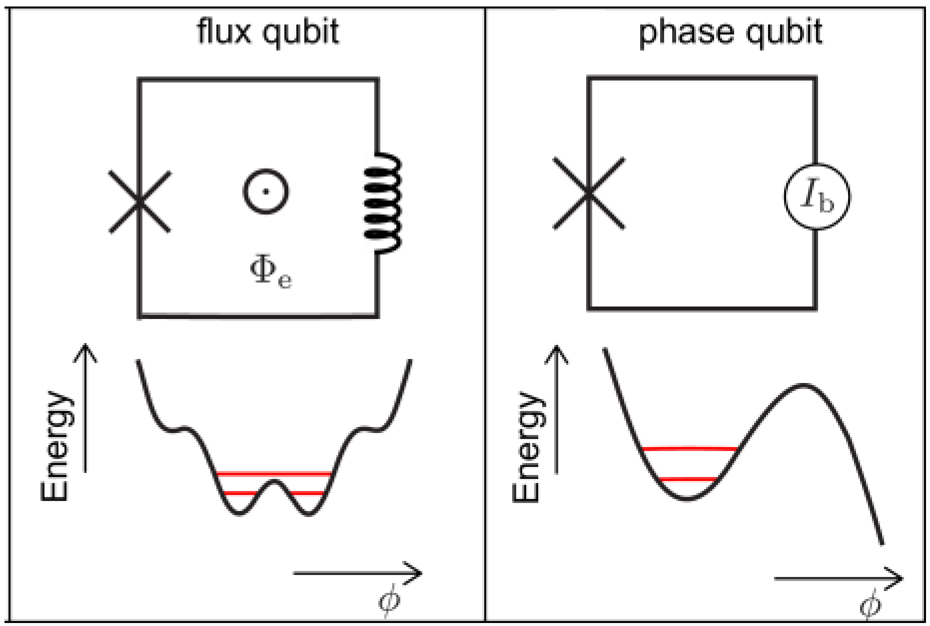
Student seminar: Flux and Phase qubit



Quantum Engineering[A.M.Zagoskin,(2011)]

Rabi Oscillations in a Large Josephson-Junction Qubit; Martinis et al., 2002

Josephson Persistent-Current Qubit; Mooij et al., 1999

·Why such devices?

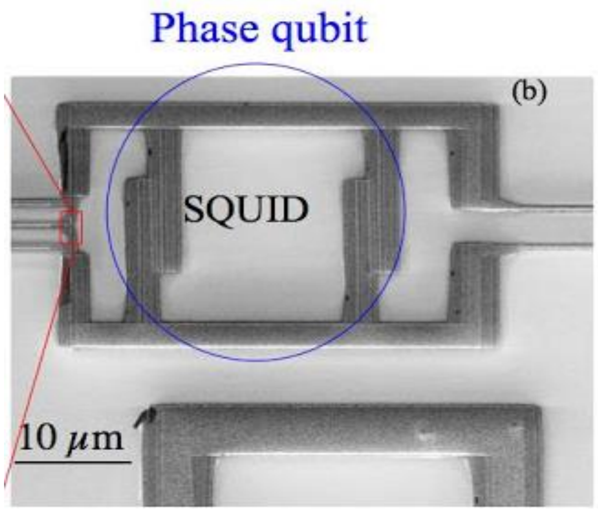
·One of the most successful realizations of solid state qubits at least in early 2000´s

·Based in the Josephson junctions phenomena

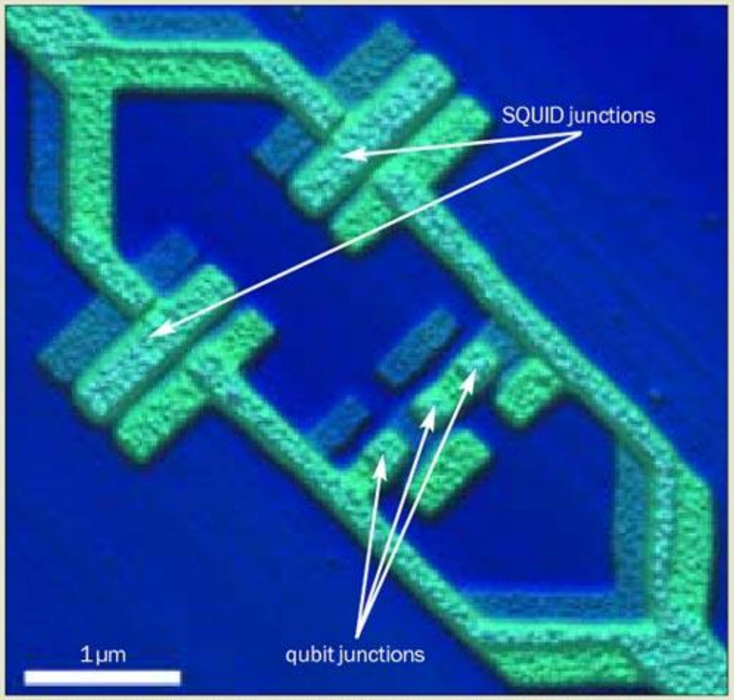
There are two main reasons:

-The systems are big and the technology is quite developed.

-Integration in an electric circuit involving simple manipulation and read-out.



http://neel.cnrs.fr/spip.php?rubrique481



4 Flux qubit

This image of a flux qubit plus SQUID system was obtained with an atomic force microscope at Delft. The qubit is formed by the small loop bottom right, and the three Josephson junctions are indicated by arrows. The SQUID is the larger loop and contains two Josephson junctions. As the SQUID and qubit loops share a large fraction of their circumference, they are tightly coupled. A technique called shadow-evaporation was used to make the circuit.

A Short Introduction to Quantum Information and Quantum Computation (2002)

Phase Qubit

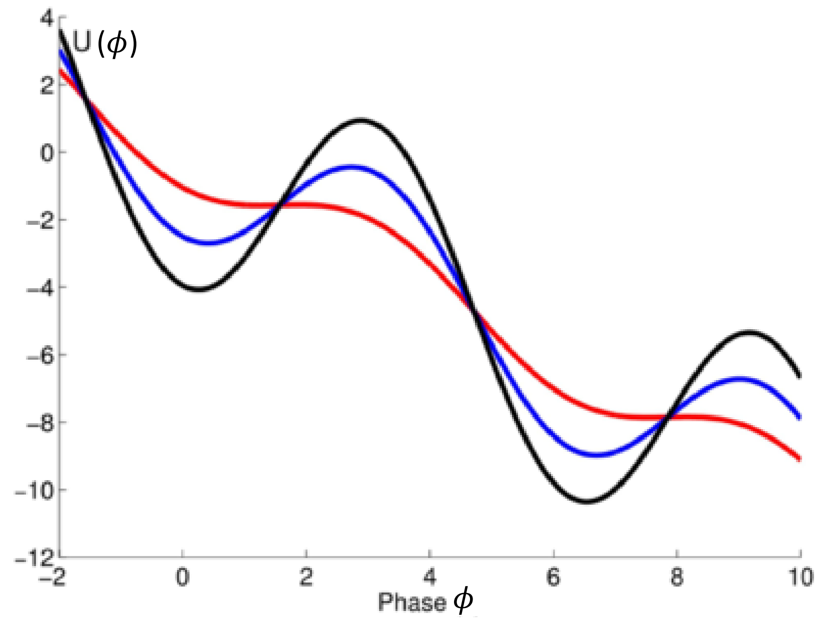
·Lagrangian for a current biased Josephson junction

$$\mathcal{L}(\phi,\dot{\phi};I) = K - U = \frac{C\dot{\phi}^2}{2}\left(\frac{\hbar}{2e}\right)^2 + \frac{I_c\Phi_0}{2\pi c}\cos\phi + \frac{I\Phi_0}{2\pi c}\phi$$

·First we will obtain the canonical momentum and operate till we get the hamiltonian in the position (ϕ) representation

$$\Theta = \frac{\partial}{\partial\dot{\phi}}\mathcal{L} = \left(\frac{\hbar}{2e}\right)^2 C\dot{\phi} \qquad C(\hbar/2e)\dot{\phi} = CV = Q$$

$$\Theta = \hbar N$$



Plot of the so called 'Washboard potential'

$$\mathcal{H}(\Theta,\phi) = \Theta\dot{\phi} - \mathcal{L} = \frac{1}{2C}\left(\frac{2e\Theta}{\hbar}\right)^2 - E_J\cos\phi - \frac{I\Phi_0}{2\pi c}\phi \equiv E_C\left(\frac{\Theta}{\hbar}\right)^2 - E_J\cos\phi - \frac{I\Phi_0}{2\pi c}\phi$$

$$H = -E_C\frac{\partial^2}{\partial\phi^2} - E_J\cos\phi - \frac{I\Phi_0}{2\pi c}\phi \equiv -E_C\frac{\partial^2}{\partial\phi^2} - E_J\left(\cos\phi + \frac{I}{I_c}\phi\right)$$

Phase Qubit

·Finally we can obtain the equations of motion for the phase and number operators

$$\imath\hbar\dot{\phi} = [\phi, H] = -E_C\left[\phi, \frac{\partial^2}{\partial\phi^2}\right] = 2E_C\frac{\partial}{\partial\phi} = 2E_C \times \imath\hat{N};$$

$$\imath\hbar\dot{\hat{N}} = [\hat{N}, H] = -E_J\left[\frac{1}{\imath}\frac{\partial}{\partial\phi}, \cos\phi + \frac{I}{I_c}\phi\right] = \frac{E_J}{\imath}\left(\sin\phi - \frac{I}{I_c}\right)$$

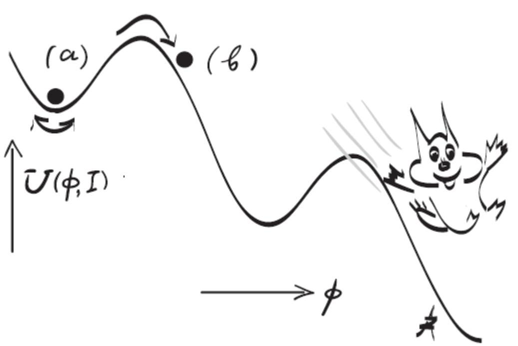
Displacement in new coordinates $\eta = \phi - \phi_{min}$

$$\ddot{\eta} \approx -\frac{2E_C E_J \cos\phi_{\min}}{\hbar^2}\eta = -\frac{2E_C E_J}{\hbar^2}\left[1-(I/I_c)^2\right]^{1/2}\eta = -\left[1-(I/I_c)^2\right]^{1/2}\omega_0^2\eta$$

What are the options for the value of the bias current I?

$\mathrm{I} \to I_C$
Potential softening

$\mathrm{I} \to 0$
Strong potential well



Phase Qubit

·We now focus on a local minimun $\frac{d}{d\phi}\left(\cos\phi + \frac{I}{I_c}\phi\right) = 0$ $\phi_{\min} = \arcsin\frac{I}{I_c}$

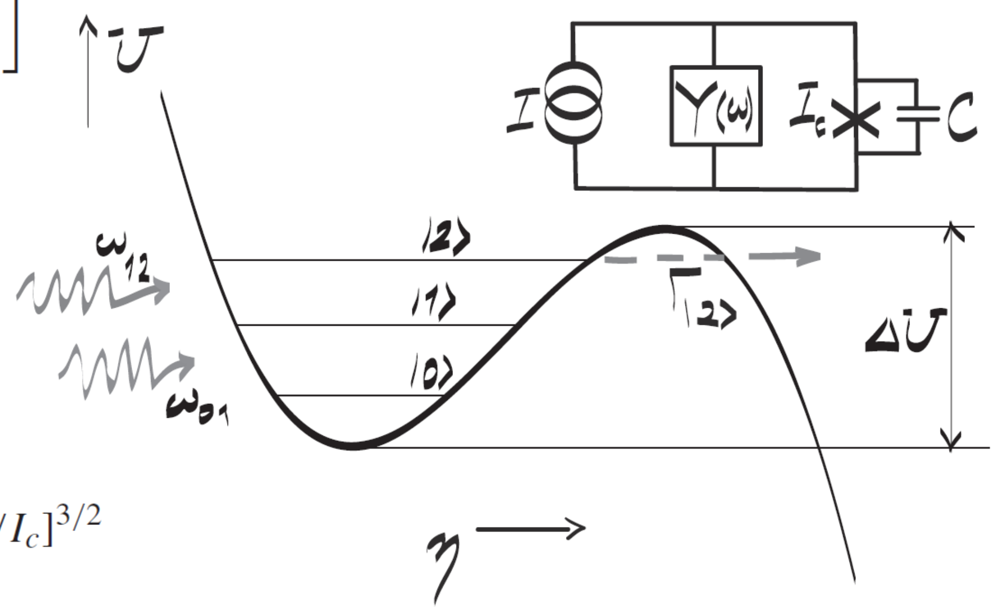
·By using we obtain the new Hamiltonian for an oscillator with cubic anharmonicity.

$$H \approx -E_C\frac{\partial^2}{\partial\eta^2} + E_J\left(\left(1-(I/I_c)^2\right)\right)^{1/2}\frac{\eta^2}{2} - \frac{I}{I_c}\frac{\eta^3}{6}\Bigg)$$

·Small oscillations lead to a plasma frequency

$$\omega_I = \omega_0\left[1-\left(\frac{I}{I_c}\right)^2\right]^{1/4} \approx \omega_0\left[2\left(1-\frac{I}{I_c}\right)\right]^{1/4}$$

It it like an LC circuit!!!



·Using the same aproximation for the height of the potential barrier

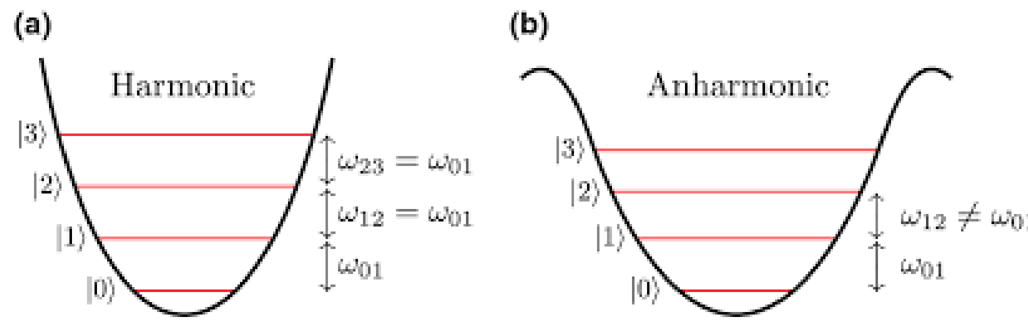
$$\Delta U = \frac{2}{3}E_J\frac{\left[1-(I/I_c)^2\right]^{3/2}}{(I/I_c)^2} \approx \frac{2^{5/2}}{3}E_J\left[1-I/I_c\right]^{3/2}$$

Phase Qubit

·The anharmonic potential leads to corrections in the level separation

$$\hbar\omega_{01} \approx \hbar\omega_I \left(1 - \frac{5}{36}\frac{\hbar\omega_I}{\Delta U}\right)$$

$$\hbar\omega_{12} \approx \hbar\omega_I \left(1 - \frac{5}{18}\frac{\hbar\omega_I}{\Delta U}\right)$$



How can we actually meassure the qubit?

Tunneling rates are the key

$$\Gamma_{n+1} \approx 1000\ \Gamma_n$$

·The qubit is manipulated through the bias current

$$I(t) = I_{\mathrm{dc}} + \Delta I(t) \equiv I_{\mathrm{dc}} + I_{\mathrm{lf}}(t) + I_{\mathrm{rf,c}}(t)\cos\omega_{01}t + I_{\mathrm{rf,s}}(t)\sin\omega_{01}t$$

·Changes in amplitudes must be slow in comparison with the anharmonicity

·The analysis continues with the aplication of the rotating wave aproximation. Leading to the Rabi oscilations induced by this resonant perturbations.

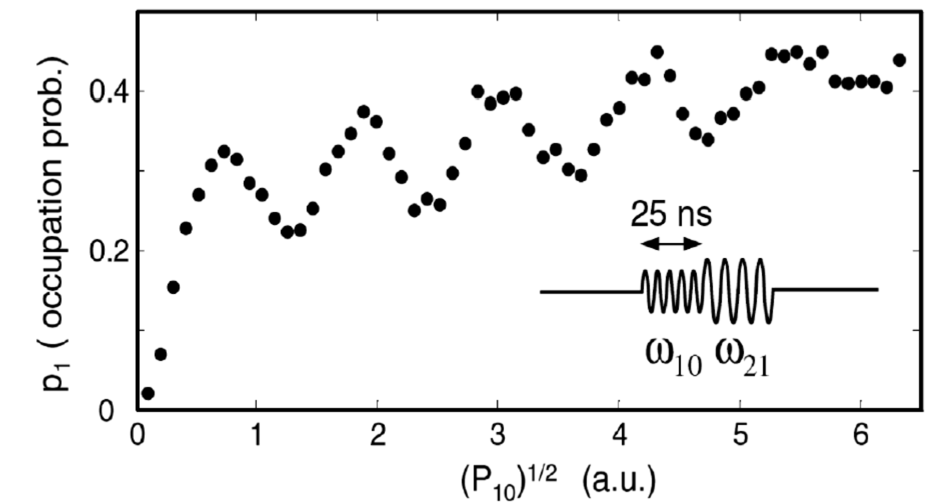
Phase Qubit

Let´s take a look at the experiment

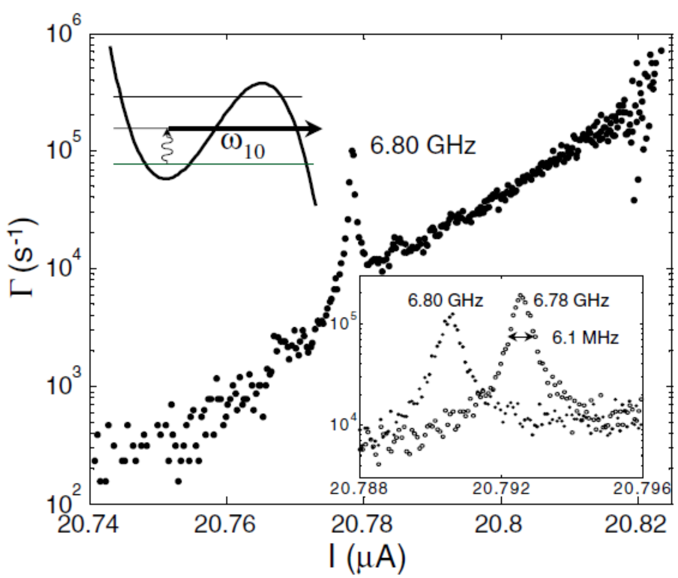
Rabi Oscillations in a Large Josephson-Junction Qubit; Martinis et al., 2002

·Experimental demonstration

·Resonant EM field at a frequency ω_{01} for a duration τ



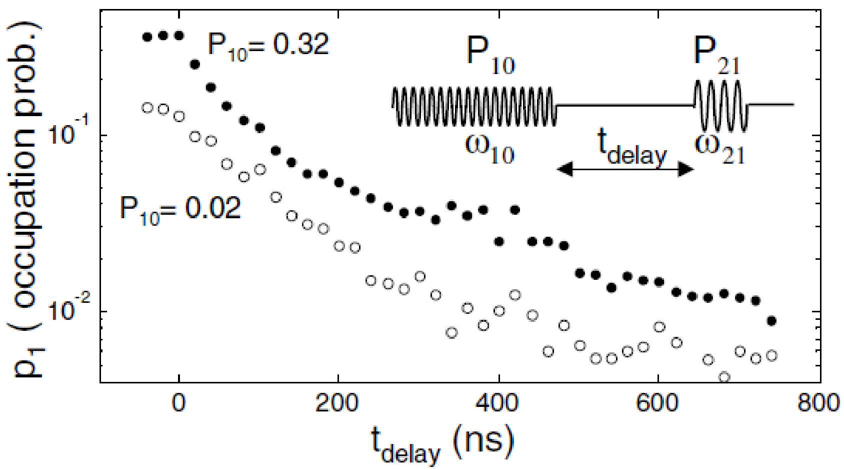
$\tau = 25ns$ $\omega_{01}/2\pi = 6.9$ GHz $\omega_{12}/2\pi = 6.28$ GHz

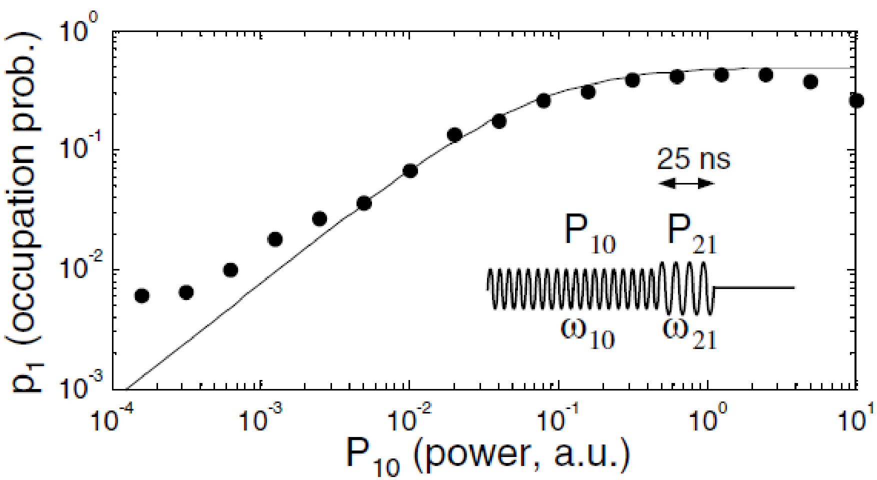


·Observation of the Rabi oscillations in spite of the relatively short decoherence time of their early device ~ 10 ns.

Later experiments yielded decoherence of ~ 80 ns
The data obtained showed high fidelity.

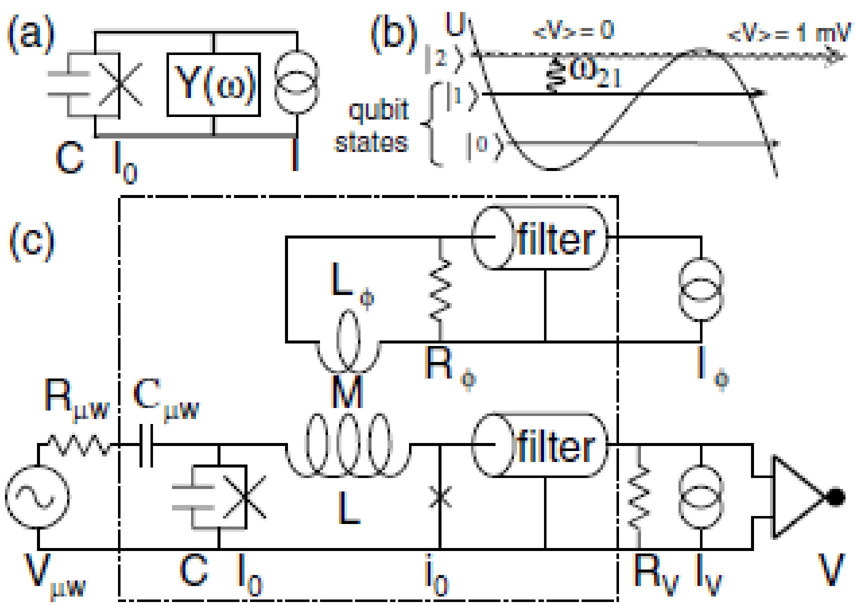
Phase Qubit





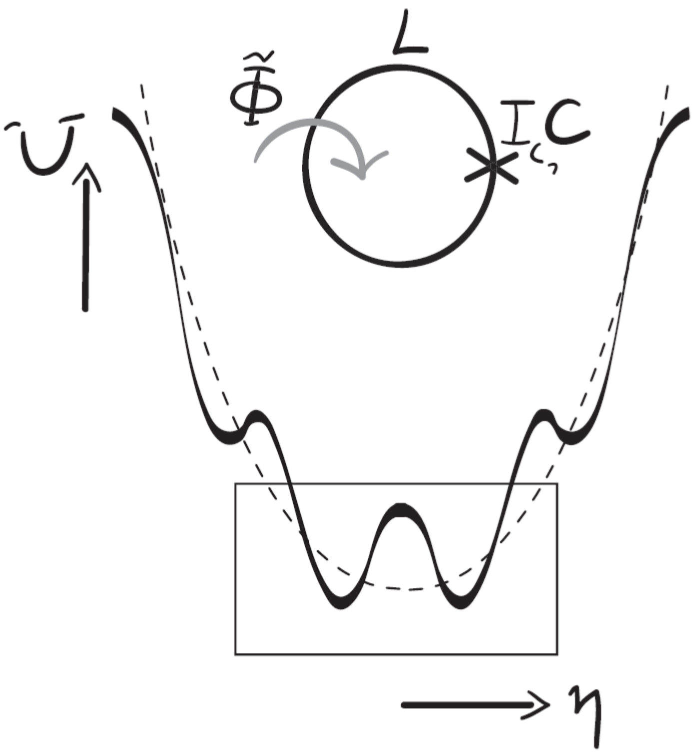
·Expected decoherence times > 1 µs after improving or changing the junction fabrication process to reduce dissipation.

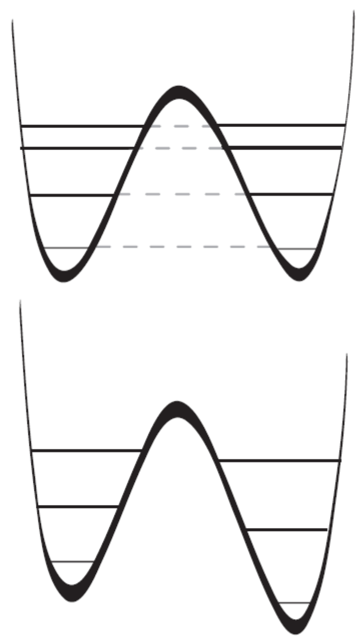
(Mainly because of trap states in the niobium oxides used in the junction)

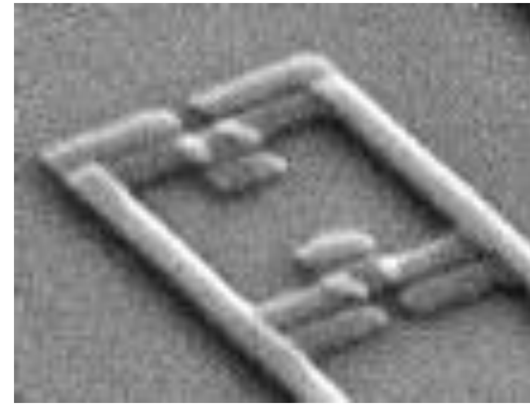


Flux Qubit

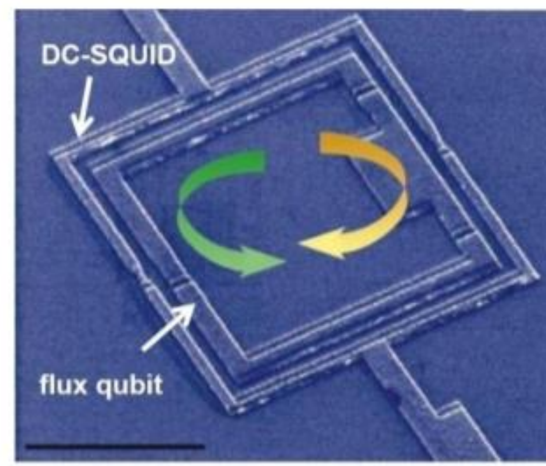
A different approach to introduce external currents to bias the Josephson junction is to use the supercurrent induced in the closed loop via a external magnetic field. That is known as a rf SQUID configuration.







SEM image of a flux qubit
https://en.wikipedia.org/wiki/Flux_qubit



Science 290, Caspar H. van der Wal et al., 2000

Flux Qubit

·In this case the Josephson potential is

$$U(\phi) = \frac{\Phi_{\text{tot}}^2}{2L} - E_J \cos\phi = \frac{\left(\Phi_0\phi/2\pi - \widetilde{\Phi}\right)^2}{2L} - E_J \cos\phi$$

·The minimum is found for $\phi = 2\pi \frac{\widetilde{\Phi} - \frac{LI_c}{c}\sin\phi}{\Phi_0}$

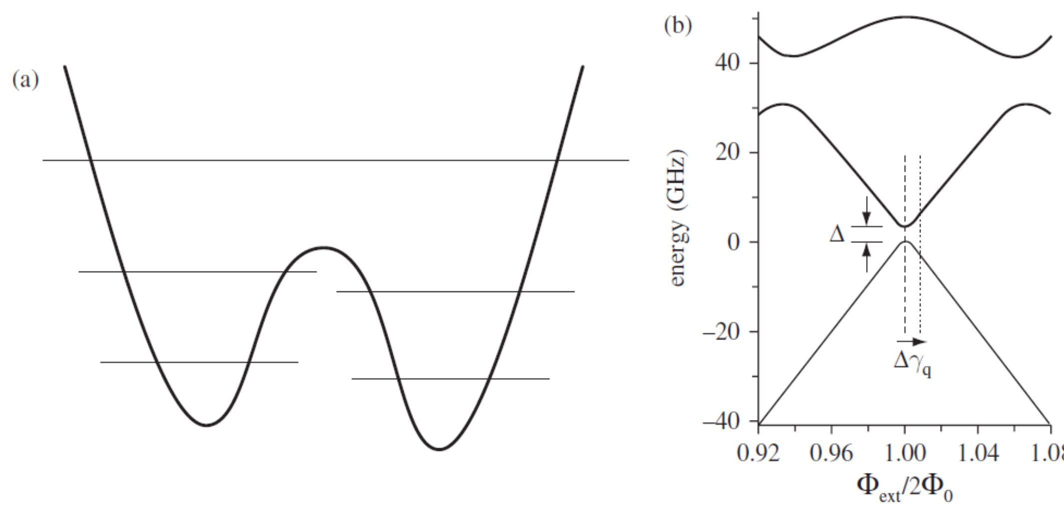
·We can continue defining $\eta = \phi - \pi$

$$U(\eta)|_{\widetilde{\Phi}=\Phi_0/2} = \frac{\Phi_0^2}{4\pi^2 L}\frac{\eta^2}{2} + E_J\cos\eta \approx -E_J\left(1 - \frac{\pi}{\beta_L}\right)\frac{\eta^2}{2} + E_J\frac{\eta^4}{24} + \text{const.}$$

·Maximal extra phase shift

$$\beta_L = 2\pi L I_c / c\Phi_0$$

$(|L\rangle \pm |R\rangle)/\sqrt{2}$ are the formed bonding-antibonding type states



Quantum Transport Introduction to Nanoscience (2009)

Flux Qubit

·Taking the system away from the degeneracy point $\delta\widetilde{\Phi}$ skews the potential, adding to it the δU part.

$$\delta\widetilde{\Phi} = \widetilde{\Phi} - \Phi_0/2 \Longrightarrow \delta U(\eta) = -\frac{\Phi_0 \delta\widetilde{\Phi}}{2\pi L}\eta + \frac{(\delta\widetilde{\Phi})^2}{2L}$$

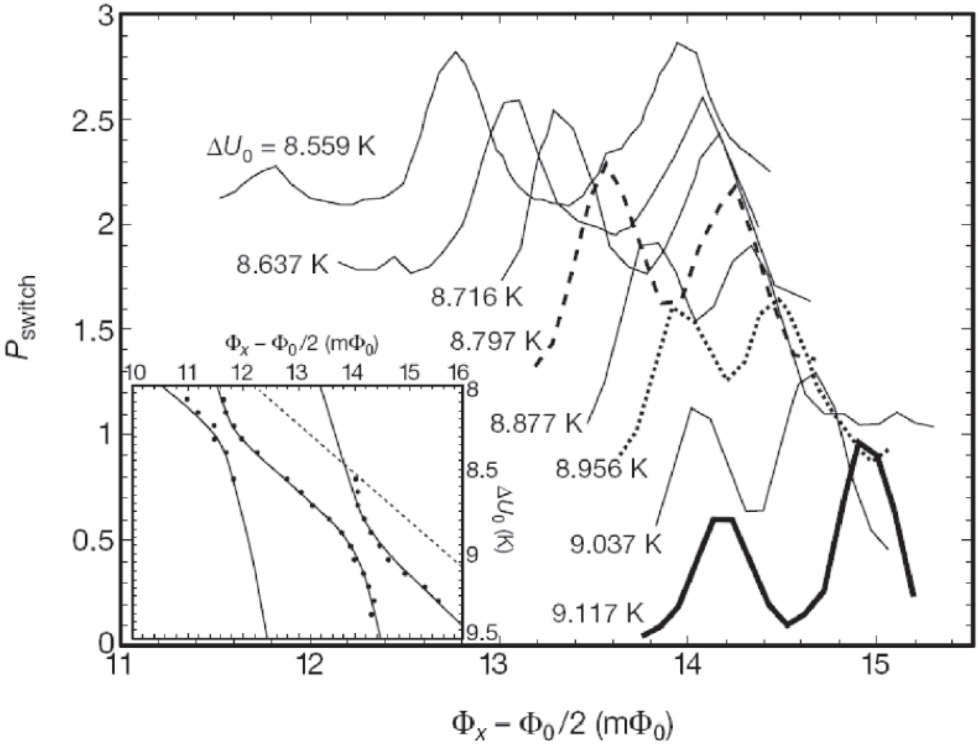
·The stablished anharmonicity allows us to separate the two bottom levels. We now look at the parameters of the rf SQUID that modulate our qubit

$$\Delta\eta = \sqrt{24(1-\pi/\beta_L)}$$

$$\Delta U = \tfrac{3}{2}(1-\pi/\beta_L)^2)$$

·Conditions:

$\beta_L > \pi$ the inductance of the SQUID loop cannot be too small. This makes the device sensitive to the external magnetic noise.



Nature 406, Friedman et al., 2000

Persistent current Flux Qubit

·How can we improve our qubit?

The idea is to create a loop of negligible inductance with three or more Josephson junctions. Leading to flux quantitation:

$$\varphi_1 + \varphi_2 + \varphi_3 + 2\pi \frac{\widetilde{\Phi}}{\Phi_0} = 2\pi q$$

$$\mathcal{L}(\Phi_1, \Phi_2; \dot{\Phi}_1, \dot{\Phi}_2) = \frac{C(1+\gamma)}{2c^2}\left(\dot{\Phi}_1^2 + \dot{\Phi}_2^2\right) + \frac{\alpha C}{2c^2}(\dot{\Phi}_1 - \dot{\Phi}_2)^2 + E\left(\cos 2\pi \frac{\Phi_1}{\Phi_0} + \cos 2\pi \frac{\Phi_2}{\Phi_0} + \alpha \cos 2\pi \frac{\Phi_1 - \Phi_2 + \Phi}{\Phi_0}\right)$$

$$\mathcal{L}(\Phi, \xi; \dot{\Phi}, \dot{\xi}) = \frac{C(1+\gamma)}{c^2}\dot{\Phi}^2 + \frac{\alpha C(1+\gamma+2\alpha)}{c^2}\dot{\xi}^2 + U(\Phi, \xi)$$

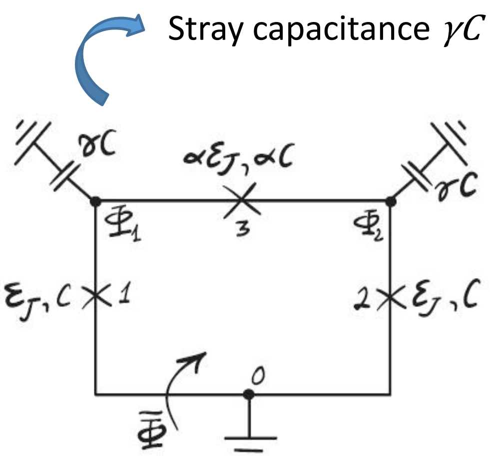
·New variables introdiced:

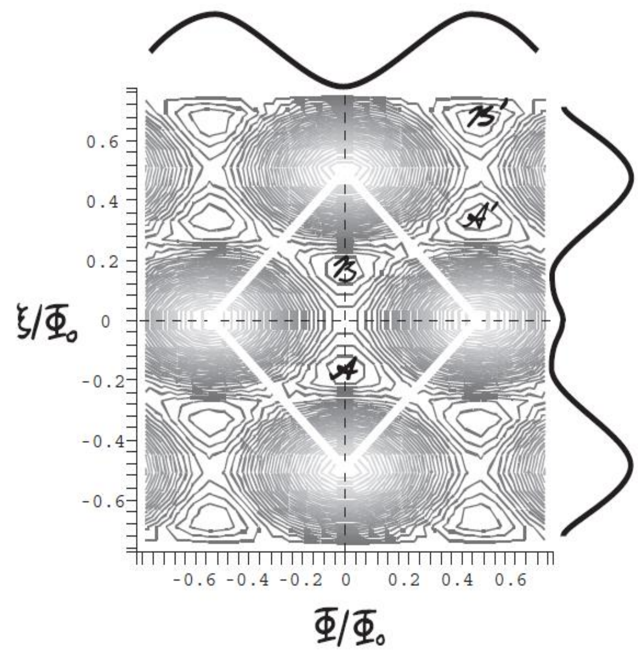
$$\Phi = (\Phi_1 + \Phi_2)/2$$

$$\xi = (\Phi_1 - \Phi_2)/2$$

·This results in equations as in a particle with anisotropic mass within periodic potential

Stray capacitance γC



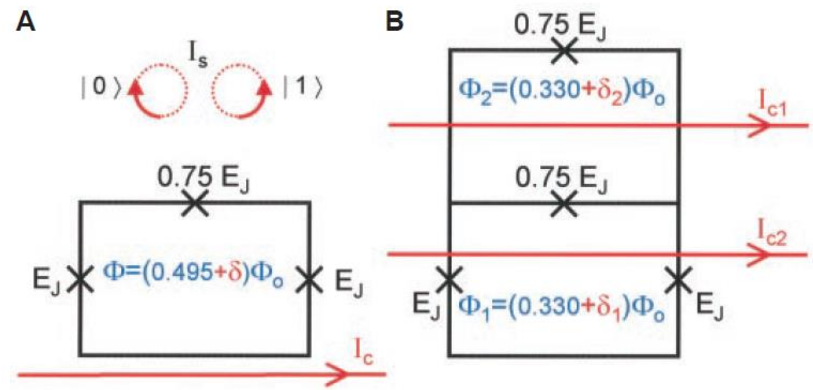


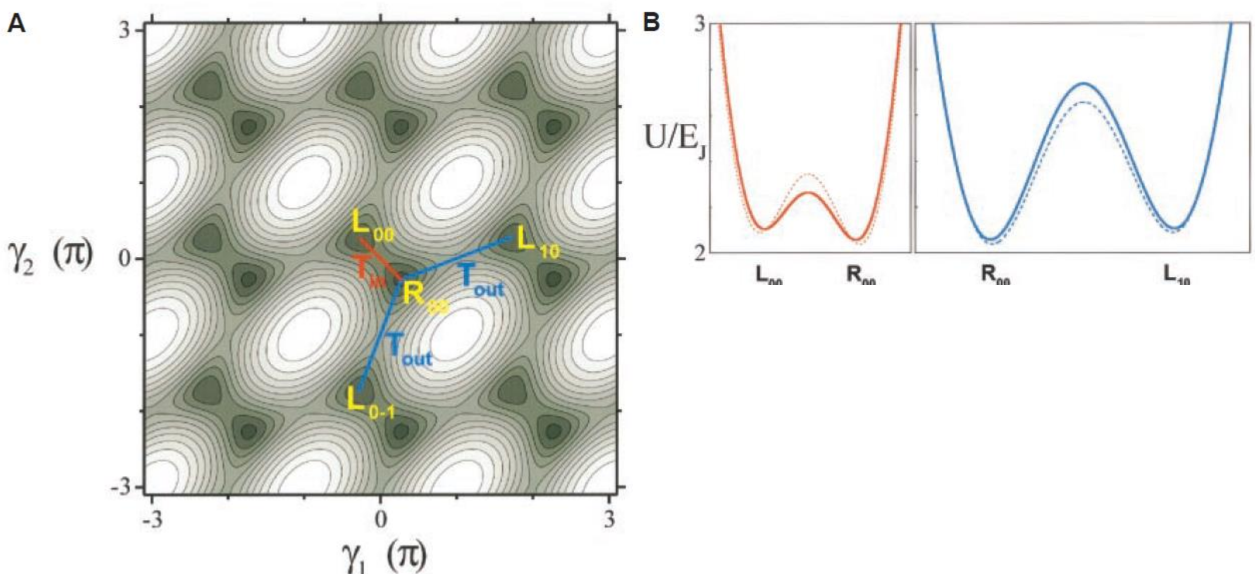
Persistent current Flux Qubit

Let´s take a look at the experiment

Josephson Persistent-Current Qubit; Mooij et al., 1999

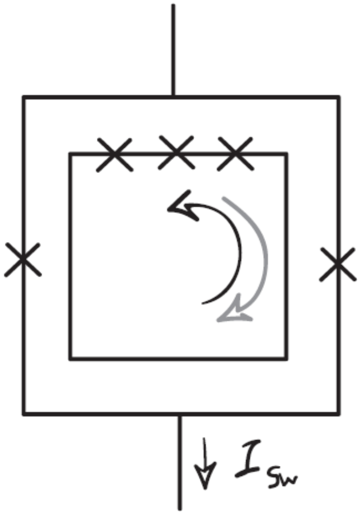
Three and four junctions qubits.

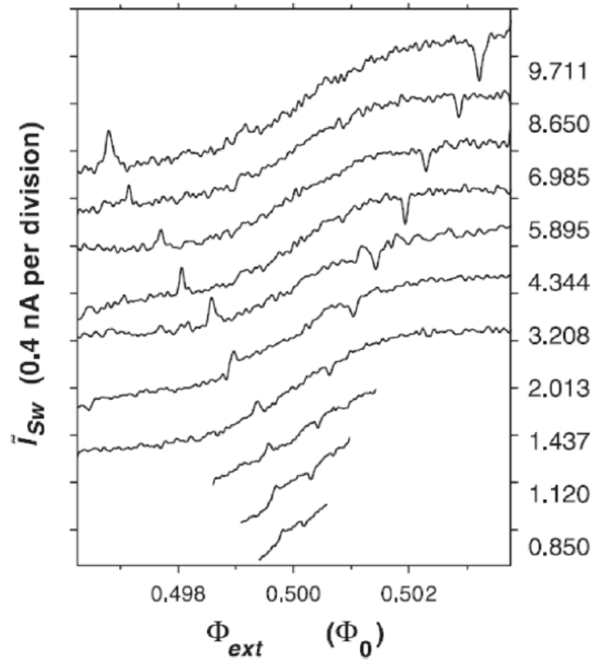




Science 285, Mooij et al., 1999

Persistent current Flux Qubit





The spikes appear in the presence of a microwave signal (frequency [GHz], on the right axis) and indicate the resonant transitions from the ground to the excited state.

This shows the transition in states and measure the quantum state.

·There are several sources contributing to the decoherence time: Photons, extra cupling to metals,...

·Typical switching times 10 to 100 ns
·Decoherence should reach 100 µs
·Overall known decoherence sources allow times above 1 ms

Summary and Conclusions

- Phase qubit
- Physical system: big Josephson junction with a capacitance.
- Goal is to house only a few quantum levels in the potential.
- Take the two lower levels as our qubit states.
- The faster decay rate of the higher states is used as read-out

- Flux qubit
- Physical system: based on an rf SQUID.
- Resulting double-well potential will house the qubit states.
- The two lowest levels of the different wells are the qubit.
- The hybridization is due to tunneling through the small separating barrier.

- Persistent current flux qubit
- Alternative implementation with several improvements.
- Minimization of the coupling with external sources of noise. Low self inductance
- Lowest-energy states are characterized by the sense of the supercurrent and is used as the working states of a qubit.

Thank you.