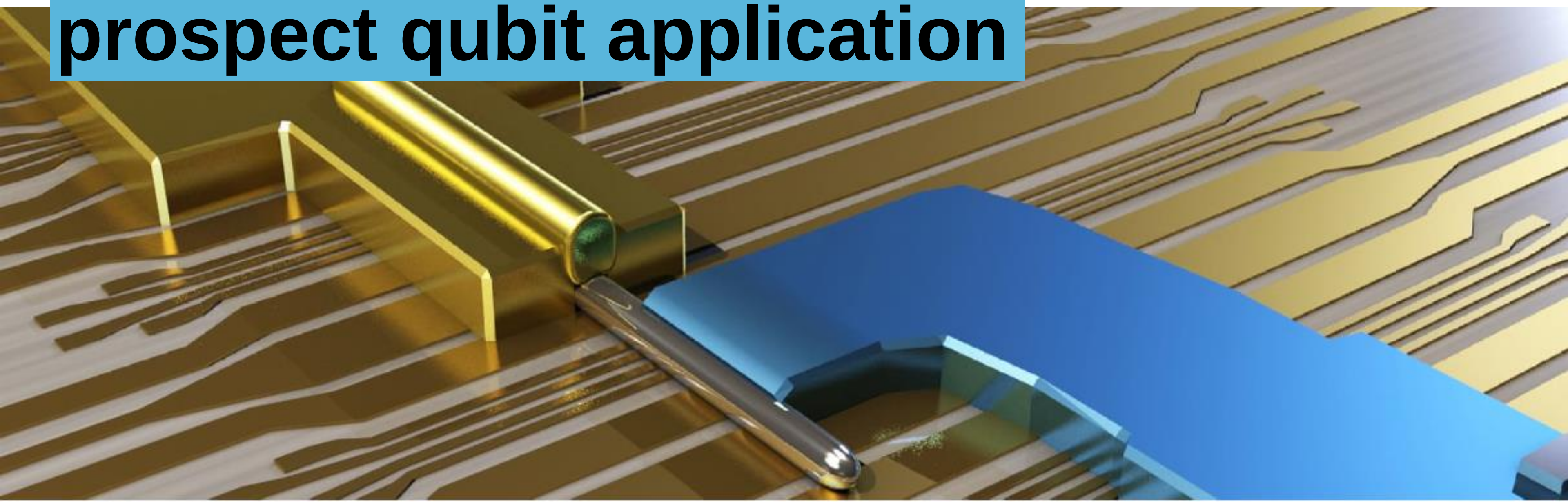
Universität Konstanz

Majorana fermions and prospect qubit application



Quelle: https://www.spektrum.de/news/exotische-teilchen-im-nanodraht/1151134

David Kaltenbrunner

Student seminar, 26.06.2020

Outline

1. Introduction
2. Majorana fermions in spinless p-wave superconductors
3. Proximity induced p-wave like gap in semiconductor
4. Quantum computation with Majorana fermions

Intro
p-wave superc.
experiment
qubit

Introduction

$$\gamma_1 = \gamma_1^\dagger$$

- Majorana fermions (MF): Fermionic particles, which are their own anti-particles
- Quasiparticle excitations in certain condensed matter states: Majorana zero mode
- Zero energy bound states and show non Abelian-exchange statistics
  - Particle exchange is no trivial operation limited to a phase factor

- Given a set of N Dirac fermions:

$$\{c_k, c_l^\dagger\} = \delta_{kl}$$
$$\{c_k, c_l\} = 0$$

↓

- One can construct a set of 2N MF:

$$\gamma_{2k-1} = c_k + c_k^\dagger$$
$$\gamma_{2k} = i(c_k^\dagger - c_k)$$

➢ Note same spin projection: p-wave pairing is necessary, but not available

Introduction

- Fermionic state can be considered as superposition of two MF

$$\gamma_{2k-1} = c_k + c_k^\dagger$$
$$\gamma_{2k} = i(c_k^\dagger - c_k)$$

$$\longrightarrow$$

$$c_k^\dagger = \frac{1}{2}(\gamma_{2k-1} - i\gamma_{2k})$$
$$c_k = \frac{1}{2}(\gamma_{2k-1} + i\gamma_{2k})$$

- Motivation to find MF:
  - If spatially separated they are expect to be immune against local perturbation and therefore decoherence-free
  - Access to perform operations due to the non-Abelian exchange statistics

- Being its own anti-particle means half electron, half hole

➢ No energy, no charge, no magnetic freedom - fingerprint: “three times nothing”

Majorana fermions in spinless p-wave superconductors

- Toy model which shows MF as eigenstates
- Hamiltonian of 1D spinless superconductor as tight-binding chain of electrons which is called "Kitaev Model"

$$\mathcal{H}_{\text{chain}} = -\mu \sum_{i=1}^{N} n_i - \sum_{i=1}^{N-1} (t c_i^\dagger c_{i+1} + \Delta c_i c_{i+1} + \text{h.c.})$$

- Rewrite this in terms of MF:

$$c_i = \tfrac{1}{2}(\gamma_{i,1} + \mathrm{i}\gamma_{i,2})$$

$$c_i^\dagger = \tfrac{1}{2}(\gamma_{i,1} - \mathrm{i}\gamma_{i,2})$$

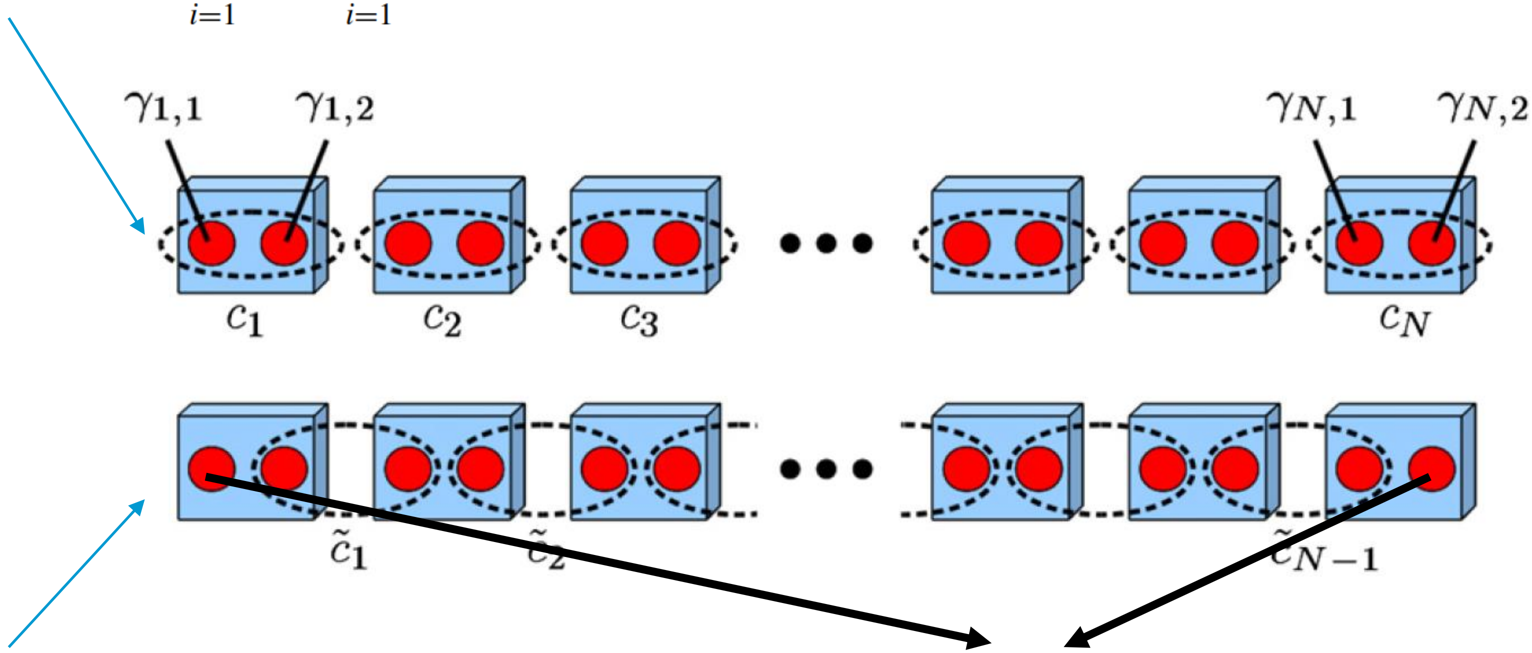
and for sake of convenience $\mu = 0, \quad t = \Delta$

- gives:

$$\mathcal{H}_{\text{chain}} = -\mathrm{i}t \sum_{i=1}^{N-1} \gamma_{i,2}\gamma_{i+1,1}$$

Majorana fermions in spinless p-wave superconductors

$$\mathcal{H}_{\text{chain}} = -\mu \sum_{i=1}^{N} n_i - \sum_{i=1}^{N-1} (t c_i^\dagger c_{i+1} + \Delta c_i c_{i+1} + \text{h.c.})$$



$$\mathcal{H}_{\text{chain}} = -\mathrm{i}t \sum_{i=1}^{N-1} \gamma_{i,2}\gamma_{i+1,1}$$

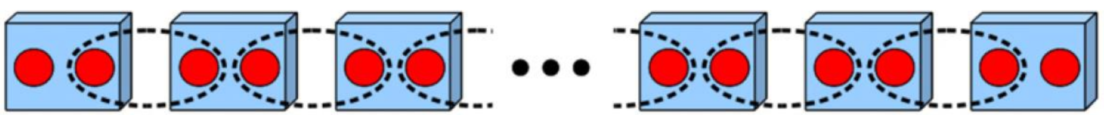
$$\tilde{c}_M = (\gamma_{N,2} + \mathrm{i}\gamma_{1,1})/2$$

Majorana zero mode

Majorana fermions in spinless p-wave superconductors

$$\mathcal{H}_{\text{chain}} = -it \sum_{i=1}^{N-1} \gamma_{i,2}\gamma_{i+1,1} \xrightarrow{\quad -i\gamma_{i,2}\gamma_{i+1,1} = 2\tilde{c}_i^\dagger \tilde{c}_i \quad} \mathcal{H}_{\text{chain}} = 2t \sum_{i=1}^{N-1} \tilde{c}_i^\dagger \tilde{c}_i$$

- Energy cost for creating a fermionic state $\tilde{c}_i^\dagger \tilde{c}_i$ is $2t$
- Arguments for $\mu = 0,\ t = \Delta$ are valid as long $|\mu| < 2t$



- MF is highly delocalized
- Fermionic parity can be $+1$ or -1 and therefore ground state is twofold degenerated

- Appearance of spatially separated MF is a topological property
- If we find another topologically equivalent Hamiltonian in terms of $|\mu| < 2t$, we are able to find a more realistic system to perform experiments

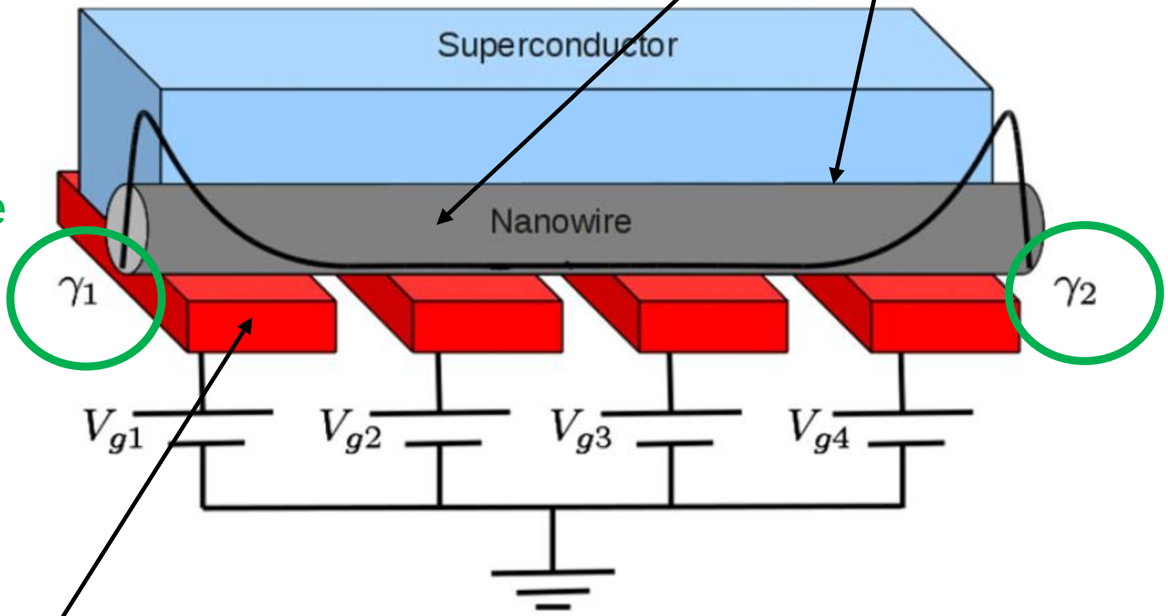
Proximity induced p-wave like gap in semiconductor

Engineering another Hamiltonian with two important ingredients:

- Spin-orbit coupling
- Zeeman splitting

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_S$$

Majorana zero mode



For adjusting chemical potential that always: $|\mu| < 2t$

Proximity induced p-wave like gap in semiconductor

Hamiltonian for proximity induced superconductivity
in a spin-orbit coupled semiconductor

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_S$$

Single electron without interaction

$$\mathcal{H}_0 = \sum_{\sigma=\uparrow,\downarrow} \int \mathrm{d}^D r \; \Psi_\sigma^\dagger(\mathbf{r}) H_0(\mathbf{r}) \Psi_\sigma(\mathbf{r})$$

$$H_0(\mathbf{r}) = \frac{\mathbf{p}^2}{2m} - \mu + V(\mathbf{r}) + \alpha\,(\mathbf{E}(\mathbf{r}) \times \mathbf{p}) \cdot \bar{\sigma} + \frac{1}{2} g \mu_B \mathbf{B}(\mathbf{r}) \cdot \bar{\sigma},$$

Induced superconductivity
(phenomenological)

$$\mathcal{H}_S = \int \mathrm{d}^D r \, \mathrm{d}^D r' \; \Psi_\downarrow(\mathbf{r}) \Delta(\mathbf{r}, \mathbf{r}') \Psi_\uparrow(\mathbf{r}') + \text{h.c.}$$

- s-wave coupling

Proximity induced p-wave like gap in semiconductor

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_S$$
$$= \frac{1}{2}\int \mathrm{d}^D r\, \mathrm{d}^D r' \;\bar{\Psi}^\dagger(\mathbf{r})\left[\bar{H}_0(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}') + \bar{\Delta}(\mathbf{r},\mathbf{r}')\right]\bar{\Psi}(\mathbf{r})$$

with using:

$$\bar{\Psi}(\mathbf{r}) = \begin{pmatrix} \Psi_\uparrow(\mathbf{r}) \\ \Psi_\downarrow(\mathbf{r}) \\ \Psi_\downarrow^\dagger(\mathbf{r}) \\ -\Psi_\uparrow^\dagger(\mathbf{r}) \end{pmatrix}$$

and properly:

$$\bar{H}_0(\mathbf{r}) = \begin{pmatrix} H_0(\mathbf{r}) & \hat{0}_\sigma \\ \hat{0}_\sigma & -\sigma_y H_0^*(\mathbf{r})\sigma_y \end{pmatrix}$$

$$\bar{\Delta}(\mathbf{r},\mathbf{r}') = \begin{pmatrix} \hat{0}_\sigma & \Delta^*(\mathbf{r},\mathbf{r}')\hat{1}_\sigma \\ \Delta(\mathbf{r},\mathbf{r}')\hat{1}_\sigma & \hat{0}_\sigma \end{pmatrix}$$

Proximity induced p-wave like gap in semiconductor

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_S$$
$$= \frac{1}{2}\int \mathrm{d}^D r\, \mathrm{d}^D r'\; \bar{\Psi}^\dagger(\mathbf{r}) \left[\bar{H}_0(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}') + \bar{\Delta}(\mathbf{r},\mathbf{r}')\right] \bar{\Psi}(\mathbf{r})$$

- Solving Bogoliubov-de Gennes-equation gives energy value for quasiparticles:

$$\bar{H}_0(\mathbf{r})\bar{\psi}_i(\mathbf{r}) + \int \mathrm{d}^D r'\; \bar{\Delta}(\mathbf{r},\mathbf{r}')\bar{\psi}_i(\mathbf{r}') = E_i\bar{\psi}_i(\mathbf{r})$$

- And with eigenvalues we can write down diagonalized Hamiltonian:

$$\mathcal{H} = \frac{1}{2}\sum_i E_i\; \Psi_i^\dagger \Psi_i, \qquad \Psi_i = \int \mathrm{d}^D r\; \bar{\psi}_i(\mathbf{r}) \cdot \bar{\Psi}(\mathbf{r})$$

Proximity induced p-wave like gap in semiconductor

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_S$$
$$= \frac{1}{2} \int \mathrm{d}^D r \, \mathrm{d}^D r' \; \bar{\Psi}^\dagger(\mathbf{r}) \left[\bar{H}_0(\mathbf{r}) \delta(\mathbf{r} - \mathbf{r}') + \bar{\Delta}(\mathbf{r}, \mathbf{r}') \right] \bar{\Psi}(\mathbf{r})$$

- Using Nambu representations with explicit hole description doubles degrees of freedom artificially

- But symmetry between eigenvalues of holes and electrons conserve original degrees of freedom:

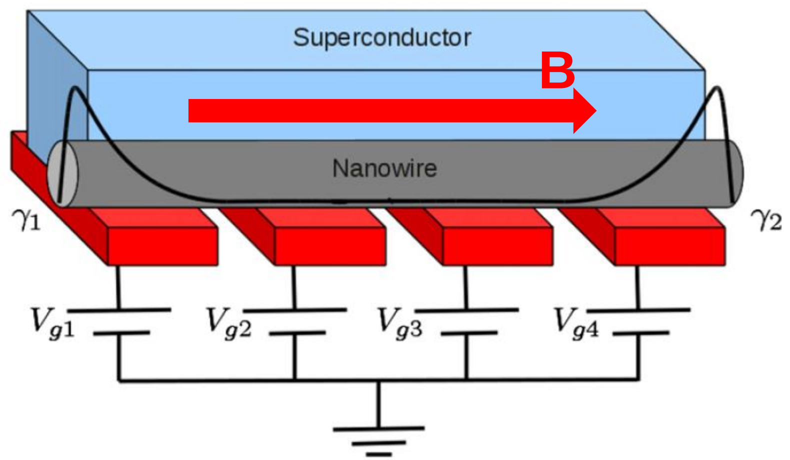
$$\bar{\Psi}(\mathbf{r}) = \begin{pmatrix} \Psi_\uparrow(\mathbf{r}) \\ \Psi_\downarrow(\mathbf{r}) \\ \Psi_\downarrow^\dagger(\mathbf{r}) \\ -\Psi_\uparrow^\dagger(\mathbf{r}) \end{pmatrix}$$

  - Creating a quasiparticle with energy E is equivalent to removing a quasiparticle with energy $-E$

  - For every eigenvector with eigenvalue E there exists another eigenvector with $-E$

Proximity induced p-wave like gap in semiconductor

- Engineering parameters for desired properties
  - Hamiltonian must be transformable continuously from spinless p-wave superconductor without closing the gap $|\mu| < 2t$



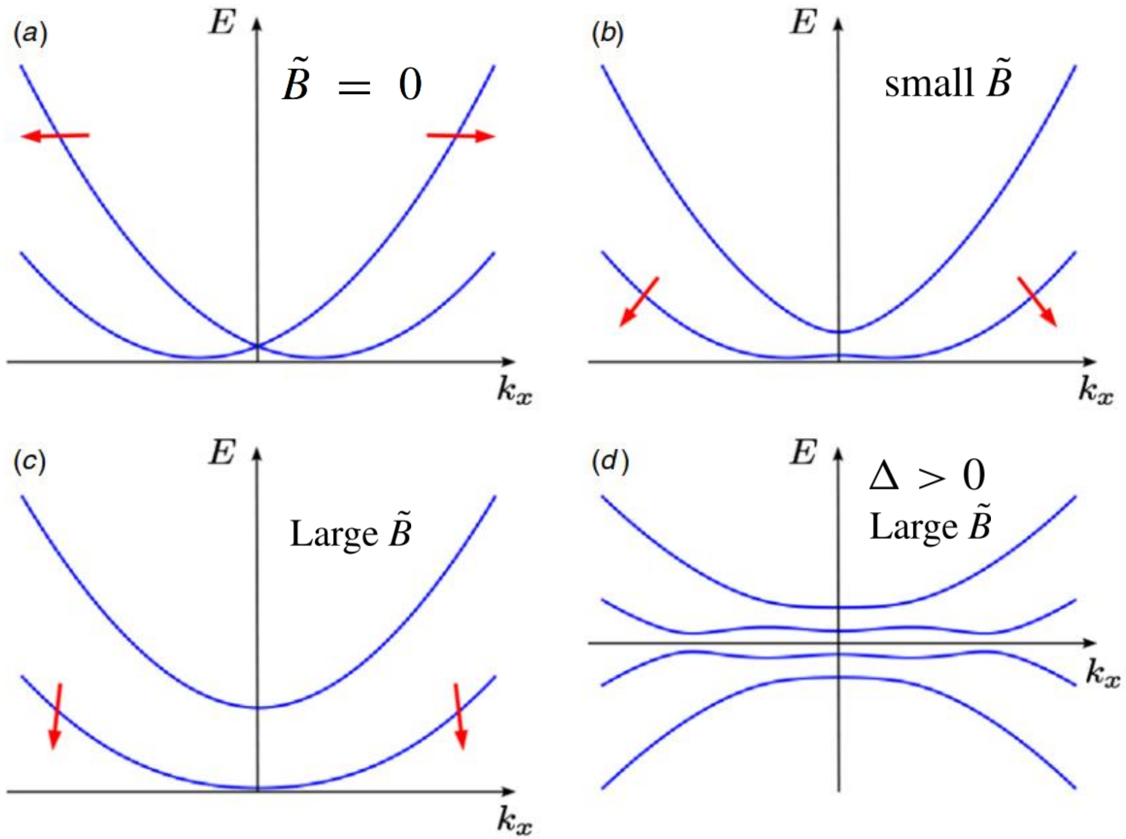
- In our special case

$$H_0(\mathbf{r}) = \frac{\mathbf{p}^2}{2m} - \mu + V(\mathbf{r}) + \alpha\,(\mathbf{E}(\mathbf{r}) \times \mathbf{p}) \cdot \bar{\sigma} + \frac{1}{2} g\mu_B \mathbf{B}(\mathbf{r}) \cdot \bar{\sigma}\,, \longrightarrow H_0(x) = \frac{k_x^2}{2m} - \mu + \tilde{\alpha} k_x \sigma_y + \frac{1}{2}\tilde{B}\sigma_z$$

$$\tilde{\alpha} = \alpha E_\perp$$
$$\tilde{B} = g\mu_B B$$

Proximity induced p-wave like gap in semiconductor

$$H_0(x) = \frac{k_x^2}{2m} - \mu + \tilde{\alpha} k_x \sigma_y + \frac{1}{2}\tilde{B}\sigma_z \longrightarrow E_\pm(k_x) = \frac{k_x^2}{2m} - \mu \pm \sqrt{(\tilde{\alpha} k_x)^2 + \tilde{B}^2}$$



Gap at zero momentum closes if:

$$|\tilde{B}| = \sqrt{\Delta^2 + \mu^2}$$

Topological phase with MF exists, as long:

$$|\tilde{B}| > \sqrt{\Delta^2 + \mu^2}$$

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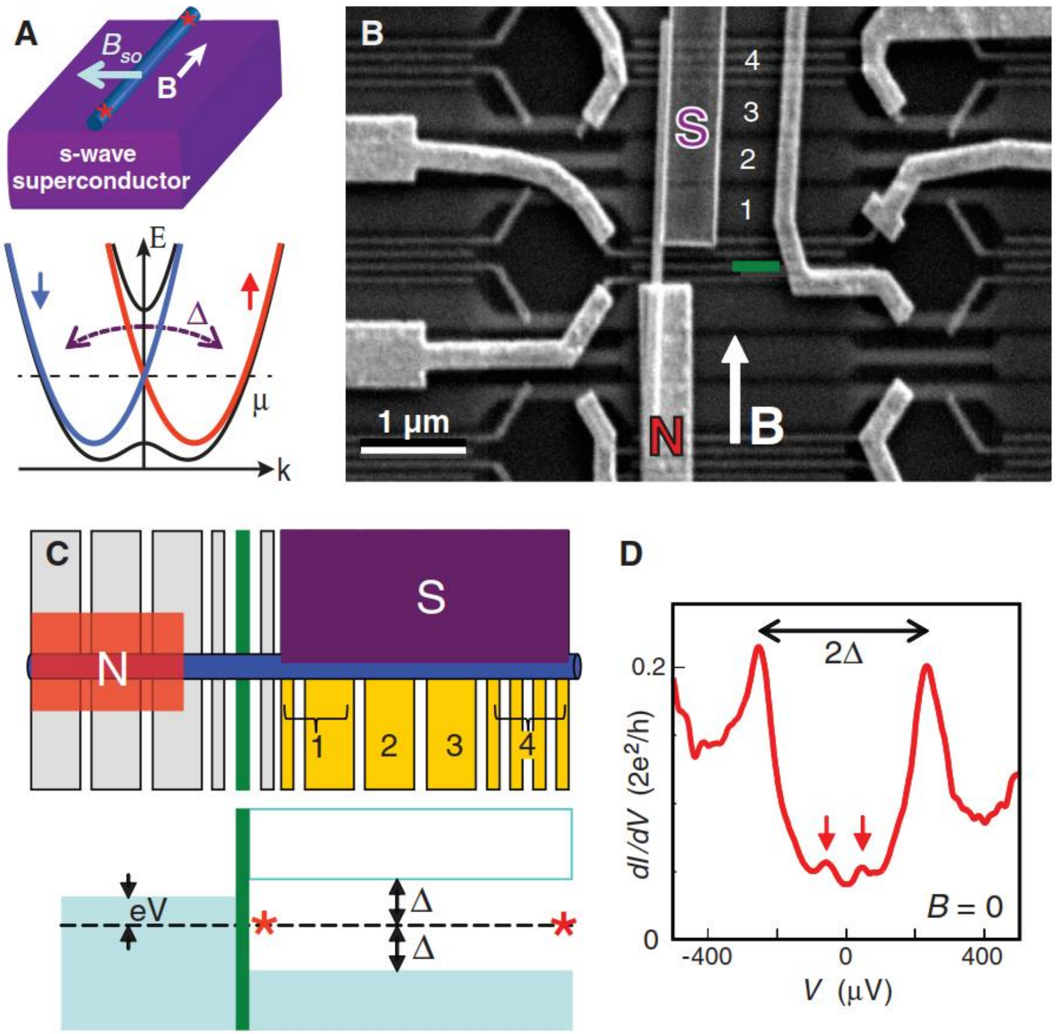
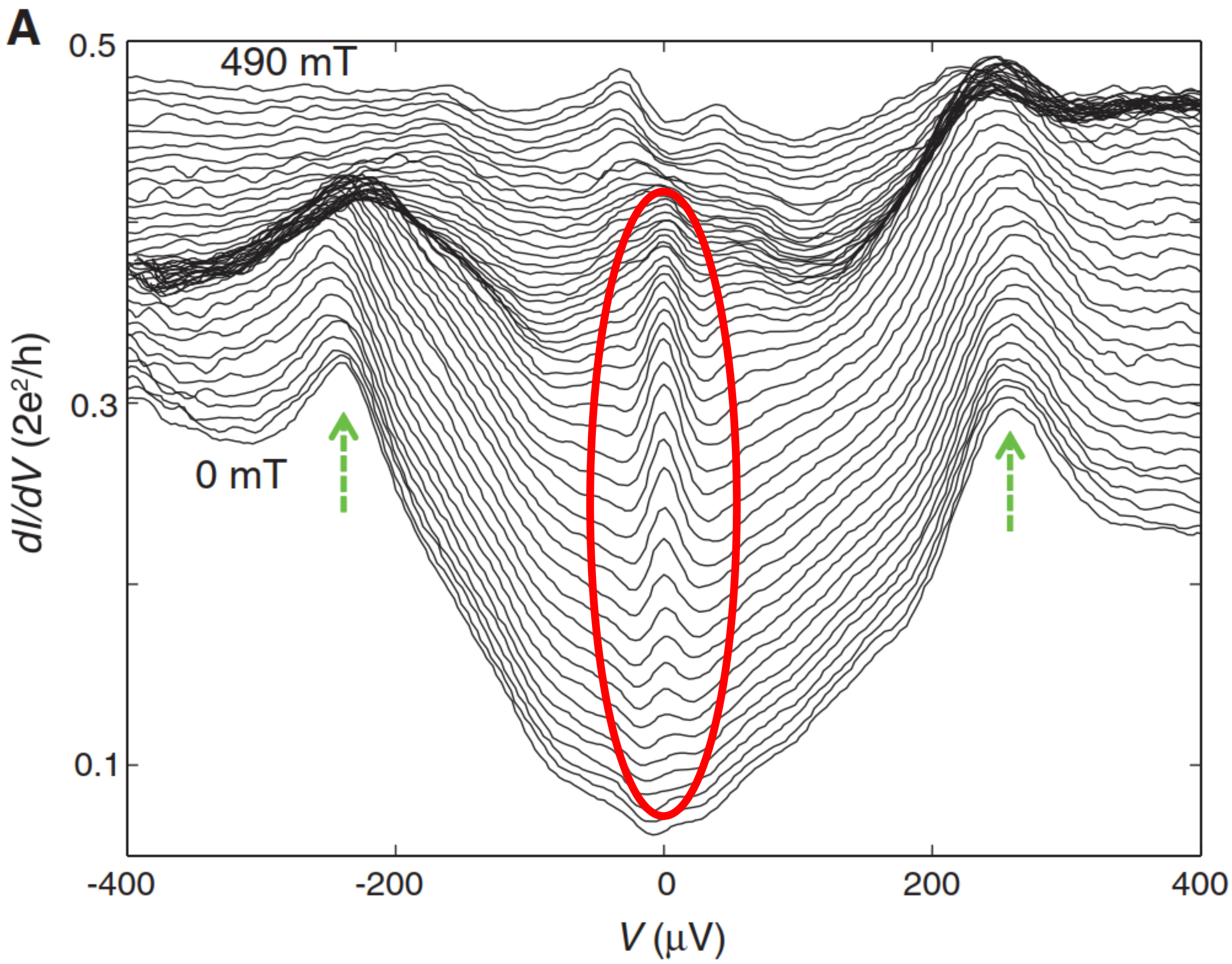
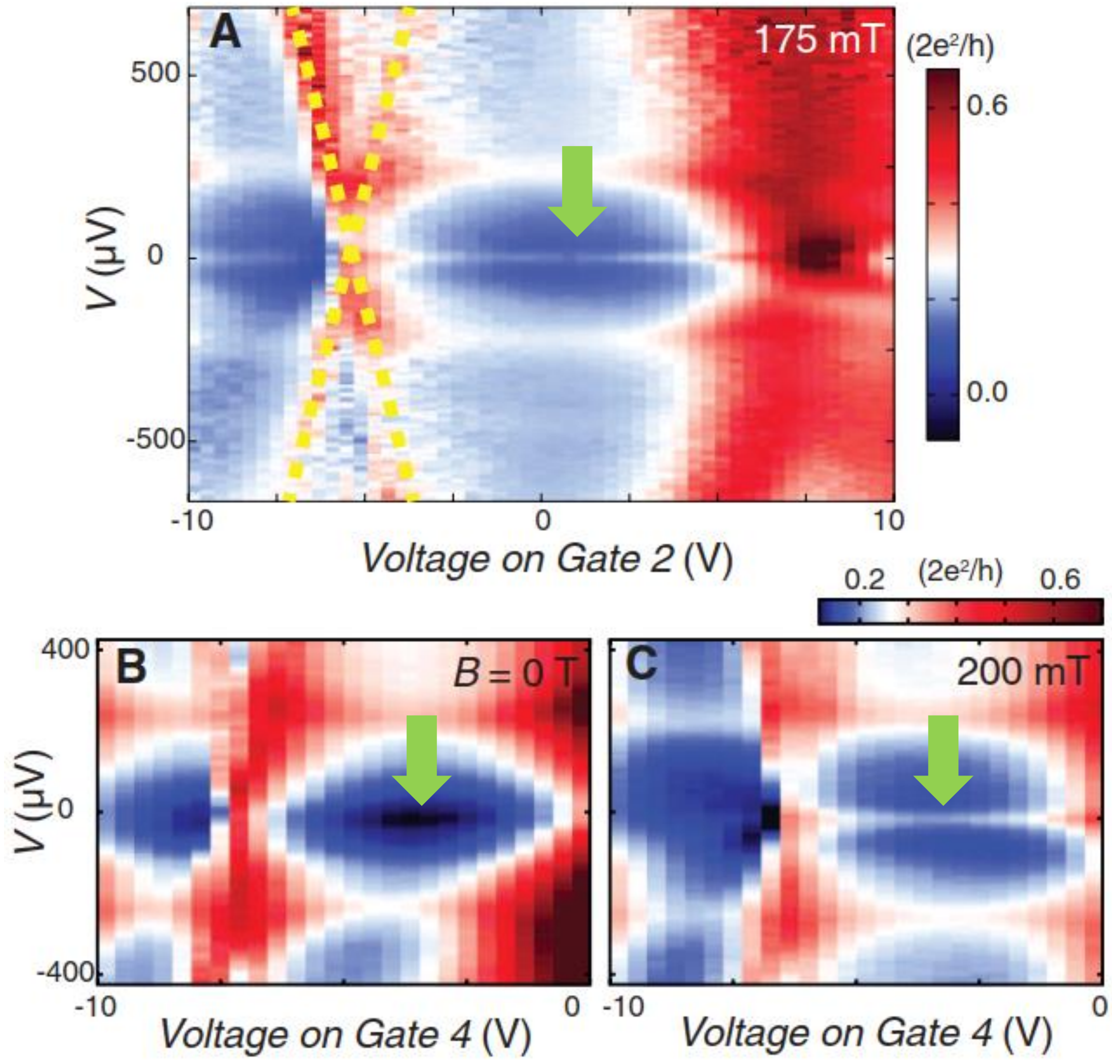
Proximity induced p-wave like gap in semiconductor



Proximity induced p-wave like gap in semiconductor



Proximity induced p-wave like gap in semiconductor



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Quantum computation with Majorana fermions

- Encoding the qubit in fermionic Fock states: $|n_1, n_2, \ldots, n_N\rangle$ with $n_j \in \{0,1\}$
- Dimension is 2^N, each of the N states is occupied or not
- Do we have N qubits?
  - …no, because of superselection rule:
    - Using parity operator: $\mathcal{P} = \prod_k \mathcal{P}_k = (-1)^{\sum_k n_k}$
    - we get for bosonic operator: $\mathcal{P} A \mathcal{P} = A$
    - due to even number of fermionic operators in A and $\mathcal{P} c_j \mathcal{P} = -c_j$
    - For two states with different parity $\mathcal{P}|\psi_\pm\rangle = \pm|\psi_\pm\rangle$ follows:

$$\langle\psi_-|A|\psi_+\rangle = \langle\psi_-|\mathcal{P}A\mathcal{P}|\psi_+\rangle = -\langle\psi_-|A|\psi_+\rangle = 0$$

➢ Out of 2^N states in fermionic Fock space only 2^{N-1} can be used for computation

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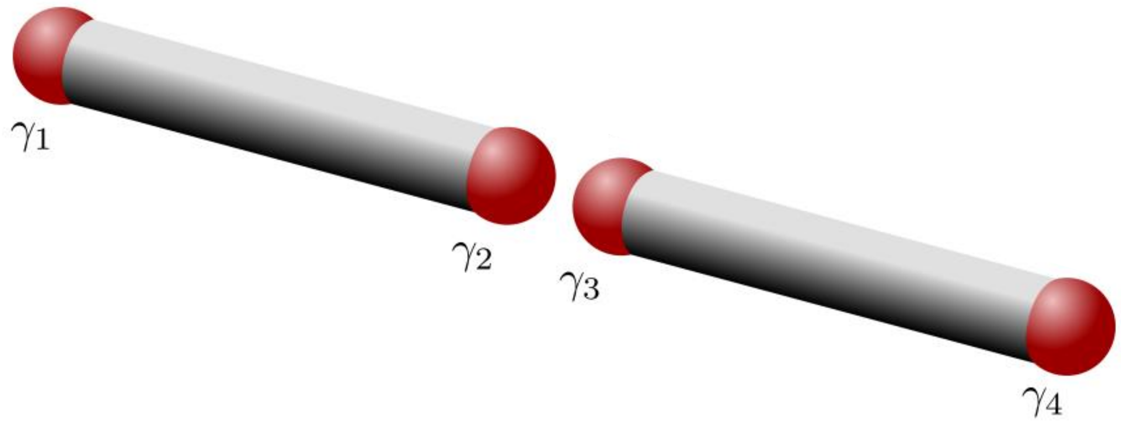
Quantum computation with Majorana fermions

- Therefore we encode one qubit as:

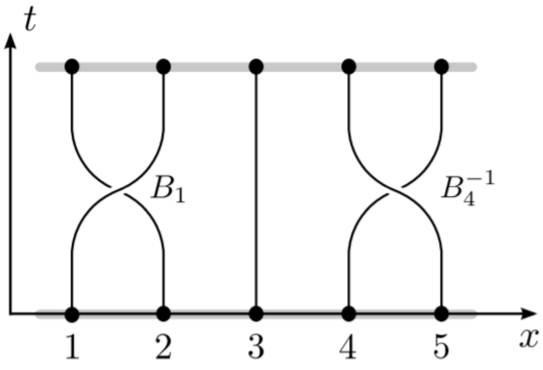
$$|\bar{0}\rangle = |00\rangle$$
$$|\bar{1}\rangle = |11\rangle$$



- ➢ "topological protected" due to delocalized states
- ➢ Environment should not provide single unpaired electrons

Quantum computation with Majorana fermions

- Idea: Operating gates on the qubit with non-Abelian exchange statists
- What means braiding?
  - Particle exchange in space-time



- Elements of the Braid group $\mathcal{B}_N$ for N particles fulfill:

$$B_k B_l = B_l B_k, \quad |k-l| \geq 2$$
$$B_k B_{k+1} B_k = B_{k+1} B_k B_{k+1}$$

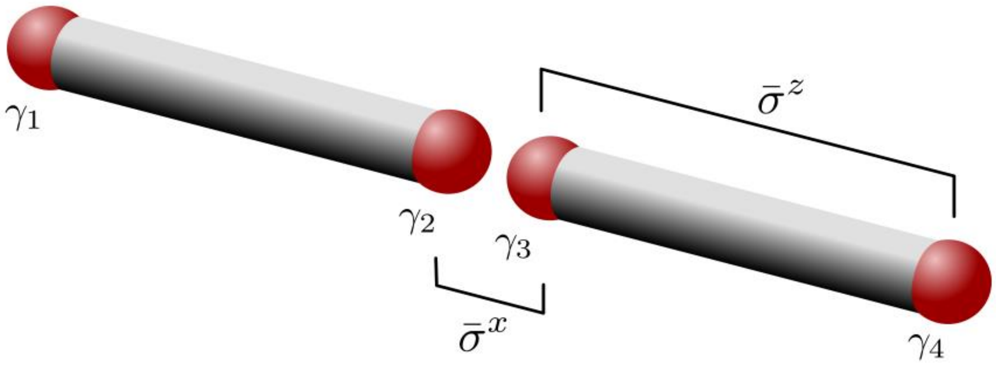
with 1D representation $e^{i\theta}$ *acting on wave function*

Quantum computation with Majorana fermions

- One can show, that these operators are representations of the Braid group:

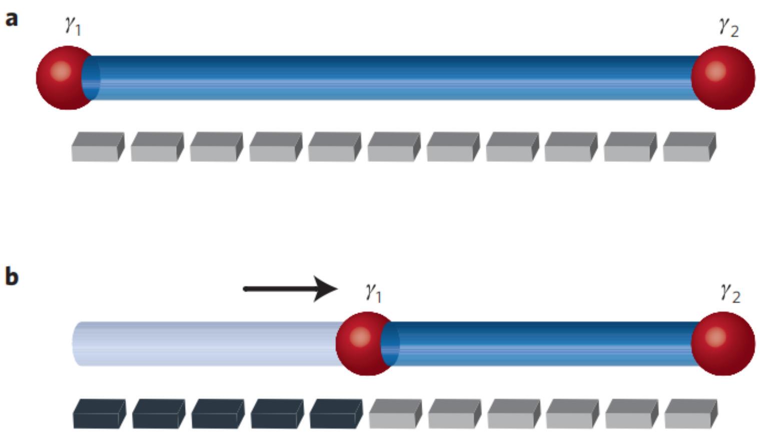
$$U_1 = U_3 = e^{\pi\gamma_1\gamma_2/4} = e^{i\pi\bar{\sigma}^z/4}$$
$$U_2 = e^{\pi\gamma_2\gamma_3/4} = e^{i\pi\bar{\sigma}^x/4}$$

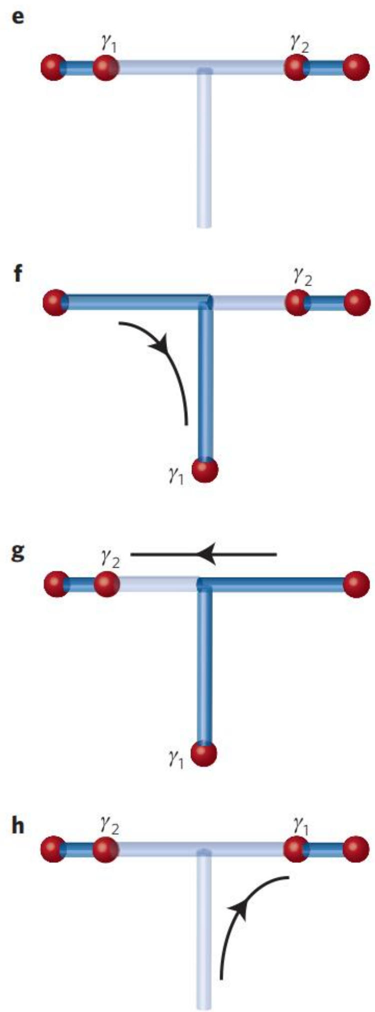
  - 2D representations and do not commute



Quantum computation with Majorana fermions

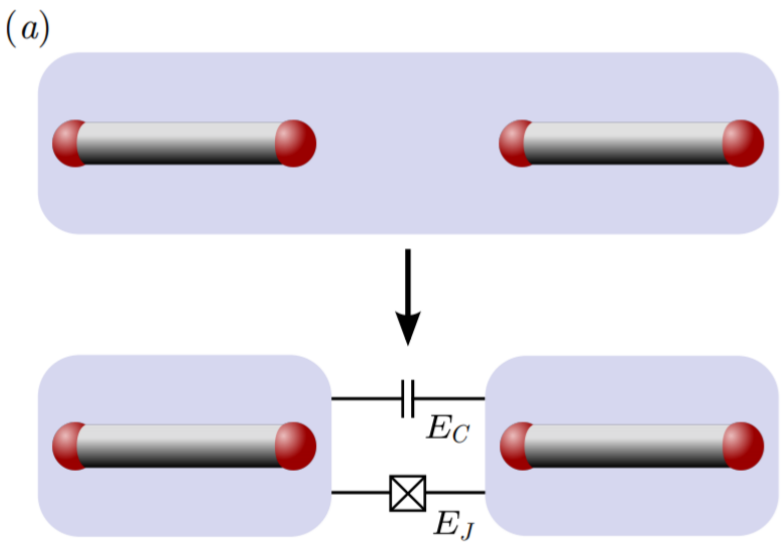
- How to realize adiabatic braiding with our nanowire?





Quantum computation with Majorana fermions

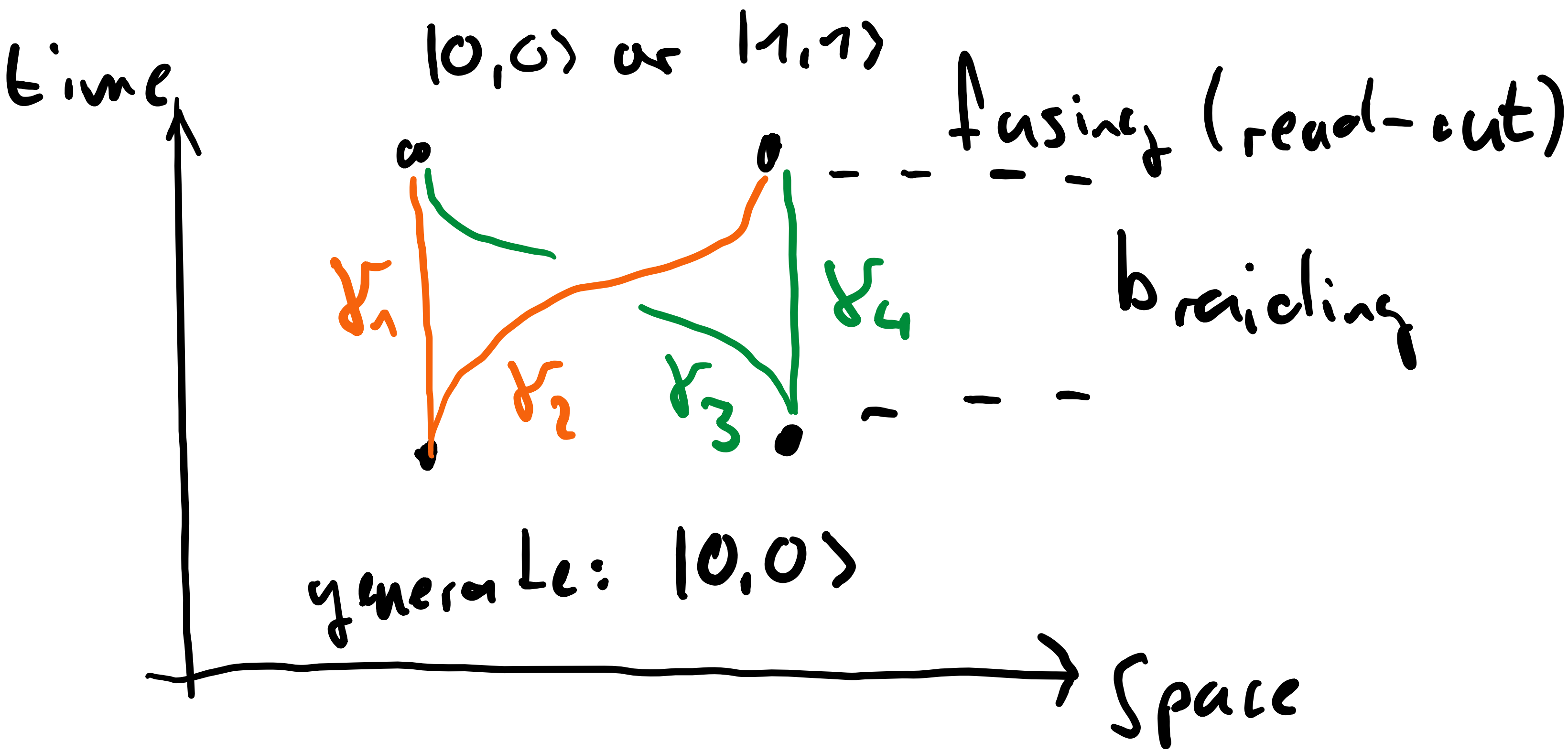
- For the read-out we fuse both pairs of MF by overlapping their wavefunctions
  - First we remove the topological protection:



  - Afterwards we perform any measurement which depends on the different energy of the qubit states

Quantum computation with Majorana fermions

- Quick review of quantum computation with MF:



TOPOLOGICAL QUANTUM COMPUTATION

A Majorana mass production line

A scalable manufacturing process for complex, high-quality superconductor/topological insulator structures could, in future, enable the production of topological quantum computation architectures.

Erwann Bocquillon

The challenge of taming decoherence impedes the advent of quantum computers: because of interactions with their environment, large quantum systems often behave more like classical objects than like quantum objects. Topological quantum computation overcomes this limitation when storing information in delocalized qubits. Theory predicts that such qubits are insensitive to local perturbations and hence can preserve coherence over extended periods. Protocols for topological quantum computation take advantage of non-local excitations called Majorana (quasi)particles. These particles emerge as, at the same time, electron- and hole-like excitations and can be considered their own antiparticle. 'Braiding' Majorana particles — that is, moving them around one another — modifies the phase of the particles, which can in turn be used to realize quantum logical operations. However, no topological qubit has yet been demonstrated, as it is extremely difficult to produce Majorana excitations. The most natural recipe is to engineer them from superconductors and topological

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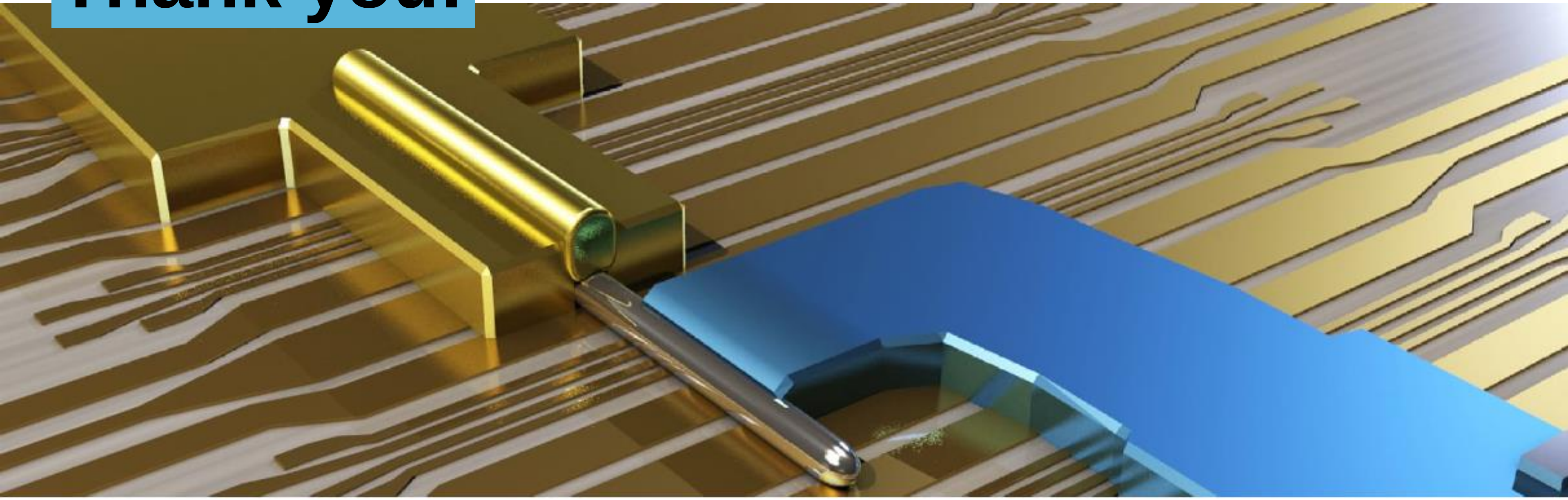
Thank you!



Quelle: https://www.spektrum.de/news/exotische-teilchen-im-nanodraht/1151134

Appendix

$$\mathcal{H}_{\text{chain}} = -\mu \sum_{i=1}^{N} n_i - \sum_{i=1}^{N-1} (t c_i^\dagger c_{i+1} + \Delta c_i c_{i+1} + \text{h.c.})$$

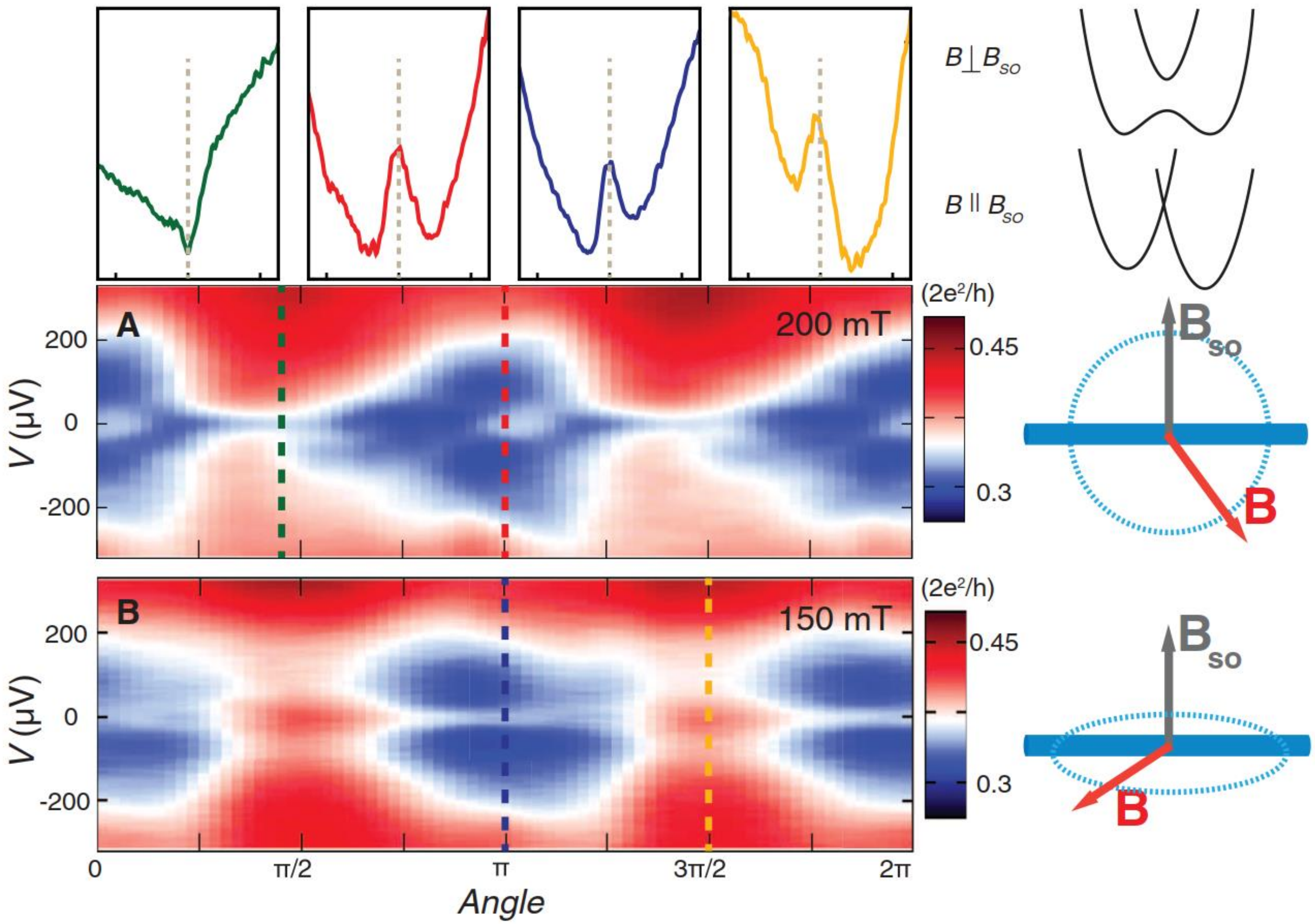
↓

$$\mathcal{H}_{1D}^{\text{pw}} = \int \mathrm{d}x \left[\Psi^\dagger(x) \left(\frac{p_x^2}{2m} - \mu \right) \Psi(x) + \Psi(x) |\Delta| \mathrm{e}^{\mathrm{i}\phi} p_x \Psi(x) + \text{h.c.} \right],$$

↓

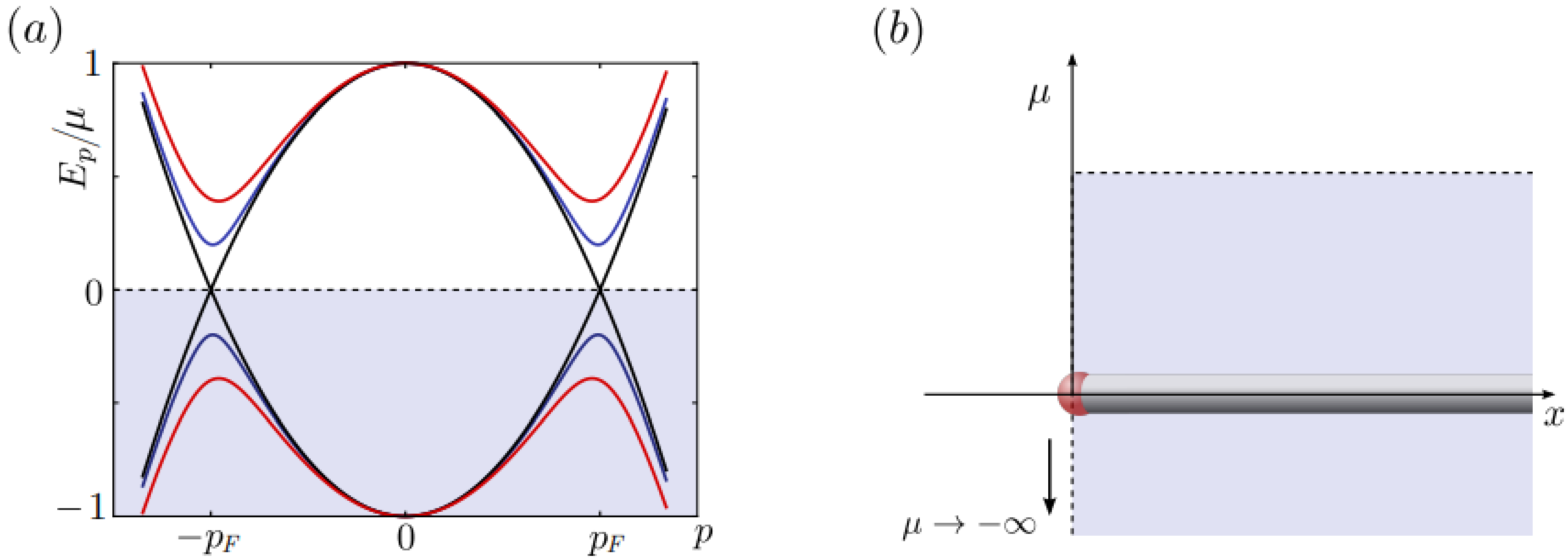
$$\mathcal{H}_{\text{eff}} = \int \mathrm{d}x \left[\Psi_-^\dagger(x) \left(\frac{k_x^2}{2m} - \frac{1}{2}\tilde{B} \right) \Psi_-(x) + \mathrm{i} \frac{\tilde{\alpha}}{\tilde{B}} \Delta \Psi_-(x) k_x \Psi_-(x) \right].$$

Appendix



B ⊥ B_so
B ∥ B_so
(2e²/h)
A
200 mT
0.45
0.3
B_so
B
V (μV)
200
0
-200
(2e²/h)
B
150 mT
0.45
0.3
B_so
B
V (μV)
200
0
-200
0
π/2
π
3π/2
2π
Angle

Appendix



Appendix