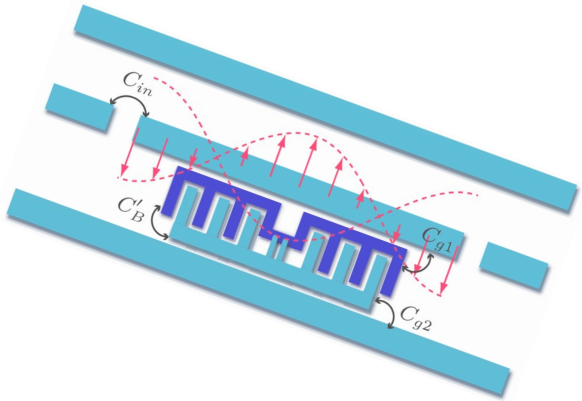
Transmon and Fluxonium Qubit



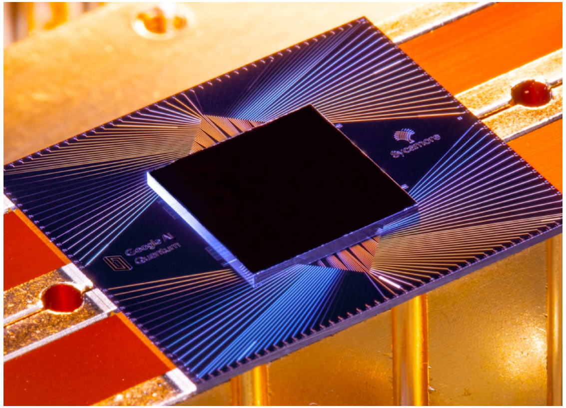
Emanuel Hubenschmid

Seminar: Superconducting quantum hardware for quantum computing

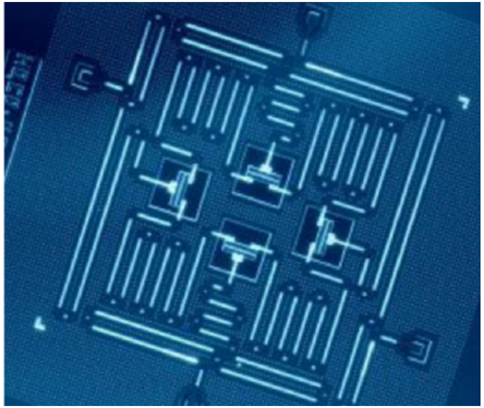
19.06.2020

What is so special?

Google (53 qubits processor Sycamore)



IBM (4 qubit prozessor)



F. Arute et al. “Quantum supremacy using a programmable superconducting processor”. *Nature* **574**.**7779** (Oct. 2019), 505.
J. M. Gambetta et al. “Building logical qubits in a superconducting quantum computing system”. *npj Quantum Information* **3**.**1** (Jan. 2017).

Contents

1 Introduction

1.1 The DiVincenzo criteria

The DiVincenzo criteria

1. Scalable system with well characterized qubits $\mathcal{H} = \mathcal{H}_{\text{qubit}} \otimes \mathcal{H}_{\text{r}}$

D. P. DiVincenzo. "The Physical Implementation of Quantum Computation". *Fortschritte der Physik* **48.9-11** (Sept. 2000), 771.

The DiVincenzo criteria

1. Scalable system with well characterized qubits $\mathcal{H} = \mathcal{H}_{\text{qubit}} \otimes \mathcal{H}_{\text{r}}$
2. Initializability of the qubits $|\Psi\rangle \in \mathcal{H}_{\text{qubit}}^{\otimes n} \to \otimes_{i=0}^{n} |0\rangle$

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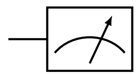
4. A "universal" set of quantum gates $-\boxed{H}-$, ...
5. A qubit-specific measurement capability

0

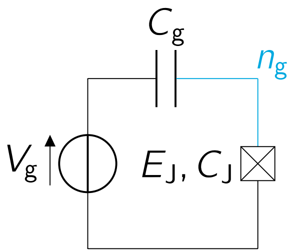
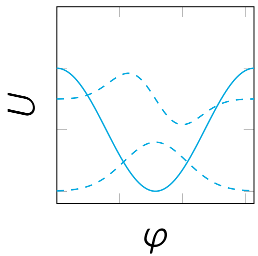
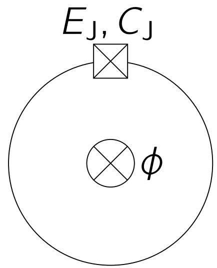
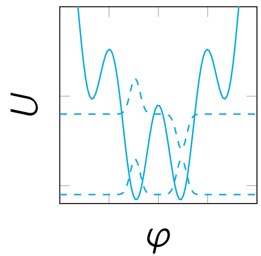


D. P. DiVincenzo. "The Physical Implementation of Quantum Computation". *Fortschritte der Physik* **48.9-11** (Sept. 2000), 771.

1 Introduction

1.2 Some familiar qubits

Comparison of qubits

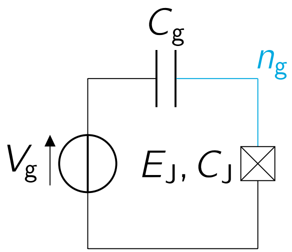
Charge qubit	Flux qubit
 	 

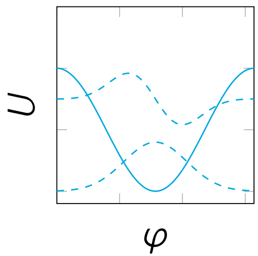
G. Catelani et al. "Relaxation and frequency shifts induced by quasiparticles in superconducting qubits". *Physical Review B* **84**.6 (Aug. 2011).

Comparison of qubits

Charge qubit	Flux qubit

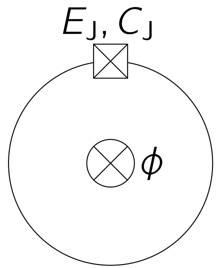
Charge qubit

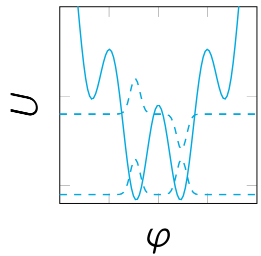




$$E_L = 0$$

Flux qubit





$$E_L \ll E_J,\ n_g = 0$$

$$\hat{H} = 4E_C(\hat{n} - n_g)^2 - E_J \cos\hat{\varphi} + \frac{1}{2}E_L(\hat{\varphi} - 2\pi\phi/\phi_0)^2$$

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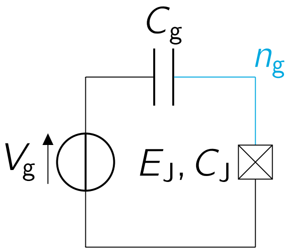
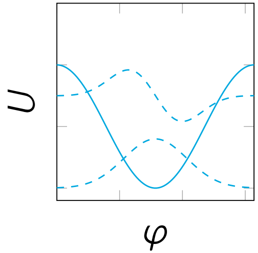
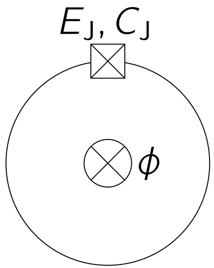
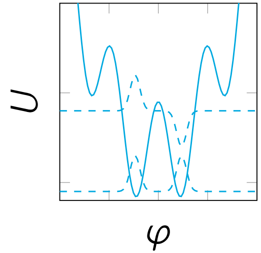
Comparison of qubits

Charge qubit	Flux qubit
 	 
$E_\mathrm{L} = 0$	$E_\mathrm{L} \ll E_\mathrm{J},\ n_\mathrm{g} = 0$
$E_\mathrm{J}/E_\mathrm{C} \sim 1$	$E_\mathrm{J}/E_\mathrm{C} \sim 10^2 - 10^3$

$$\hat{H} = 4E_\mathrm{C}(\hat{n} - n_\mathrm{g})^2 - E_\mathrm{J}\cos\hat{\varphi} + \frac{1}{2}E_\mathrm{L}(\hat{\varphi} - 2\pi\phi/\phi_0)^2$$

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Comparison of qubits

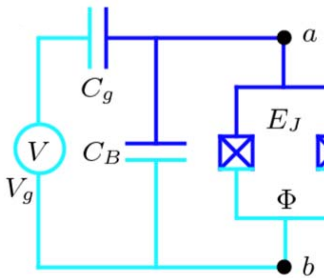
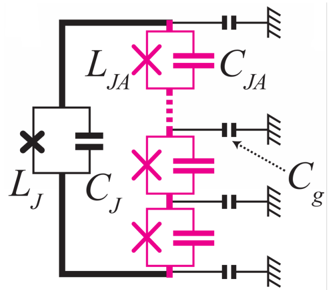
Problem: Single Cooper pair circuits decohere due to low frequency offset charge noise

J. Koch et al. "Charge-insensitive qubit design derived from the Cooper pair box". *Physical Review A* **76**.**4** (Oct. 2007).
V. E. Manucharyan et al. "Fluxonium: Single Cooper-Pair Circuit Free of Charge Offsets". *Science* **326**.**5949** (Oct. 2009), 113.

Comparison of qubits

Problem: Single Cooper pair circuits decohere due to low frequency offset charge noise

Solution:

Transmon qubit	Fluxonium qubit
Increase E_J/E_C with large shunting capacitance	Small junction shunted by an JJ array
	
$E_J/E_C \sim 100$	$E_J/E_C \sim 1-10$

J. Koch et al. "Charge-insensitive qubit design derived from the Cooper pair box". *Physical Review A* **76.4** (Oct. 2007).
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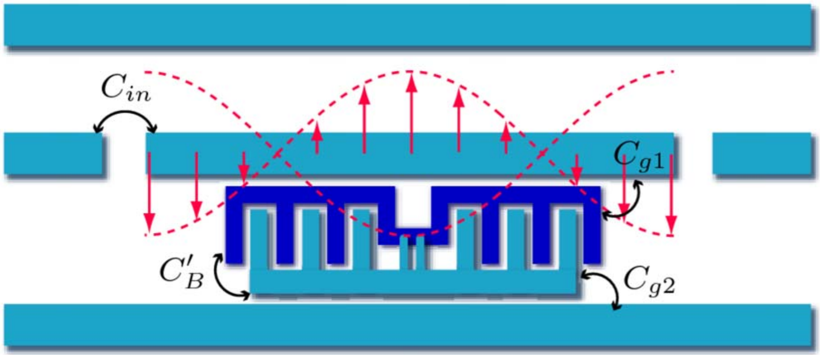
2 The transmon qubit

2.1 Characterization of the qubit

The device



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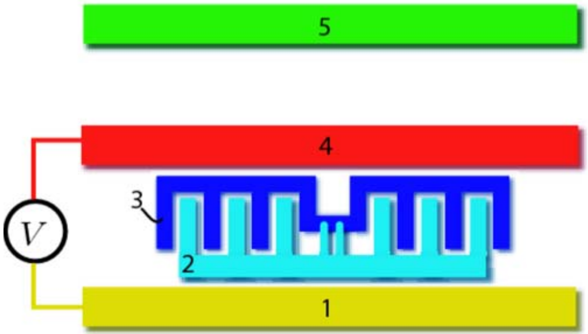
The capacitance network



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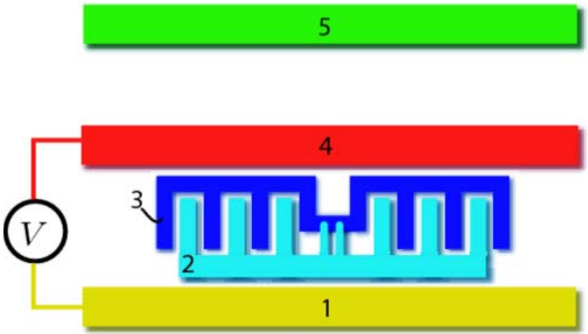

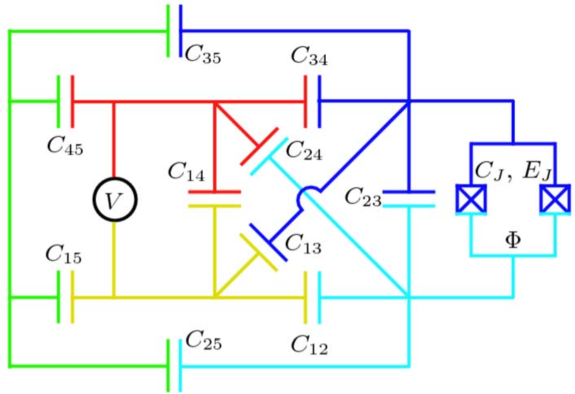

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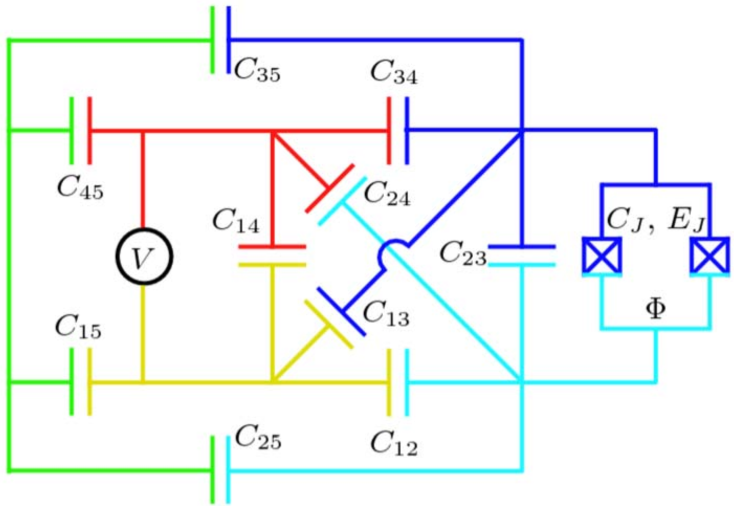

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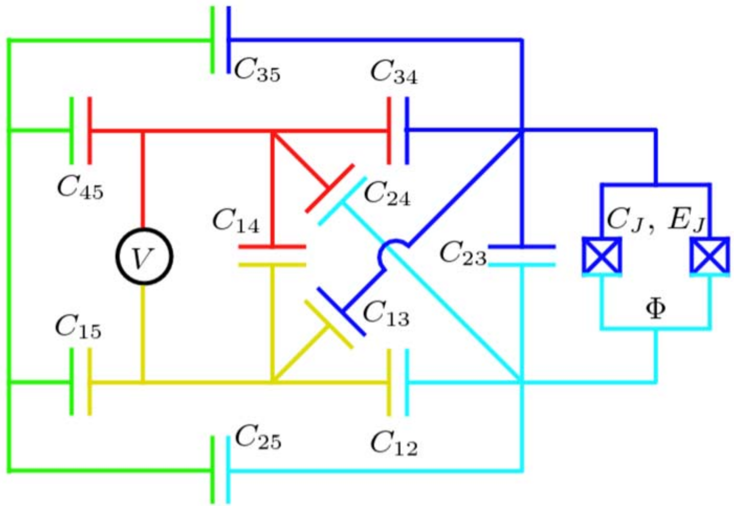
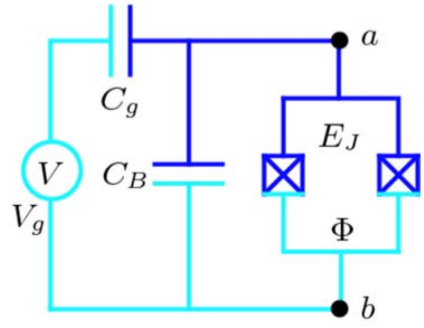


Thévenin-Theorem
$\longrightarrow$



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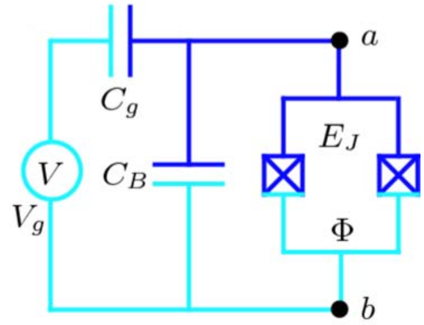
The capacitance network

$$\hat{H} = 4E_\mathrm{C}(\hat{n} - n_\mathrm{g})^2 - E_\mathrm{J}\cos\hat{\varphi}$$



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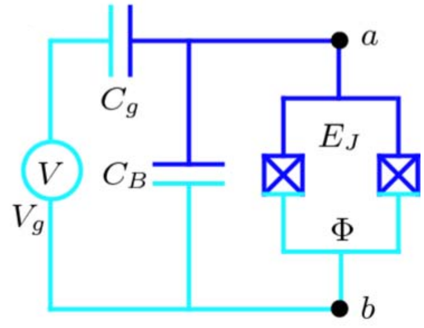
Charging energy:
$E_{\mathrm{C}} = e^2/2C_{\Sigma}$
$C_{\Sigma} = C_{\mathrm{J}} + C_{\mathrm{B}} + C_{\mathrm{g}}$



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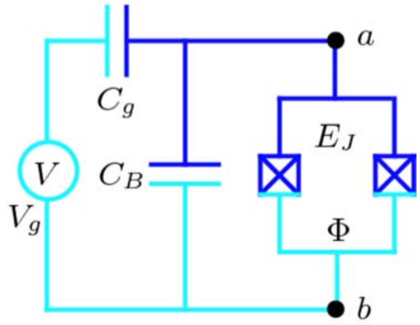
$C_\Sigma = C_\mathrm{J} + C_\mathrm{B} + C_\mathrm{g}$

Josephson energy:

$E_\mathrm{J} = E_\mathrm{J,max}\,|\cos(\pi\Phi/\Phi_0)|$



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The capacitance network

Effective offset charge:

$n_{\mathrm{g}} = Q_{\mathrm{r}}/2e + C_{\mathrm{g}}V_{\mathrm{g}}/2e$

$$\hat{H} = 4E_{\mathrm{C}}(\hat{n} - n_{\mathrm{g}})^2 - E_{\mathrm{J}} \cos\hat{\varphi}$$

Charging energy:

$E_{\mathrm{C}} = e^2/2C_{\Sigma}$

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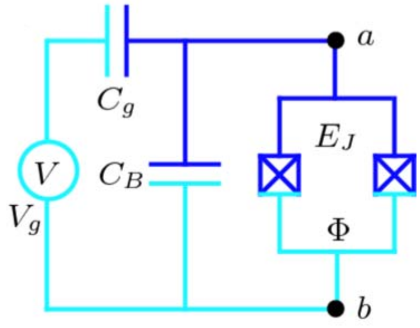
$E_{\mathrm{J}} = E_{\mathrm{J,max}} \left|\cos(\pi\Phi/\Phi_0)\right|$



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Eigenenergies

$$E_m(n_\mathrm{g})\Psi(\varphi) = \left(4E_\mathrm{C}\left(\frac{\mathrm{d}}{\mathrm{d}\varphi} - n_\mathrm{g}\right)^2 - E_\mathrm{J}\cos\varphi\right)\Psi(\varphi)$$

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Eigenenergies

$$\Psi(\varphi + 2\pi) = \Psi(\varphi)$$

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Eigenenergies

$$\Psi(\varphi + 2\pi) = \Psi(\varphi) \Rightarrow \Psi(\varphi) = e^{i n_g \varphi} g(\varphi/2)$$

$$\downarrow$$

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$$\Rightarrow g''(x) + \left(\frac{E}{E_\mathrm{C}} + \frac{E_\mathrm{J}}{E_\mathrm{C}}\cos(2x)\right) g(x) = 0$$

$$\varphi/2$$

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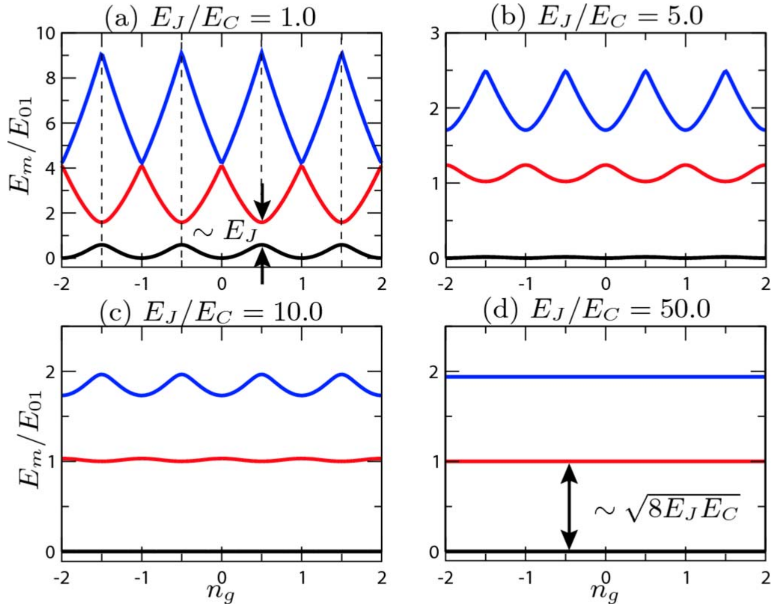

J. Koch et al. "Charge-insensitive qubit design derived from the Cooper pair box". *Physical Review A* **76.4** (Oct. 2007).

Anharmonicity vs. charge dispersion

Peak to peak charge dispersion:

$$\epsilon = E_m(n_{\mathrm{g}} = 1/2) - E_m(n_{\mathrm{g}} = 0)$$

J. Koch et al. "Charge-insensitive qubit design derived from the Cooper pair box". *Physical Review A* **76**.4 (Oct. 2007).

Anharmonicity vs. charge dispersion

Peak to peak charge dispersion:

$$\epsilon = E_m(n_\mathrm{g} = 1/2) - E_m(n_\mathrm{g} = 0) \simeq$$
$$\simeq (-1)^m E_\mathrm{C} \frac{2^{4m+5}}{m!} \sqrt{\frac{2}{\pi}} \left(\frac{E_\mathrm{J}}{2E_\mathrm{C}}\right)^{m/2+3/4} e^{-\sqrt{8E_\mathrm{J}/E_\mathrm{C}}}$$

J. Koch et al. "Charge-insensitive qubit design derived from the Cooper pair box". *Physical Review A* **76**.4 (Oct. 2007).

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Relative anharmonicity (charge degeneracy $n_\mathrm{g} = 1/2$):

$$\alpha_\mathrm{r} = \frac{E_{12} - E_{01}}{E_{01}}$$

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Relative anharmonicity (charge degeneracy $n_\mathrm{g} = 1/2$):

$$\alpha_\mathrm{r} = \frac{E_{12} - E_{01}}{E_{01}} \simeq -(8E_\mathrm{J}/E_\mathrm{C})^{-1/2}$$

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Anharmonicity vs. charge dispersion

Peak to peak charge dispersion:

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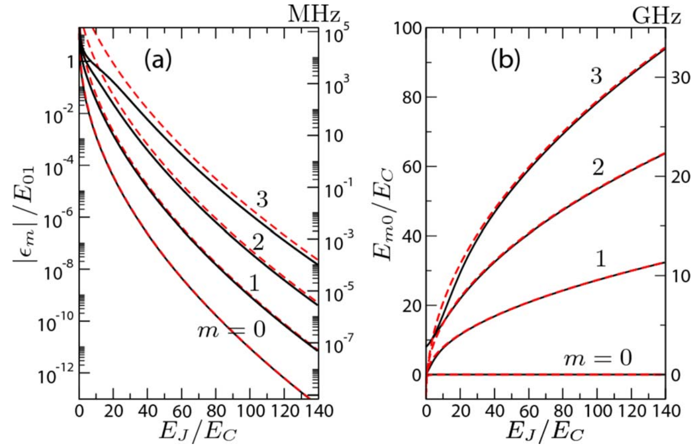
Relative anharmonicity (charge degeneracy $n_g = 1/2$):

$$\alpha_r = \frac{E_{12} - E_{01}}{E_{01}} \simeq -(8E_J/E_C)^{-1/2}$$

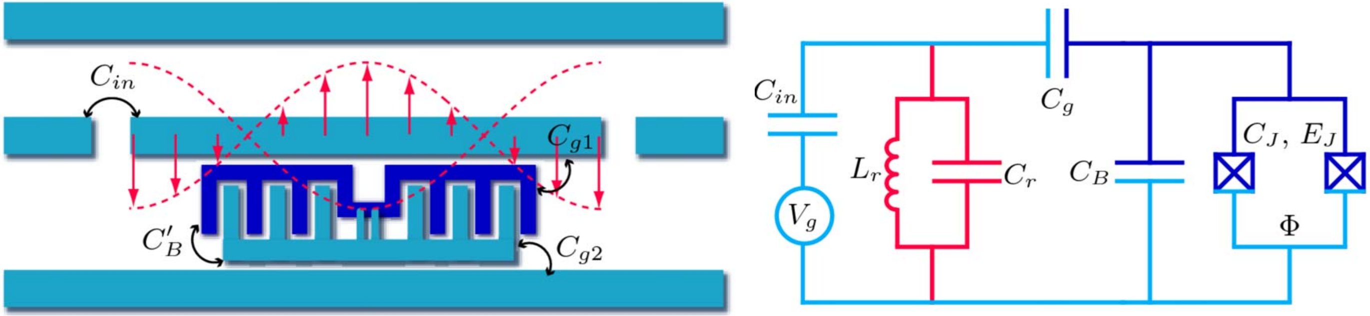


J. Koch et al. "Charge-insensitive qubit design derived from the Cooper pair box". *Physical Review A* **76**.4 (Oct. 2007).

2 The transmon qubit

2.2 Initialization, control and read-out

Circuit QED for the transmon



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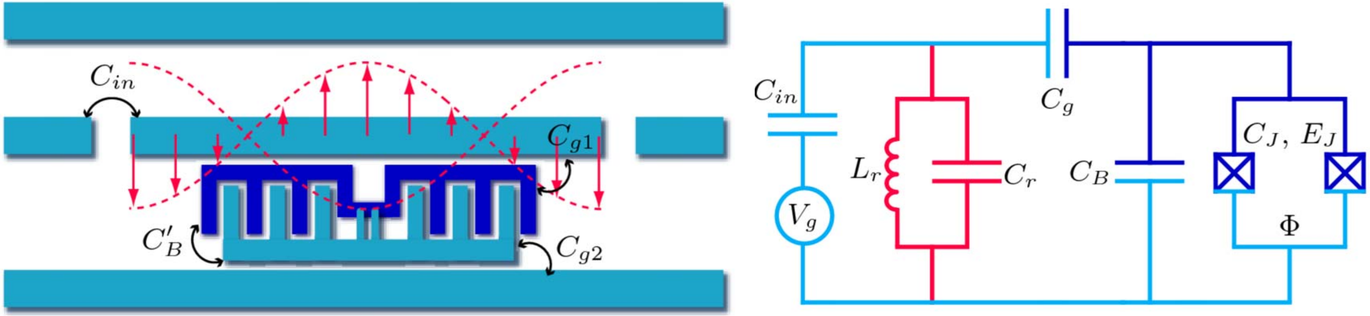
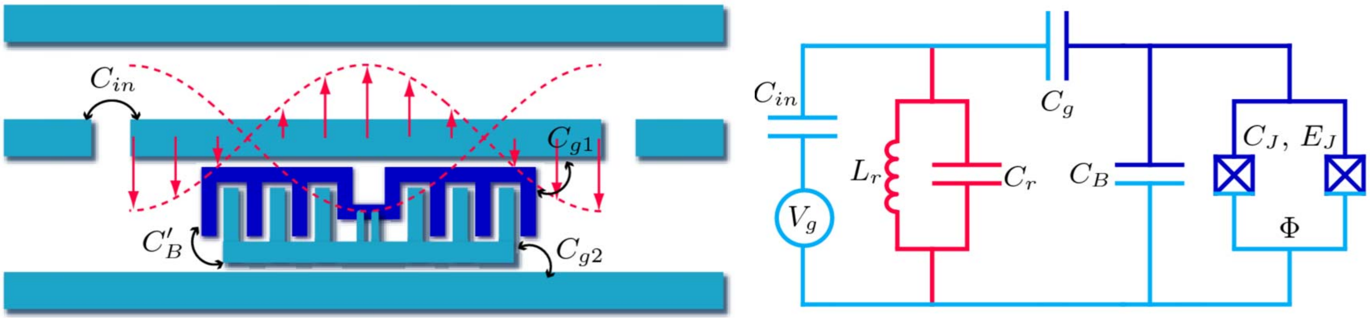
Circuit QED for the transmon



$$\hat{H} = 4E_\mathrm{C}(\hat{n} - n_\mathrm{g})^2 - E_\mathrm{J}\cos\hat{\varphi} + \hbar\omega_\mathrm{r}\hat{a}^\dagger\hat{a} + 2\beta e\hat{v}\hat{n}$$

J. Koch et al. "Charge-insensitive qubit design derived from the Cooper pair box". *Physical Review A* **76**.4 (Oct. 2007).

Circuit QED for the transmon



$$\hat{H} = 4E_\mathrm{C}(\hat{n} - n_\mathrm{g})^2 - E_\mathrm{J}\cos\hat{\varphi} + \hbar\omega_\mathrm{r}\hat{a}^\dagger\hat{a} + 2\beta e\hat{v}\hat{n}$$

"$V_\mathrm{g} \rightarrow V_\mathrm{g} + v$"

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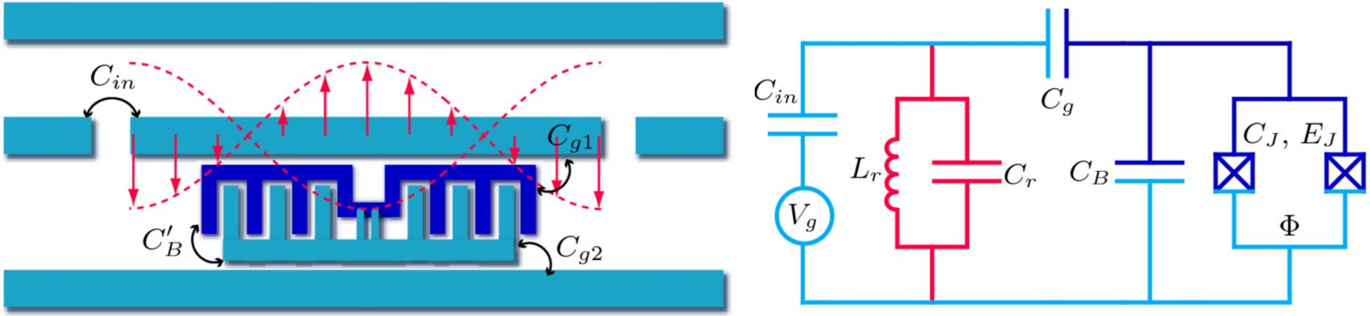
Circuit QED for the transmon



Resonator frequency

$$\hat{H} = 4E_C(\hat{n} - n_g)^2 - E_J \cos\hat{\varphi} + \hbar\omega_r \hat{a}^\dagger \hat{a} + 2\beta e \hat{v}\hat{n}$$

$"V_g \rightarrow V_g + v"$

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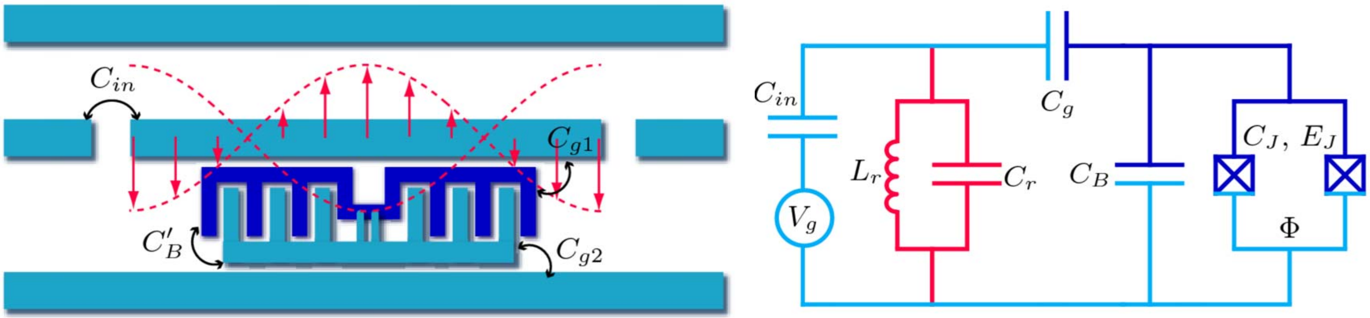

Resonator frequency

C_g/C_Σ

$$\hat{H} = 4E_\mathrm{C}(\hat{n} - n_\mathrm{g})^2 - E_\mathrm{J}\cos\hat{\varphi} + \hbar\omega_\mathrm{r}\hat{a}^\dagger\hat{a} + 2\beta e\hat{v}\hat{n}$$

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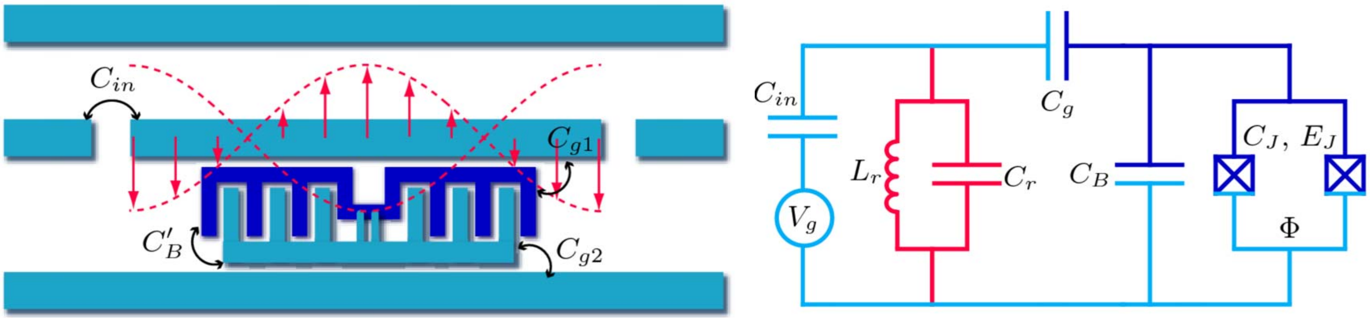

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"$V_\mathrm{g} \rightarrow V_\mathrm{g} + v$"

$V^0_\mathrm{rms}(\hat{a} + \hat{a}^\dagger)$

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Circuit QED for the transmon

$$\hat{H} = 4E_\mathrm{C}(\hat{n} - n_\mathrm{g})^2 - E_\mathrm{J}\cos\hat{\varphi} + \hbar\omega_\mathrm{r}\hat{a}^\dagger\hat{a} + 2\beta eV^0_\mathrm{rms}\hat{n}(\hat{a} + \hat{a}^\dagger)$$

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Circuit QED for the transmon

$\sum_j |j\rangle\langle j| = \mathbb{1}, \; |j\rangle$: Transmon eigenvectors

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$$\hat{H} = \hbar\sum_j \omega_j |j\rangle\langle j| + \hbar\omega_r\hat{a}^\dagger\hat{a} + \sum_{i,j}\hbar g_{i,j}|i\rangle\langle j|(\hat{a} + \hat{a}^\dagger)$$

$2\beta eV^0_\mathrm{rms}\langle i|\hat{n}|j\rangle \to 0\ \forall i > j+1$

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$$\hat{H} \approx \hbar\sum_j \omega_j |j\rangle\langle j| + \hbar\omega_r\hat{a}^\dagger\hat{a} + \left(\hbar\sum_i g_{i,i+1}\left(|i\rangle\langle i+1| + |i+1\rangle\langle i|\right)\hat{a}^\dagger + \mathrm{h.c.}\right)$$

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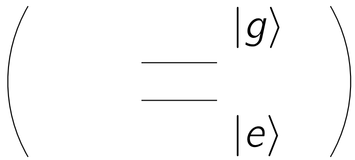
Jaynes Cummings model



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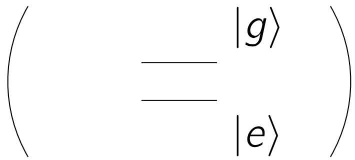
$\propto (E_\mathrm{J}/E_\mathrm{C})^{1/4}$

Jaynes Cummings model



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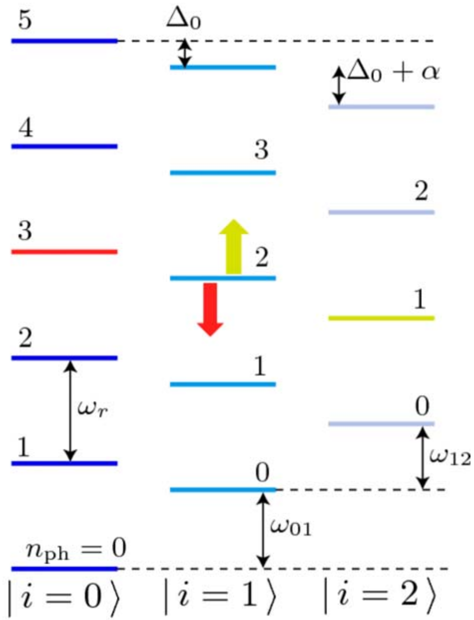
The dispersive limit

$$\hat{D}\hat{H}\hat{D}^{\dagger} \approx \frac{\hbar\omega'_{01}}{2}\hat{\sigma}_z + (\hbar\omega'_r + \hbar\chi\hat{\sigma}_z)\hat{a}^{\dagger}\hat{a} + \mathcal{O}\left(\frac{g_{ij}^2}{\Delta_i^2}\right)$$



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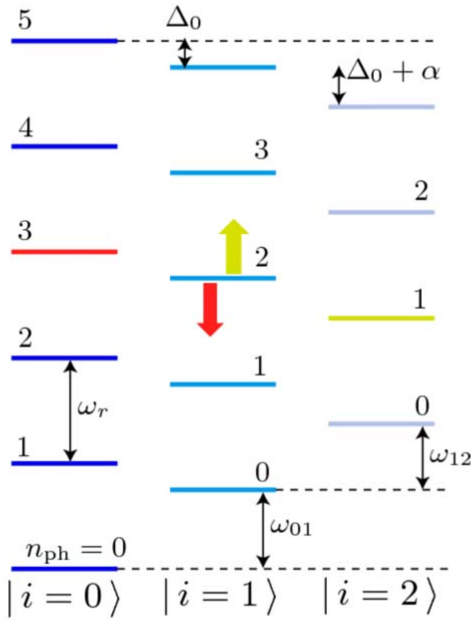
$$\hat{D}\hat{H}\hat{D}^{\dagger} \approx \frac{\hbar\omega_{01}^{'}}{2}\hat{\sigma}_z + (\hbar\omega_r^{'} + \hbar\chi\hat{\sigma}_z)\hat{a}^{\dagger}\hat{a} + \mathcal{O}\left(\frac{g_{ij}^2}{\Delta_i^2}\right)$$

$$\uparrow$$

$\hat{D} = \exp\left(\hat{S} - \hat{S}^{\dagger}\right);\ \hat{S} = \sum_i \frac{g_{i,i+1}}{\Delta_i}\hat{a}\,|i+1\rangle\,\langle i|$



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The dispersive limit

Effective desperive shift

$$\chi = \chi_{01} - \chi_{12}/2,\ \chi_{ij} = \frac{g_{ij}^2}{\omega_{ij-\omega_r}}$$

$$\hat{D}\hat{H}\hat{D}^\dagger \approx \frac{\hbar\omega_{01}'}{2}\hat{\sigma}_z + (\hbar\omega_r' + \hbar\chi\hat{\sigma}_z)\hat{a}^\dagger\hat{a} + \mathcal{O}\left(\frac{g_{ij}^2}{\Delta_i^2}\right)$$

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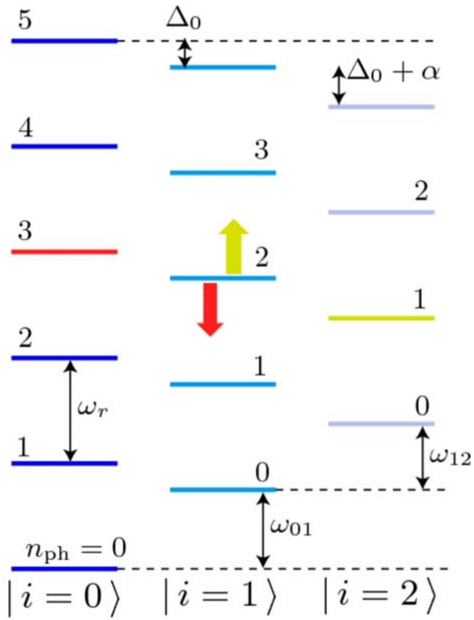

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2 The transmon qubit

2.3 Relaxation and decoherence

Relaxation time T_1

J. Koch et al. "Charge-insensitive qubit design derived from the Cooper pair box". *Physical Review A* **76**.4 (Oct. 2007).

Relaxation time T_1

(1) Spontaneous emission

$$P = \frac{1}{4\pi\epsilon_0}\frac{d^2\omega^4}{3c^3} \Rightarrow T_1^{\mathrm{rad}} = \frac{\hbar\omega_{01}}{P} \sim 0.3\,\mathrm{ms}$$

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(2) Purcell effect

$$\Gamma_{i\rightarrow f} = \frac{2\pi}{\hbar}\rho(\omega_k)\left|\langle 1, f|\hbar\textstyle\sum_k \lambda_k(\hat{b}_k^\dagger\hat{a} + \text{h.c.})|0, i\rangle\right|^2$$

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$$= \kappa\left|\langle f|\hat{a}|i\rangle\right|^2 \Rightarrow T_1^{\text{pur}} \sim 16\,\mu\text{s}$$

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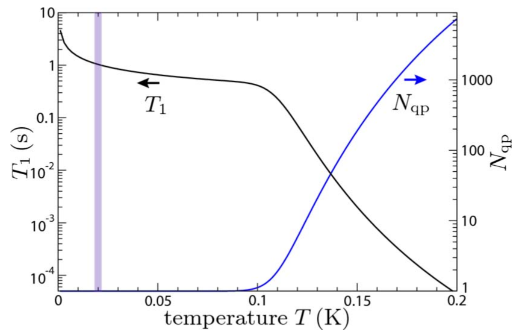

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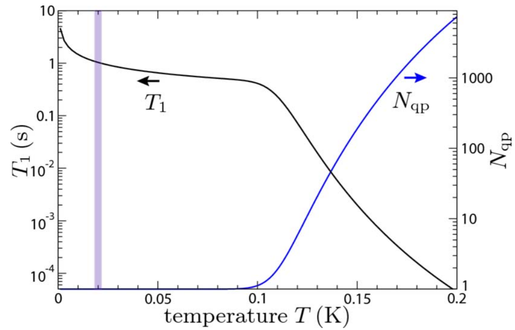
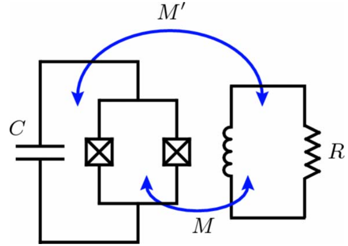
(3) Quasiparticle tunneling $T_1^{\text{qp}} \sim 1\,\text{s}$

(4) Flux coupling $T_1^{\text{flux}} \sim 70\,\text{ms}$





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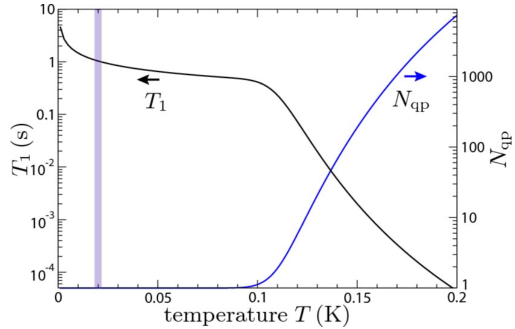
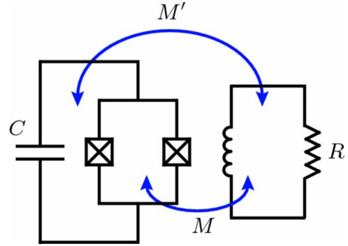
(5) Dielectric losses





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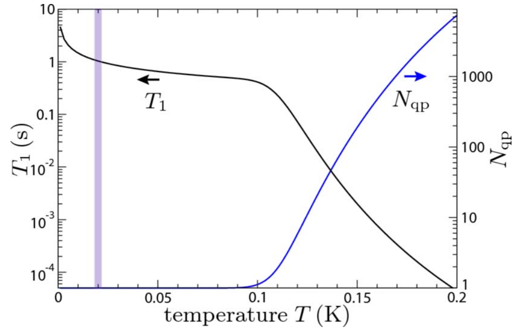
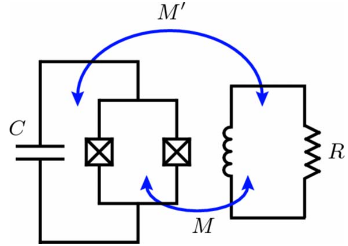
(5) Dielectric losses

$$\Rightarrow T_1 = \left(\frac{1}{T_1^{\text{rad}}} + \frac{1}{T_1^{\text{pur}}} + \frac{1}{T_1^{\text{qp}}} + \frac{1}{T_1^{\text{flux}}}\right)^{-1} \sim 15\,\mu\text{s}$$





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Decoherence time T_2

$$T_2 \simeq \frac{\hbar}{A}\left|\frac{\partial E_{01}}{\partial \lambda}\right|^{-1}$$

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$$S(\omega) = \frac{2\pi A^2}{|\omega|} \qquad n_{\mathrm{g}}, \Phi, I_{\mathrm{c}}$$

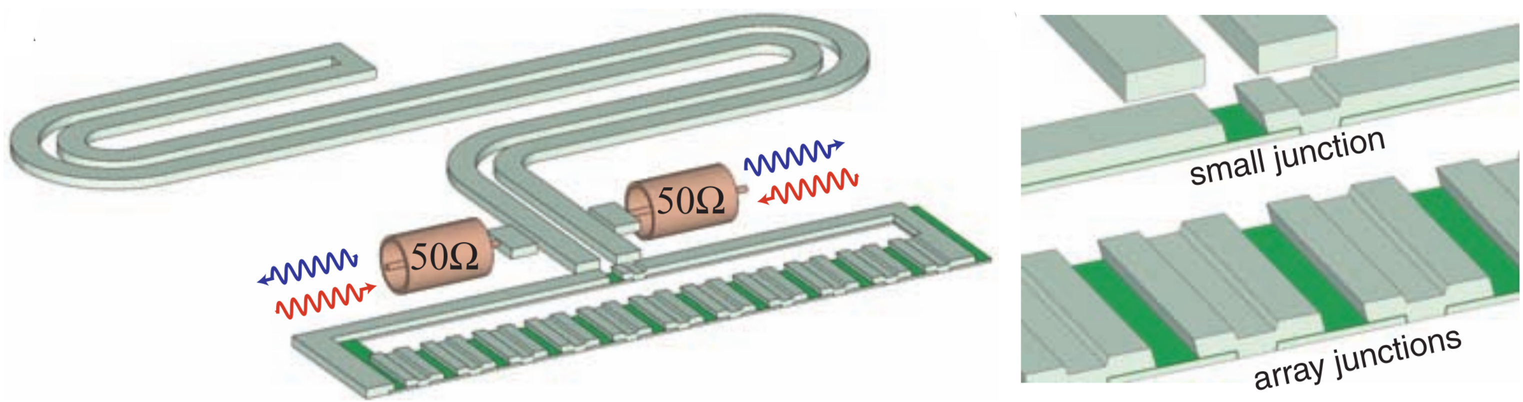
		Transmon ($E_{\mathrm{J}}/E_{\mathrm{C}} = 85$)	CPB ($E_{\mathrm{J}}/E_{\mathrm{C}} = 1$)
Noise source	$1/f$ amplitude A	T_2 in ns	T_2 in ns
Charge	$10^{-4} - 10^{-3}e$	400′000	1′000
Flux	$10^{-6} - 10^{-5}\Phi_0$	3′600′000	1′000′000
Critical current	$10^{-7} - 10^{-6}I_{\mathrm{c}}$	35′000	17′000

J. Koch et al. "Charge-insensitive qubit design derived from the Cooper pair box". *Physical Review A* **76.4** (Oct. 2007).

3 The fluxonium qubit

3.1 Characterization of the qubit

The device



V. E. Manucharyan et al. "Fluxonium: Single Cooper-Pair Circuit Free of Charge Offsets". *Science* **326.5949** (Oct. 2009), 113.

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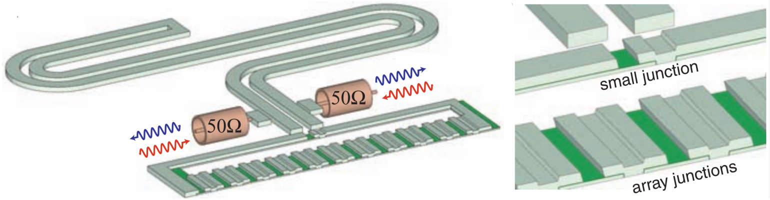

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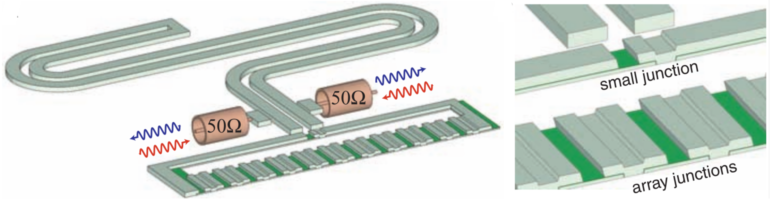

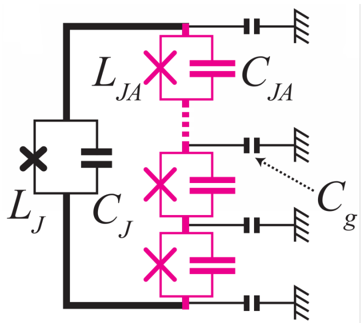

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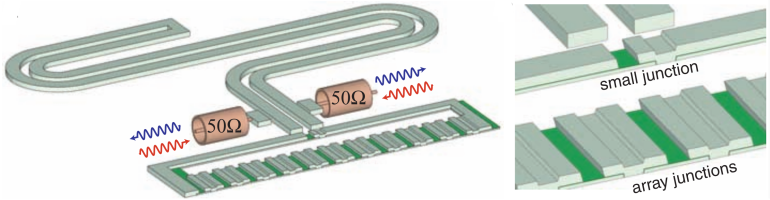
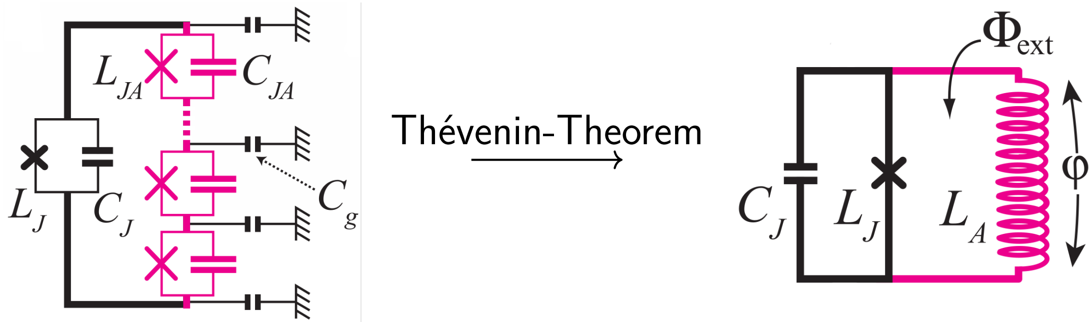
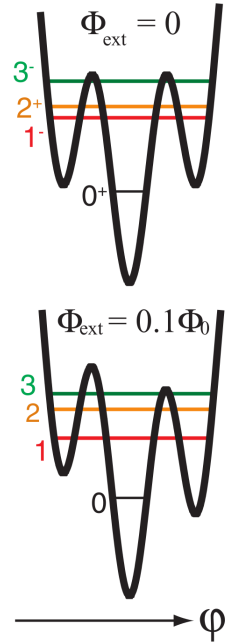




V. E. Manucharyan et al. "Fluxonium: Single Cooper-Pair Circuit Free of Charge Offsets". *Science* **326**.**5949** (Oct. 2009), 113.

The Eigenstates

$$H = 4E_\mathrm{C}\hat{n}^2 + \frac{1}{2}E_\mathrm{L}\hat{\varphi}^2 - E_\mathrm{J}\cos(\hat{\varphi} - 2\pi\Phi_\mathrm{ext}/\Phi_0)$$



V. E. Manucharyan et al. "Fluxonium: Single Cooper-Pair Circuit Free of Charge Offsets". *Science* **326.5949** (Oct. 2009), 113.

The Eigenstates

$$E_\mathrm{C} = e^2/2C_\mathrm{J}$$

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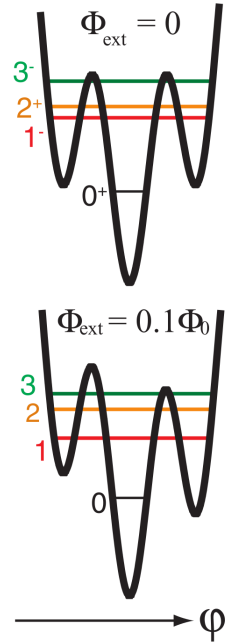

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The Eigenstates

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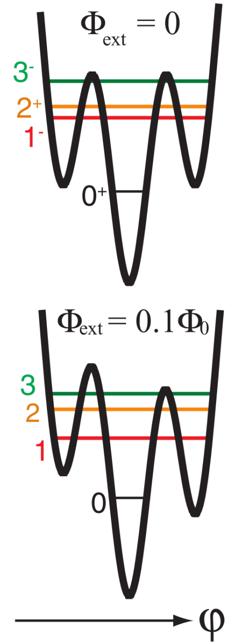
$E_\mathrm{L} = (\Phi_0/2\pi)^2/L_\mathrm{A}$



V. E. Manucharyan et al. "Fluxonium: Single Cooper-Pair Circuit Free of Charge Offsets". *Science* **326.5949** (Oct. 2009), 113.

The Eigenstates

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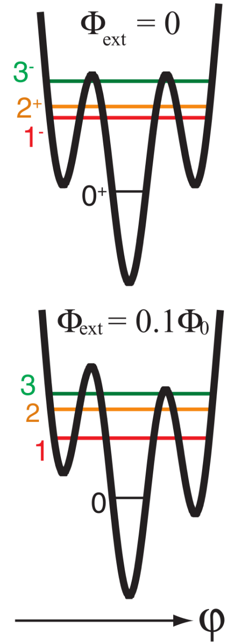
$$H = 4E_{\mathrm{C}}\hat{n}^2 + \frac{1}{2}E_{\mathrm{L}}\hat{\varphi}^2 - E_{\mathrm{J}}\cos(\hat{\varphi} - 2\pi\Phi_{\mathrm{ext}}/\Phi_0)$$

$$E_{\mathrm{L}} = (\Phi_0/2\pi)^2/L_{\mathrm{A}}$$



V. E. Manucharyan et al. "Fluxonium: Single Cooper-Pair Circuit Free of Charge Offsets". *Science* **326.5949** (Oct. 2009), 113.

The Eigenstates

$E_\mathrm{C} = e^2/2C_\mathrm{J}$ $\qquad$ $E_\mathrm{J} = (\Phi_0/2\pi)^2/L_\mathrm{J}$

$$H = 4E_\mathrm{C}\hat{n}^2 + \frac{1}{2}E_\mathrm{L}\hat{\varphi}^2 - E_\mathrm{J}\cos(\hat{\varphi} - 2\pi\Phi_\mathrm{ext}/\Phi_0)$$

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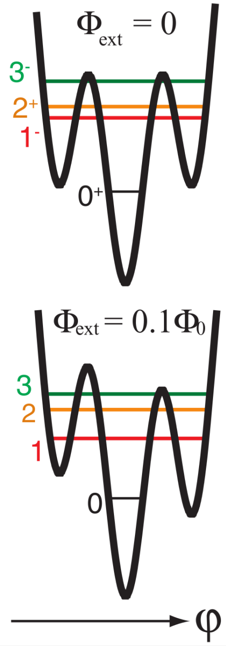

(1) Small charge fluctuations $\Rightarrow$ Large flux fluctuations $\Rightarrow$ $NL_\mathrm{JA} \gg L_\mathrm{J}$

V. E. Manucharyan et al. "Fluxonium: Single Cooper-Pair Circuit Free of Charge Offsets". *Science* **326.5949** (Oct. 2009), 113.

The Eigenstates

$$E_{\mathrm{C}} = e^2/2C_{\mathrm{J}} \qquad E_{\mathrm{J}} = (\Phi_0/2\pi)^2/L_{\mathrm{J}}$$

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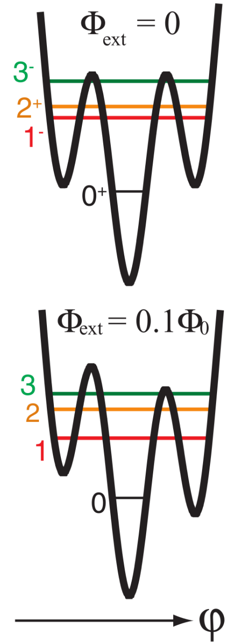
$$E_{\mathrm{L}} = (\Phi_0/2\pi)^2/L_{\mathrm{A}}$$



(1) Small charge fluctuations $\Rightarrow$ Large flux fluctuations $\Rightarrow NL_{\mathrm{JA}} \gg L_{\mathrm{J}}$

(2) Reduce offset charge $\Rightarrow e^{-8R_{\mathrm{Q}}/Z_{\mathrm{JA}}} < \varepsilon \ll 1$

V. E. Manucharyan et al. "Fluxonium: Single Cooper-Pair Circuit Free of Charge Offsets". *Science* **326.5949** (Oct. 2009), 113.

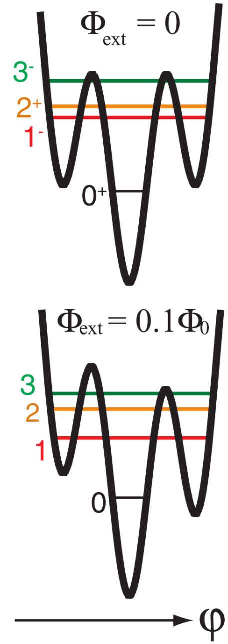
The Eigenstates

$E_C = e^2/2C_J$

$E_J = (\Phi_0/2\pi)^2/L_J$

$$H = 4E_C\hat{n}^2 + \frac{1}{2}E_L\hat{\varphi}^2 - E_J\cos(\hat{\varphi} - 2\pi\Phi_{ext}/\Phi_0)$$

$E_L = (\Phi_0/2\pi)^2/L_A$



(1) Small charge fluctuations $\Rightarrow$ Large flux fluctuations $\Rightarrow NL_{JA} \gg L_J$

(2) Reduce offset charge $\Rightarrow e^{-8R_Q/Z_{JA}} < \varepsilon \ll 1$

(3) Suppress quantum phase slips $\Rightarrow Ne^{-8R_Q/Z_{JA}} \ll e^{-8R_Q/Z_J}$

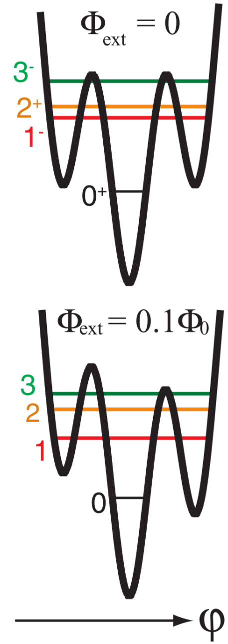
V. E. Manucharyan et al. “Fluxonium: Single Cooper-Pair Circuit Free of Charge Offsets”. *Science* **326.5949** (Oct. 2009), 113.

The Eigenstates

$$E_\mathrm{C} = e^2/2C_\mathrm{J} \qquad E_\mathrm{J} = (\Phi_0/2\pi)^2/L_\mathrm{J}$$

$$H = 4E_\mathrm{C}\hat{n}^2 + \frac{1}{2}E_\mathrm{L}\hat{\varphi}^2 - E_\mathrm{J}\cos(\hat{\varphi} - 2\pi\Phi_\mathrm{ext}/\Phi_0)$$

$$E_\mathrm{L} = (\Phi_0/2\pi)^2/L_\mathrm{A}$$



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(4) Reduce parasitic capacitance to ground $\Rightarrow N < (C_\mathrm{JA}/C_\mathrm{g})^{1/2}$

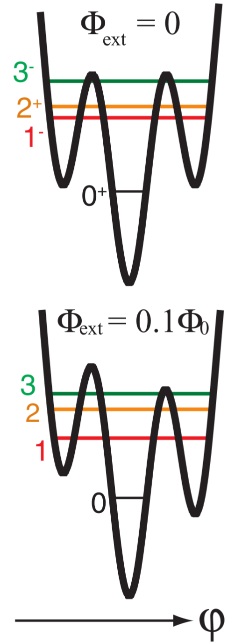
V. E. Manucharyan et al. "Fluxonium: Single Cooper-Pair Circuit Free of Charge Offsets". *Science* **326.5949** (Oct. 2009), 113.

The Eigenstates

$$E_\mathrm{C} = e^2/2C_\mathrm{J} = 2.5\,\mathrm{GHz} \qquad E_\mathrm{J} = (\Phi_0/2\pi)^2/L_\mathrm{J} = 9.0\,\mathrm{GHz}$$

$$H = 4E_\mathrm{C}\hat{n}^2 + \frac{1}{2}E_\mathrm{L}\hat{\varphi}^2 - E_\mathrm{J}\cos(\hat{\varphi} - 2\pi\Phi_\mathrm{ext}/\Phi_0)$$

$$E_\mathrm{L} = (\Phi_0/2\pi)^2/L_\mathrm{A} = 0.52\,\mathrm{GHz}$$



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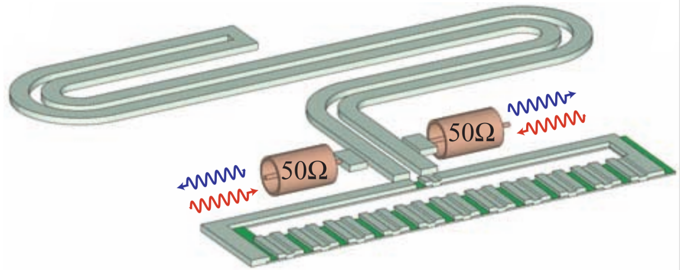
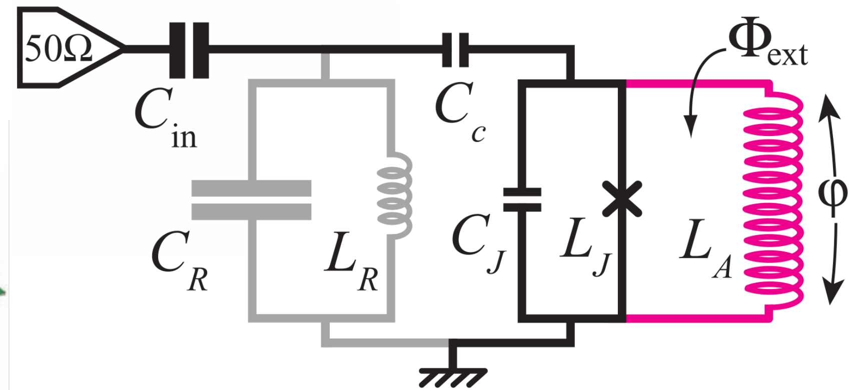
V. E. Manucharyan et al. "Fluxonium: Single Cooper-Pair Circuit Free of Charge Offsets". *Science* **326.5949** (Oct. 2009), 113.

3 The fluxonium qubit

3.2 Initialization, control and read-out

Circuit QED





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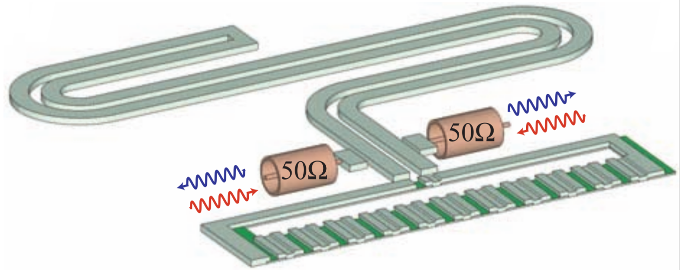
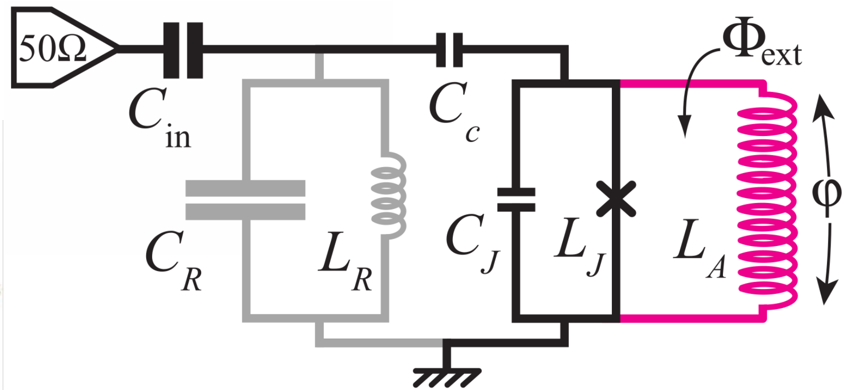
Circuit QED

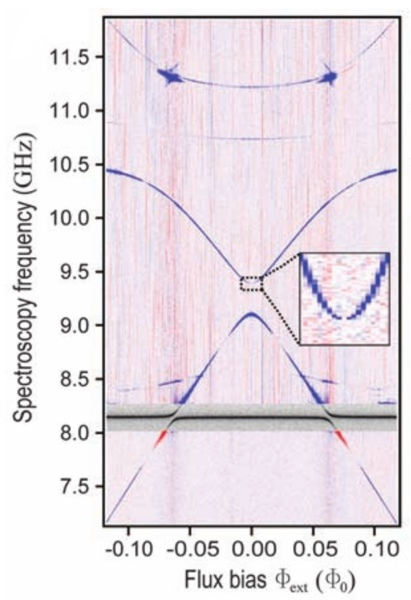




$$H = 4E_\mathrm{C}\hat{n}^2 + \frac{1}{2}E_\mathrm{L}\hat{\varphi}^2 - E_\mathrm{J}\cos(\hat{\varphi} - 2\pi\Phi_\mathrm{ext}/\Phi_0) + g\hat{n}(\hat{a} + \hat{a}^\dagger) + \hbar\omega_\mathrm{R}\hat{a}^\dagger\hat{a}$$

V. E. Manucharyan et al. "Fluxonium: Single Cooper-Pair Circuit Free of Charge Offsets". *Science* **326.5949** (Oct. 2009), 113.

Microwave spectroscopy



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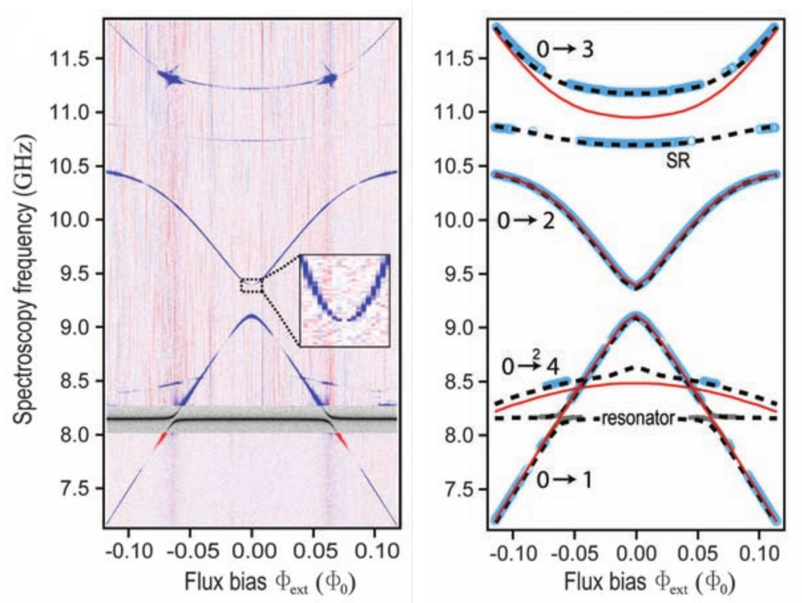

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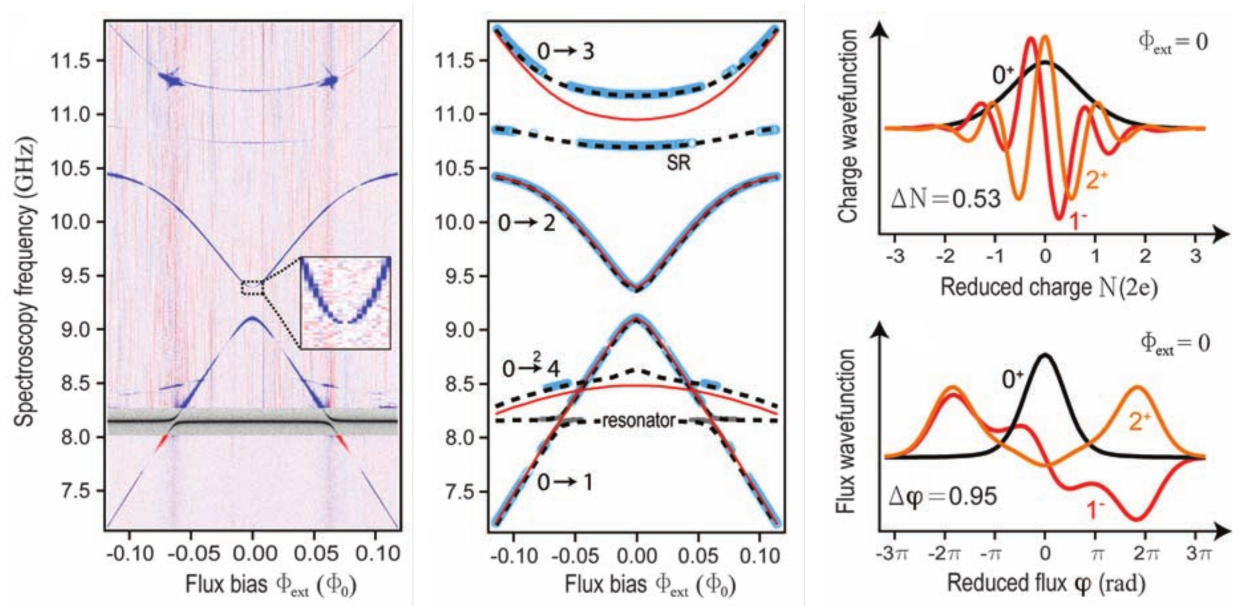
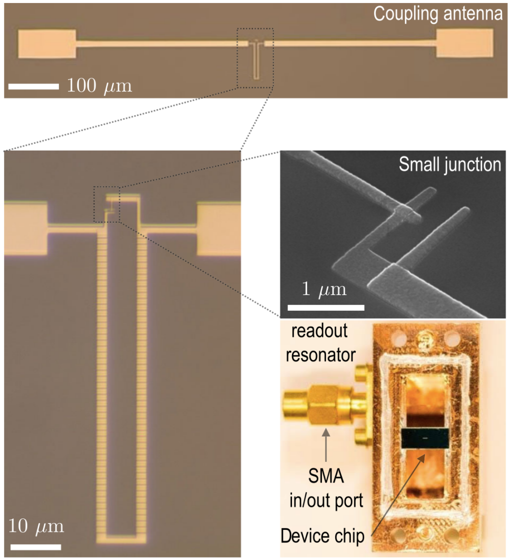


V. E. Manucharyan et al. "Fluxonium: Single Cooper-Pair Circuit Free of Charge Offsets". *Science* **326**.5949 (Oct. 2009), 113.

3 The fluxonium qubit

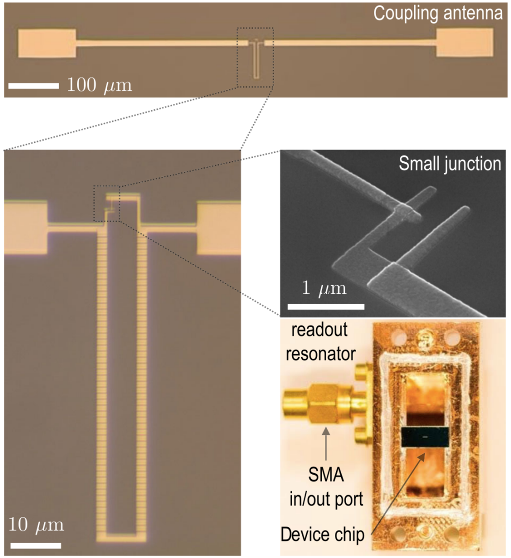
3.3 Relaxation and decoherence

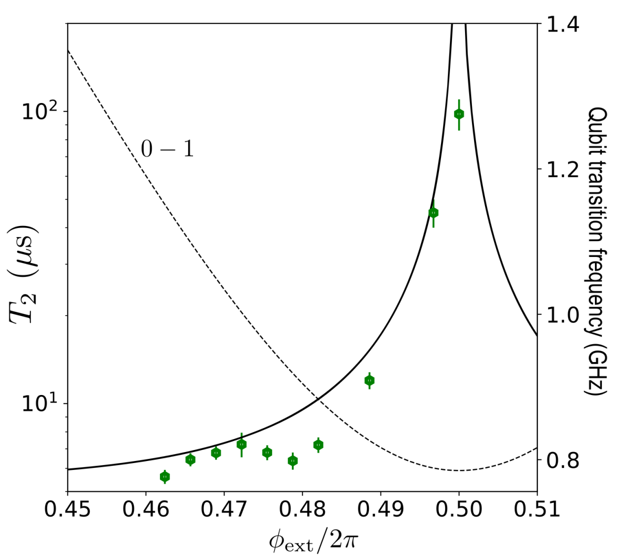
High coherence device



L. B. Nguyen et al. "High-Coherence Fluxonium Qubit". *Physical Review X* **9**.4 (Nov. 2019).

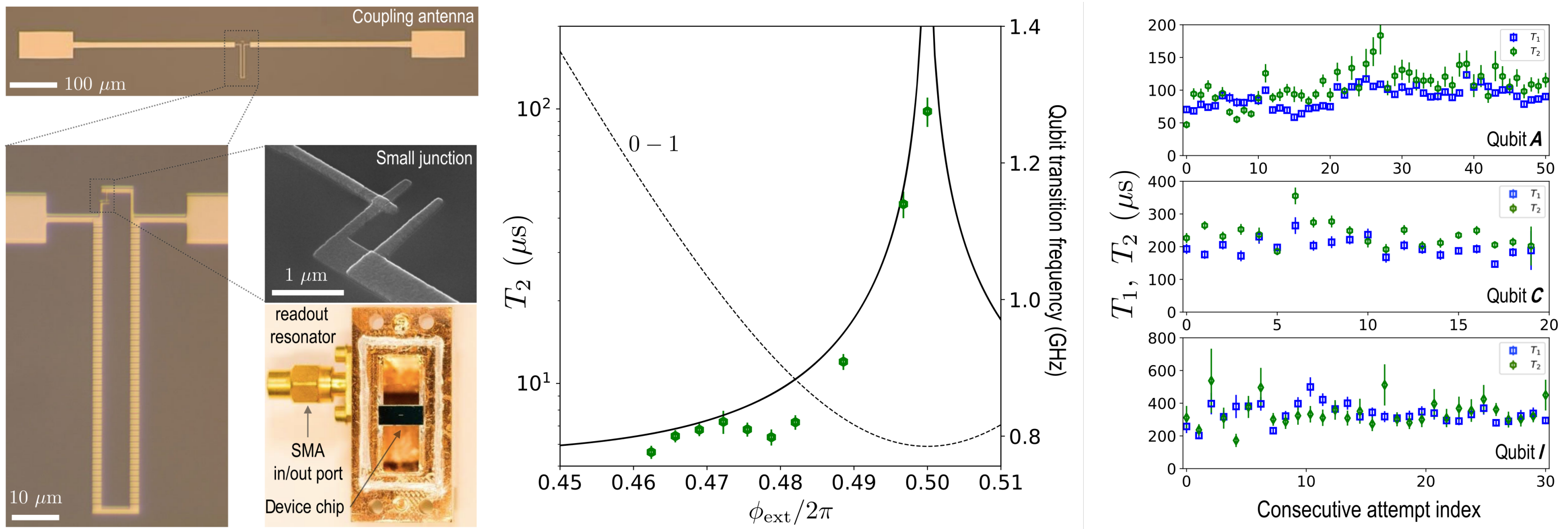
High coherence device





L. B. Nguyen et al. "High-Coherence Fluxonium Qubit". *Physical Review X* **9**.4 (Nov. 2019).

High coherence device



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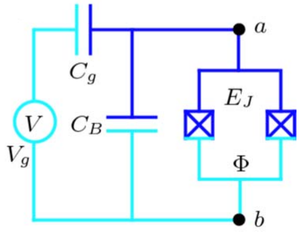
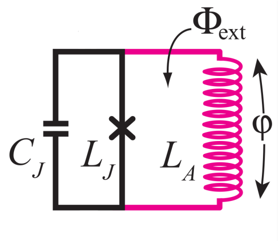
4 Summary

Summary

The transmon qubit

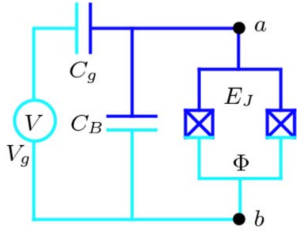


The fluxonium qubit



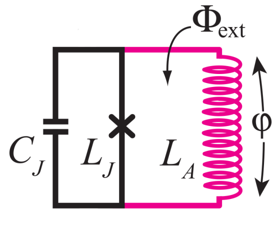
Summary

The transmon qubit



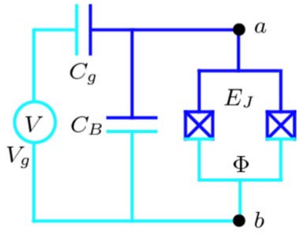
- CPB with effective shunting capacitance

The fluxonium qubit



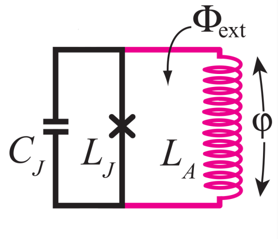
Summary

The transmon qubit



- CPB with effective shunting capacitance
- Charge dispersion $\propto e^{-\sqrt{8E_\mathrm{J}/E_\mathrm{C}}}$
  $\Rightarrow$ Exponential insentivity to $1/f$ noise

The fluxonium qubit



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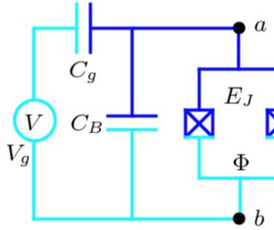
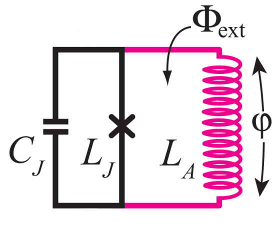
The transmon qubit



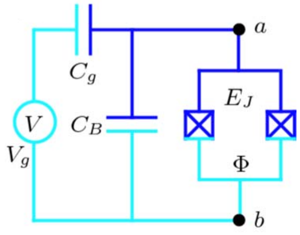
- CPB with effective shunting capacitance
- Charge dispersion $\propto e^{-\sqrt{8E_\mathrm{J}/E_\mathrm{C}}}$
  $\Rightarrow$ Exponential insentivity to $1/f$ noise
- Anharmonicity $\propto -(8E_\mathrm{J}/E_\mathrm{C})^{-1/2}$

The fluxonium qubit



Summary

The transmon qubit



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  $\Rightarrow$ Exponential insentivity to $1/f$ noise
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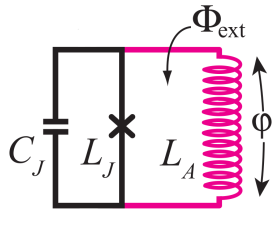
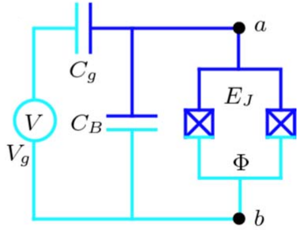
- Less sensitive to flux or crit. current noise as CPB

The fluxonium qubit



Summary

The transmon qubit



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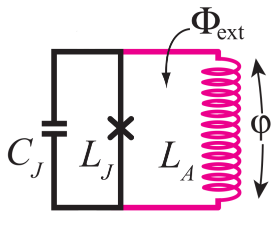
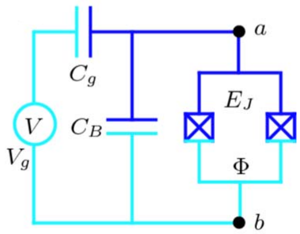
The fluxonium qubit



- Short-ciruit offset charge noise with JJ array

Summary

The transmon qubit



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- Charge dispersion $\propto e^{-\sqrt{8E_\mathrm{J}/E_\mathrm{C}}}$
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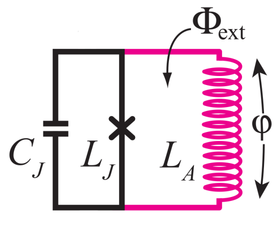
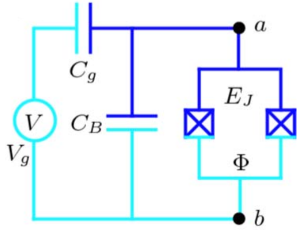
The fluxonium qubit



- Short-ciruit offset charge noise with JJ array
- Large JJ array protects small JJ from large flux fluctuations

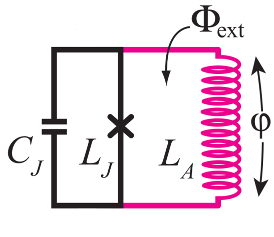
Summary

The transmon qubit



- CPB with effective shunting capacitance
- Charge dispersion $\propto e^{-\sqrt{8E_J/E_C}}$
  $\Rightarrow$ Exponential insentivity to $1/f$ noise
- Anharmonicity $\propto -(8E_J/E_C)^{-1/2}$
- Less sensitive to flux or crit. current noise as CPB

The fluxonium qubit



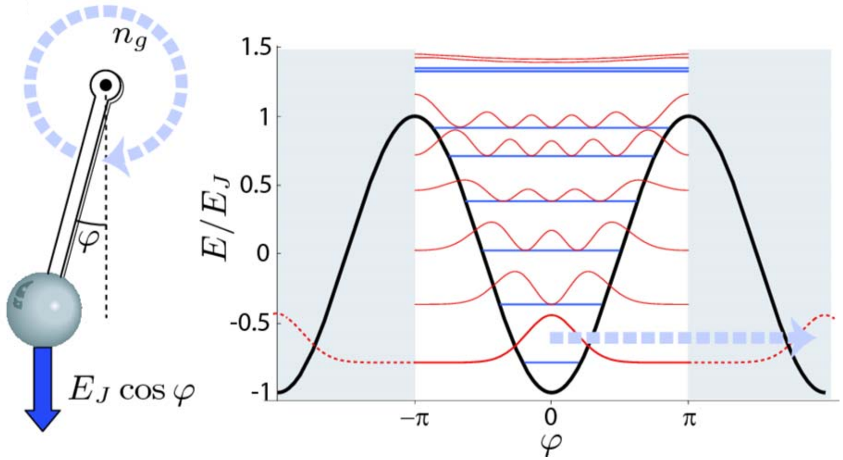
- Short-ciruit offset charge noise with JJ array
- Large JJ array protects small JJ from large flux fluctuations
- Unharmonic as the flux qubit but as insesitive to flux noise as the transmon

Thank You
For Your Attention!
Do You Have Any Questions?

Analogy to the rotor

$$\hat{H} = 4E_\mathrm{C}(\hat{n} - n_\mathrm{g})^2 - E_\mathrm{J}\cos\hat{\varphi}$$

$$H_\mathrm{rot} = \frac{\hat{L_z}^2}{2ml^2} - mgl\cos\hat{\varphi}$$

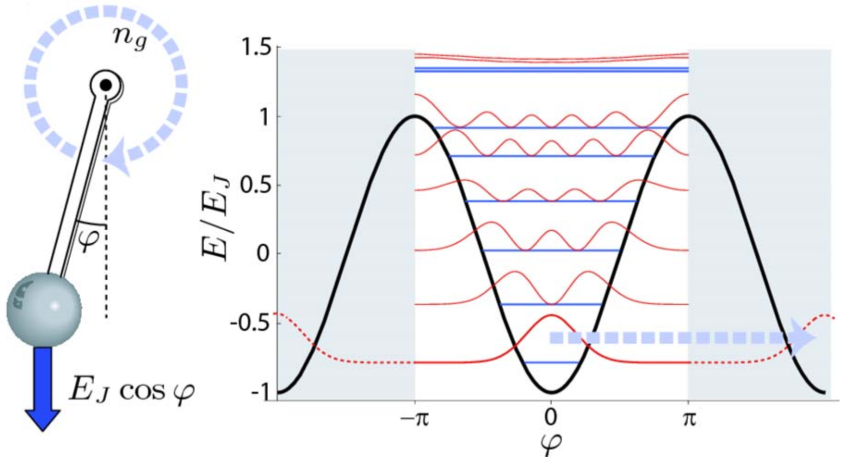


J. Koch et al. “Charge-insensitive qubit design derived from the Cooper pair box”. *Physical Review A* **76**.4 (Oct. 2007).

Analogy to the rotor

$$\hat{H} = 4E_\mathrm{C}(\hat{n} - n_\mathrm{g})^2 - E_\mathrm{J}\cos\hat{\varphi}$$

$$H_\mathrm{rot} = \frac{(\hat{L_z} + qB_0 l^2/2)^2}{2ml^2} - mgl\cos\hat{\varphi}$$



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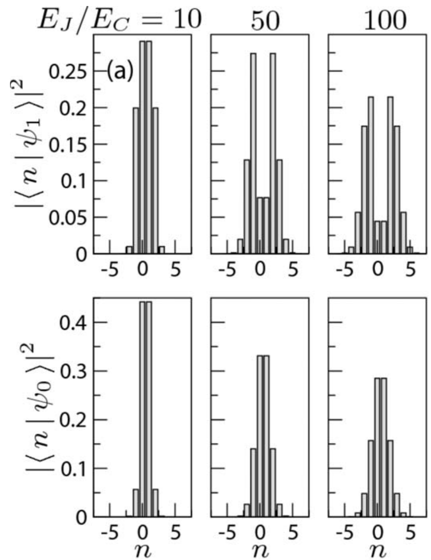
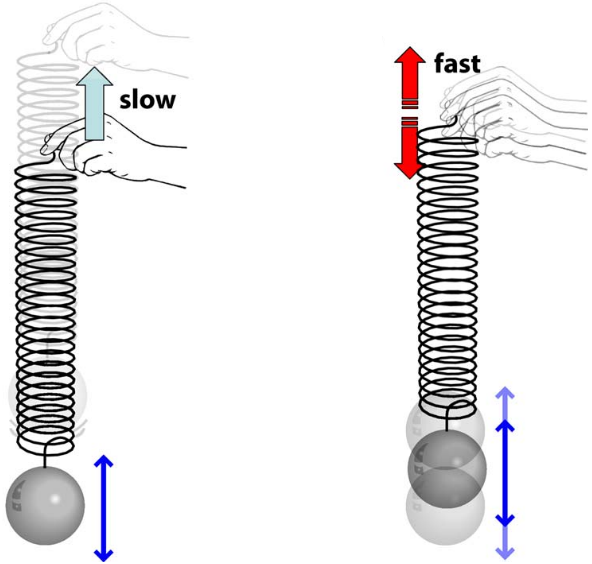
Charge number states vs. transmon eigenstates



Ac vs dc noise



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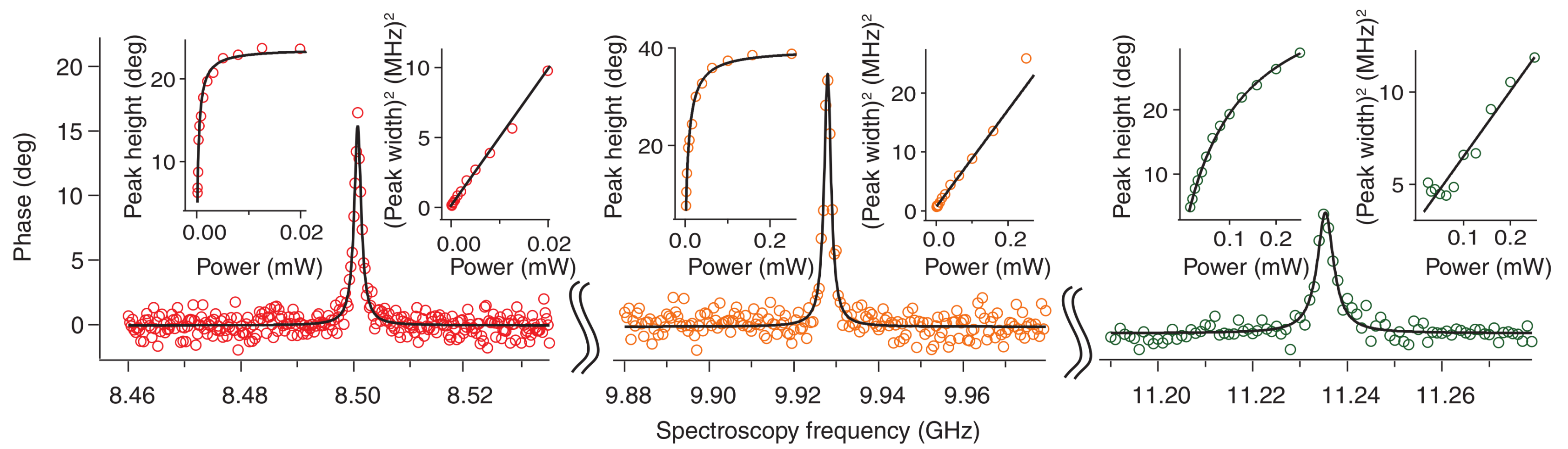
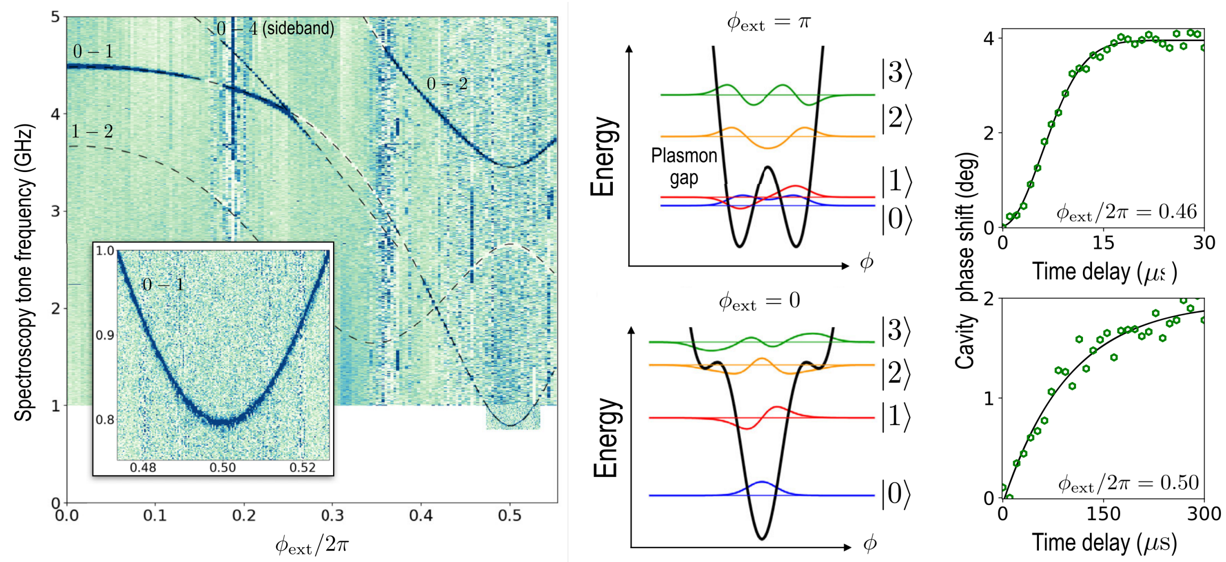
Microwave spectroscopy



V. E. Manucharyan et al. "Fluxonium: Single Cooper-Pair Circuit Free of Charge Offsets". *Science* **326.5949** (Oct. 2009), 113.

Measured relaxation and coherence time



L. B. Nguyen et al. "High-Coherence Fluxonium Qubit". *Physical Review X* **9**.4 (Nov. 2019).