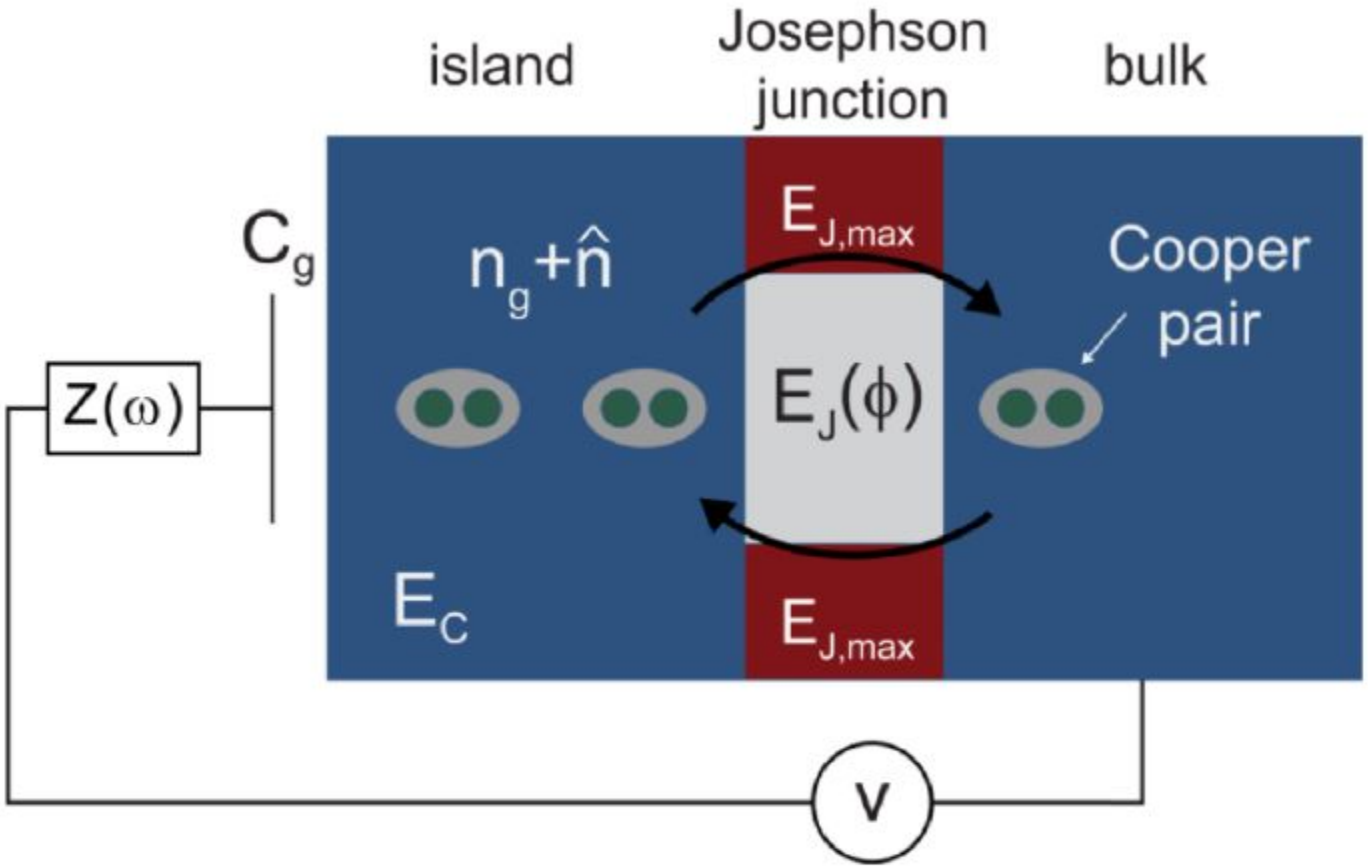
Cooper-pair box and Quantronium



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Finite Superconductors

- Consider Δ and Φ as classical variables (infinite superconductors)

$$|\mathrm{BCS}\rangle = \prod_{\mathbf{k}} \left(u_{\mathbf{k}} + v_{\mathbf{k}} a^{\dagger}_{\uparrow\mathbf{k}} a^{\dagger}_{\downarrow,-\mathbf{k}} \right) |0\rangle.$$

- Now also justified for finite small superconductors

$$|N\rangle = \int_0^{\phi} \mathrm{d}\phi\, \mathrm{e}^{-\imath N\phi} |\mathrm{BCS}\rangle = \int_0^{\phi} \mathrm{d}\phi\, \mathrm{e}^{-\imath N\phi} \prod_{\mathbf{k}} \left(|u_{\mathbf{k}}| + |v_{\mathbf{k}}| \mathrm{e}^{\imath\phi} a^{\dagger}_{\uparrow\mathbf{k}} a^{\dagger}_{\downarrow,-\mathbf{k}} \right) |0\rangle$$

$$\mathrm{e}^{\imath M\phi} a^{\dagger}_{\uparrow\mathbf{k}_1} a^{\dagger}_{\downarrow,-\mathbf{k}_1} \ldots a^{\dagger}_{\uparrow\mathbf{k}_M} a^{\dagger}_{\downarrow,-\mathbf{k}_M} |0\rangle.$$

- Phase is now totally undefinied

- Now: Josephson junction
- Phase difference induce Josephson current, but electrons can move freely: get uncertainty in number of electrons
- Some cases number of particle can't move freely: consider Δ and Φ as qm variable

$$H = \hbar\omega_0 \left(b^\dagger b + \frac{1}{2} \right) \equiv \hbar\omega_0 \left(\hat{N} + \frac{1}{2} \right)$$

- With approximations (Zagoskin 2.2.1):

$$\hat{\phi}|\phi\rangle = \phi|\phi\rangle; \quad \hat{N}|\phi\rangle = \frac{1}{\imath}\frac{\partial}{\partial\phi}|\phi\rangle,$$

$$\hat{\phi}|N\rangle = -\frac{1}{\imath}\frac{\partial}{\partial N}|N\rangle \quad \hat{N}|N\rangle = N|N\rangle,$$

$$|\phi\rangle = \sum_N e^{\imath N\phi}|N\rangle; \quad |N\rangle = \int_0^{2\pi} \frac{d\phi}{2\pi} e^{-\imath N\phi}|\phi\rangle.$$

Charge regime: normal conductors

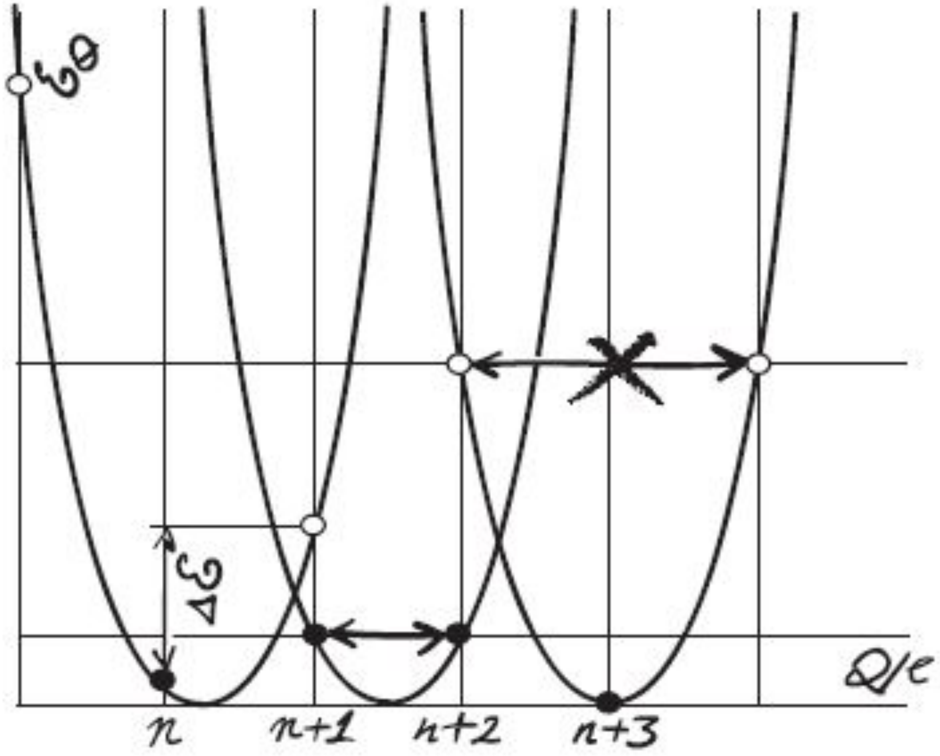
- Consider small normal conductive island connecting to external leads by tunnelling barriers and capacitors (SET)
- Situation: tunnelling energy smaller than charging energy of the island

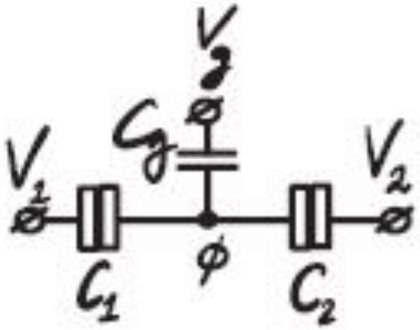
$$E_Q(Q, Q^*) = \frac{1}{2C_\Sigma}(Q - Q^*)^2 - \frac{1}{2}\sum_j C_j V_j^2. \qquad Q^* = -\sum_j C_j V_j$$

- Q have to be a multiple of e

⇒ allowed states of the system can only have discrete value of electrostatic energy

⇒ Coulomb blockade





- When coulomb blockade is lifted, resonant tunnelling through the island is allowed (single electron oszillation)

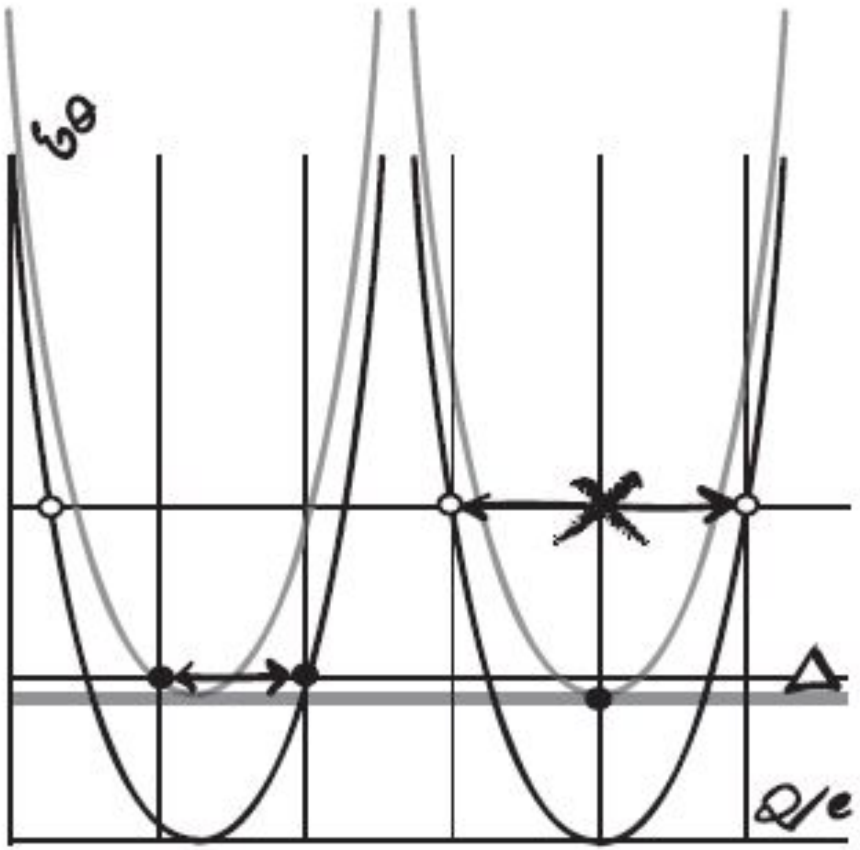
Charge regime: superconductors

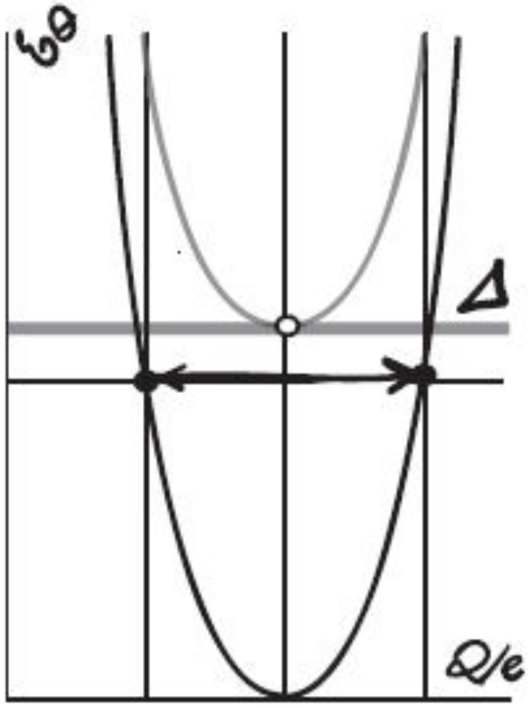
- Now: island is superconducting (SSET), gap must be considered
- Small island $\Rightarrow$ number of electrons and parity matters

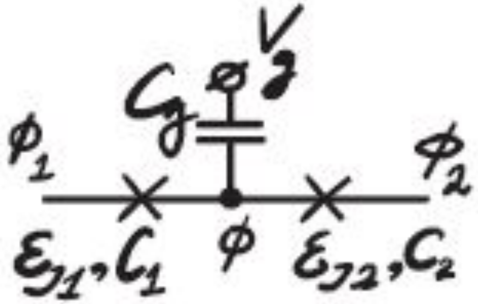
$$E_Q(Q, Q^*) = \frac{(Q - Q^*)^2}{2C_\Sigma} + p\left(\frac{Q}{2e}\right)\Delta$$

- Resonant tunnelling of cooper pairs is possible as long as: $\Delta > e^2/2C_\Sigma$

$$H = E_C\left(\frac{1}{1}\frac{\partial}{\partial\phi} - \frac{Q^*}{2e} + \frac{p}{2}\right)^2 + p\Delta - E_{J1}\cos(\phi - \phi_1) - E_{J2}\cos(\phi - \phi_2)$$


Δ
Q/e


Δ
Q/e


V_g
C_g
φ_1
φ_2
φ
E_J1, C_1
E_J2, C_2

- Assumptions: no voltage drop between island and superconducting bank $Q^* = -C_g V_g$, J-couplings are the same $\phi_1 = -\phi_2 = \phi_0/2$
- As long as gap is bigger than charging energy:

$$H = \frac{E_Q(N, V_g) + E_Q(N+1, V_g)}{2} + \frac{E_Q(N, V_g) - E_Q(N+1, V_g)}{2}\sigma_z - E_J \cos(\phi_0/2)\sigma_x,$$

- One get for energy and current: $E_{0,1}(V_g, \phi_0) = \frac{E_Q(N, V_g) + E_Q(N+1, V_g)}{2} \mp \left[\left(\frac{E_Q(N, V_g) - E_Q(N+1, V_g)}{2} \right)^2 + E_J^2 \cos^2(\phi_0/2) \right]^{1/2}$

$$I(\phi_0) = \frac{2e}{\hbar} \frac{E_J/4}{\left[\left(\frac{E_Q(N,V_g) - E_Q(N+1,V_g)}{2E_J} \right)^2 + \cos^2(\phi_0/2) \right]^{1/2}} \sin\phi_0$$

Charge qubit (Cooper-pair box)

- Hamiltonian from SSET is up to constant term, similar to qubit Hamiltonian in external field

$$H(t) = -\frac{1}{2}(\Delta\sigma_x + \epsilon(t)\sigma_z)$$

- 'left-right state' $|L\rangle \equiv |N\rangle$ and $|R\rangle \equiv |N+1\rangle$ are ground state and first exited state, with the energies seen before
- Qubit can be built with only one JJ, if also the Hamiltonian is reduced to the qubit form in the subspace, spanned by the two states, we get:

$$H(n^*) = -\frac{1}{2}\left(E_C(1-n^*)\sigma_z + E_J\sigma_x\right)$$

$$Q^* = en^*$$

- In practice: single JJ in SSET is replaced by two parallel junctions in tunable dc SQUID configuration

- Disadvantage: Very sensitive to charge noise

- Decoherence time of simple charge qubit: ~2ns

Quantronium

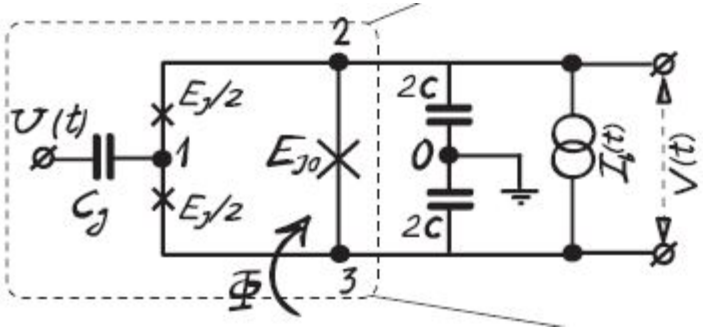
- At induced charge $n^*e = -C_g V_g + Q^*$ near to degeneracy point $n^* = 1$:

$$E_{0,1}(n^*) = \mp \frac{E_C}{2} \sqrt{(1 - n^*)^2 + (E_J/E_C)^2}.$$

- Ratio E_J/E_C very large leads to less sensitivity

- Develop design of quantronium to decrease sensitivity of noise and improve readout
- Sweet spot: $n^* = 1$ $\psi = 0$
- In quantronium don't read out charge but phase

- Device: basically SSET with tunable junctions and a dc SQUID loop closed by another large JJ



$$\psi = \phi_2 - \phi_3$$

$$H = E_C(\hat{N} - 2n^*)^2 - E_J \cos\frac{\psi}{2}\cos\theta$$

$$\theta = (\phi_2 - \phi_1) - (\phi_1 - \phi_3)$$

$$I_{0(1)} = \pm\frac{2e}{\hbar}\frac{E_J}{4}\sin\frac{\psi}{2},$$

- Phase difference readout at large JJ: $\gamma = -\psi - 2\pi\frac{\tilde{\Phi}}{\Phi_0}$

Thank you