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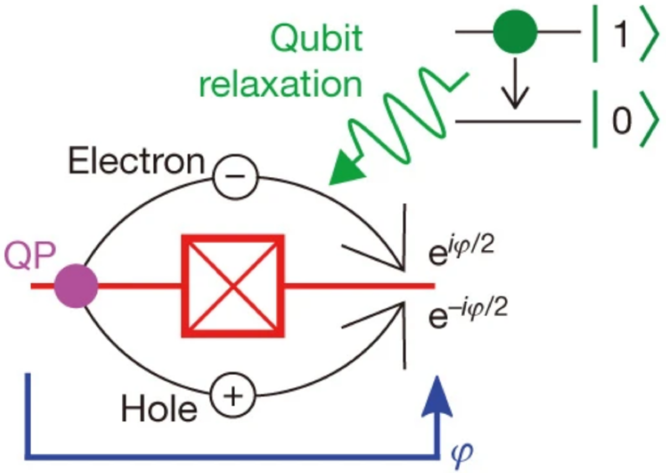
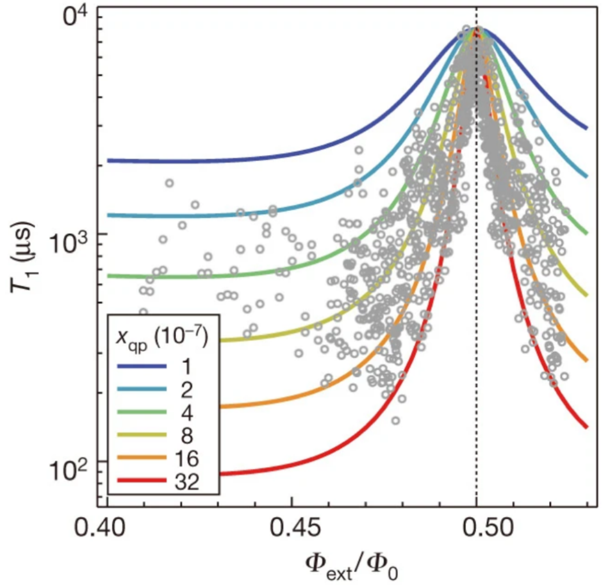
Lecture 3: Relaxation induced by quasiparticles in superconducting qubits





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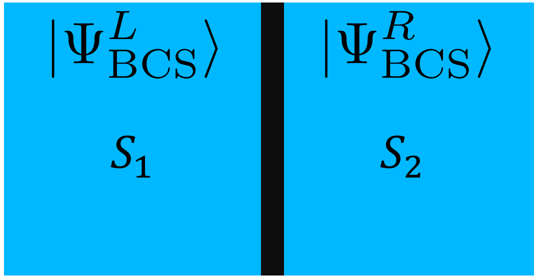
Content:

- Josephson effect derived from perturbation theory
- Qubit-Quasiparticle interaction
- Qubit relaxation induced by quasiparticle tunneling

- Josephson effect



$$|\Psi^{L/R}_{BCS}\rangle = \prod_{k=k_1,\dots,k_M} \left(|u_k| + |v_k| e^{i\varphi_{L/R}} c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}\right)|\Phi\rangle$$

$$H_T = \tilde{t}\sum_{m,n}\sum_{\sigma}(c^{\dagger L}_{m\sigma} c^R_{n\sigma} + H.c.),$$

$\tilde{t}$: constant tunneling matrix element

Bogoliubov quasiparticles:

$$c^\dagger_{n\downarrow} = -v^*_n \gamma_{n\uparrow} + u_n \gamma^\dagger_{n\downarrow} \qquad c_{n\uparrow} = u^*_n \gamma_{n\uparrow} + v_n \gamma^\dagger_{n\downarrow}$$

$$H_T = H_{qp} + H_p$$

$$H_{qp} = \tilde{t}\sum_{m,n,\sigma}\left(u_m v^*_n - v_m u^*_n\right)\gamma^{\dagger L}_{m\sigma}\gamma^R_{n\sigma} + h.c.$$

$$H_p = \tilde{t}\sum_{m,n,\sigma}\left(u_m v_n + v_m u_n\right)\gamma^{\dagger L}_{m\sigma}\gamma^{\dagger R}_{n\sigma} + h.c.$$

- Josephson effect

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$$H_{qp} = \tilde{t} \sum_{m,n,\sigma} \left(u_m v_n^* - v_m u_n^* \right) \gamma^{\dagger L}_{m\sigma} \gamma^{R}_{n\sigma} + h.c.$$

$$H_p = \tilde{t} \sum_{m,n,\sigma} \left(u_m v_n + v_m u_n \right) \gamma^{\dagger L}_{m\sigma} \gamma^{\dagger R}_{n\sigma} + h.c.$$

We take these two terms as perturbation, and aim to find the correction to the energy.

$$H = H_0 + \epsilon H_1 \qquad E_n(\epsilon) = E_n^{(0)} + \epsilon \langle n^{(0)} | H_1 | n^{(0)} \rangle + \epsilon^2 \sum_{k \neq n} \frac{\langle k^{(0)} | H_1 | n^{(0)} \rangle^2}{E_n^{(0)} - E_k^{(0)}}$$

What is the energy correction due to H_{qp}?

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H_{qp} gives NO correction to the energy, up to the second order perturbation theory.

What is the energy correction due to H_p?

The first order correction of H_p is zero

$$H_p = \tilde{t} \sum_{m,n,\sigma} (u_m v_n + v_m u_n) \gamma^{\dagger L}_{m\sigma} \gamma^{\dagger R}_{n\sigma} + h.c.$$

$$u_{m/n} = |u_{m/n}| e^{-i \varphi_{L/R}/2} \qquad v_{m/n} = |v_{m/n}| e^{+i \varphi_{L/R}/2}$$

$$u_m v_n = |u_m v_n| e^{i(\varphi_R - \varphi_L)/2}$$

$$E_n(\epsilon) = E_n^{(0)} + \epsilon \langle n^{(0)} | H_1 | n^{(0)} \rangle + \epsilon^2 \sum_{k \neq n} \frac{\langle k^{(0)} | H_1 | n^{(0)} \rangle^2}{E_n^{(0)} - E_k^{(0)}}$$

$$v_m u_n = |v_m u_n| e^{-i(\varphi_R - \varphi_L)/2}$$

- Josephson effect

$$H_p = \tilde{t} \sum_{m,n,\sigma} \left\{ |u_m v_n| e^{\frac{i\varphi}{2}} + |v_m u_n| e^{-\frac{i\varphi}{2}} \right\} \gamma^{+L}_{m\sigma} \gamma^{+R}_{n\sigma} + h.c$$

$$\delta E = -\tilde{t}^2 \sum_{|K\rangle \neq |\psi^L_{BCS}, \psi^R_{BCS}\rangle} \frac{|\langle K^{(0)} | H_p | \psi^L_{BCS}, \psi^R_{BCS} \rangle|^2}{E^{(0)}_K}$$

$$|K^{(0)}_1\rangle = \gamma^{+L}_{m\sigma} \gamma^{+R}_{n\sigma} |\psi^L_{BCS}, \psi^R_{BCS}\rangle$$

$$|\langle K^{(0)}_1 | H_p | \psi^L_{BCS} \psi^R_{BCS} \rangle|^2 = \underbrace{|u_m v_n|^2 + |v_m u_n|^2}_{\text{constant}} + |u_m||v_m||u_n||v_n| \left\{ e^{i\varphi} + e^{-i\varphi} \right\}$$

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$$|v_n|^2 = \frac{1}{2}\left(1 - \frac{\epsilon_n}{E_n}\right) \qquad |u_n|^2 = \frac{1}{2}\left(1 + \frac{\epsilon_n}{E_n}\right) \qquad E_n = \sqrt{\epsilon_n^2 + \Delta^2}$$

$$|v_n||u_n| = \frac{1}{2}\left(1 - \frac{\epsilon_n^2}{E_n^2}\right) = \frac{1}{2}\frac{\Delta}{E_n}$$

$$|u_m||v_m||u_n||v_n|\{e^{i\phi} - e^{-i\phi}\} = \frac{1}{2}\frac{\Delta^R}{E_n}\frac{\Delta^L}{E_m}\cos\phi$$

$$\delta E \propto -\boxed{\tilde{t}^2 \sum_{m,n} \frac{1}{E_n + E_m}\frac{\Delta^R}{E_n}\frac{\Delta^L}{E_m}} \cos\varphi$$

$$U = -E_J \cos\phi$$

$$E_J = \frac{1}{8}\frac{g_T}{g_K}\Delta$$

$g_T = 4\pi e^2 N_0^L N_0^R \tilde{t}^2$ Junction conductance

$g_K = e^2/2\pi$ conductance quantum

- Josephson effect

This energy is associated with a supercurrent that is driven by the phase difference across the junction:

$$I_J = \frac{2\pi}{\Phi_0}\frac{\partial U}{\partial \phi} = \frac{\pi}{2}\frac{\Delta}{e} g_T \sin\phi,$$

the AC-Josephson effect:

$$V = \frac{\Phi_0}{2\pi}\frac{d\phi}{dt}$$

$$L_J = V / \frac{dI_J}{dt} = \frac{1}{\pi \Delta g_T}\frac{1}{\cos\phi}$$

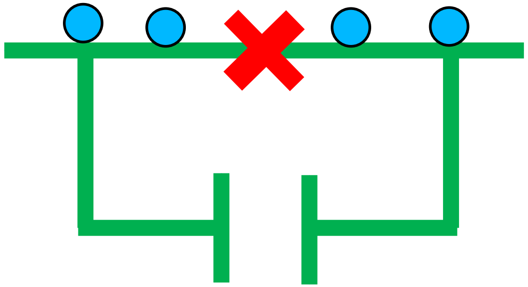
Inductance is the tendency of an electrical conductor to oppose a change in the electric current flowing through it.

changing current induces a voltage in the conductor.

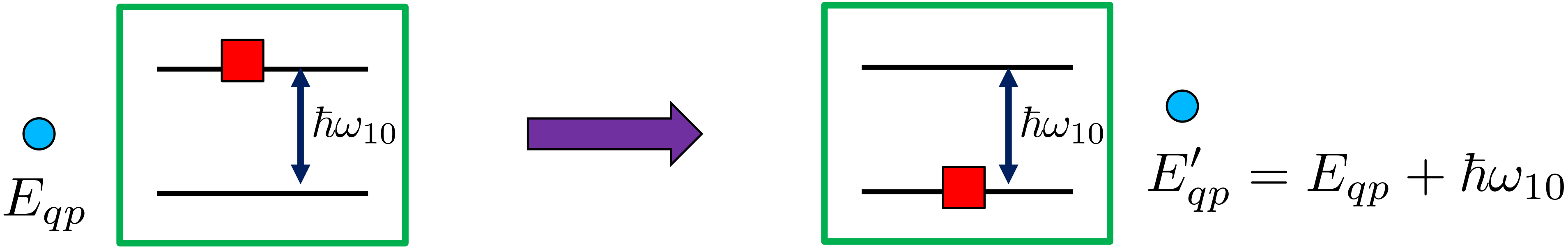
Nonlinearity + the ultra-low dissipation provided by superconductivity makes Josephson junctions promising candidates to build qubits.

- Qubit-Quasiparticle interaction

All types of superconducting qubits work based on the quantum coherence of the ground state of the superconductor.



What can happen if they are superconducting excitations (quasiparticle) in the circuit?



System initial state

System final state

- Qubit-Quasiparticle interaction

The system Hamiltonian in presence of quasiparticles can be divided into three parts:

$$H = H_{qubit} + \sum_{S=L,R} \sum_{k,\sigma} E_k \gamma^{S+}_{k\sigma} \gamma^{S}_{k\sigma} + H_{int}$$

$$H_{int} = H_{qp} = \tilde{t} \sum_{m,n,\sigma} \left\{ |u_m v_n| e^{\frac{i\varphi}{2}} - |v_m u_n| e^{\frac{-i\varphi}{2}} \right\} \gamma^{+L}_{m\sigma} \gamma^{R}_{n\sigma} + h.c.$$

$$|u_m| = \sqrt{\frac{1}{2}} \sqrt{1 + \frac{\epsilon_m}{\sqrt{\epsilon_m^2 + \Delta^2}}} \qquad |v_n| = \sqrt{\frac{1}{2}} \sqrt{1 - \frac{\epsilon_n}{\sqrt{\epsilon_n^2 + \Delta^2}}}$$

$$\epsilon_m, \epsilon_n \ll \Delta \qquad |u_m| = |v_n| \simeq \frac{1}{2}$$

- Qubit relaxation induced by quasiparticle tunneling

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$$H_{\text{int}} = \tilde{t} \sum_{m,n,\sigma} i \sin(\frac{\phi}{2}) \gamma^{\dagger L}_{m,\sigma} \gamma^{R}_{n,\sigma} + h.c.$$

We calculate the qubit relaxation rate using Fermi's golden role

$$\frac{1}{T_1} = \Gamma_{10} + \Gamma_{01}$$

$$\Gamma_{10} = 2\pi \langle\!\langle \sum_{\{\lambda\}_{\text{qp}}} |\underbrace{\langle 0, \{\lambda\}_{\text{qp}}|}_{\text{system final state}} H_{\text{int}} \underbrace{|1, \{\eta\}_{\text{qp}}\rangle}_{\text{system initial state}}|^2 \delta(E_{\lambda,\text{qp}} - E_{\eta,\text{qp}} - \omega_{10}) \rangle\!\rangle,$$

- double angular brackets $\langle\langle ... \rangle\rangle$ denote averaging over initial quasiparticle states
- the summation is over all quasiparticle states

- Qubit relaxation induced by quasiparticle tunneling

$$H_{\mathrm{int}} = \tilde{t} \sum_{k,k',\sigma} i \sin\frac{\hat{\phi}}{2} \gamma^{L\dagger}_{k\sigma} \gamma^{R}_{k'\sigma} + \mathrm{H.c.}$$

$$\Gamma_{10} = 2\pi \langle\!\langle \sum_{\{\lambda\}_{\mathrm{qp}}} |\langle 0, \{\lambda\}_{\mathrm{qp}} | H_{\mathrm{int}} | 1, \{\eta\}_{\mathrm{qp}} \rangle|^2 \delta(E_{\lambda,\mathrm{qp}} - E_{\eta,\mathrm{qp}} - \omega_{10}) \rangle\!\rangle,$$

The transition rate then factorizes into terms that separately account for qubit dynamics and quasiparticle kinetics.

$$\Gamma_{10} = |\langle 0 | \sin\frac{\hat{\phi}}{2} | 1 \rangle|^2 S_{\mathrm{qp}}(\omega_{10})$$

$$S_{\mathrm{qp}}(\omega) = 2\pi \tilde{t}^2 \langle\!\langle \sum_{k,k',\sigma} \sum_{\{\lambda\}_{\mathrm{qp}}} |\langle \{\lambda\}_{\mathrm{qp}} | \gamma^{L\dagger}_{k\sigma} \gamma^{R}_{k'\sigma} + \gamma^{R\dagger}_{k'\sigma} \gamma^{L}_{k\sigma} | \{\eta\}_{\mathrm{qp}} \rangle|^2 \delta(E_{\lambda,\mathrm{qp}} - E_{\eta,\mathrm{qp}} - \omega) \rangle\!\rangle$$

- Qubit relaxation induced by quasiparticle tunneling

$$S_{\mathrm{qp}}(\omega) = 2\pi\tilde{t}^2 \langle\!\langle \sum_{k,k',\sigma} \sum_{\{\lambda\}_{\mathrm{qp}}} |\langle\{\lambda\}_{\mathrm{qp}}|\gamma^{L\dagger}_{k\sigma}\gamma^{R}_{k'\sigma} + \gamma^{R\dagger}_{k'\sigma}\gamma^{L}_{k\sigma}|\{\eta\}_{\mathrm{qp}}\rangle|^2 \delta(E_{\lambda,\mathrm{qp}} - E_{\eta,\mathrm{qp}} - \omega)\rangle\!\rangle$$

$$\sum_{\{\lambda\}_{qp}} |\langle\{\lambda\}_{qp}|\gamma^{L\dagger}_{k\sigma}\gamma^{R}_{k'\sigma'} + \gamma^{R\dagger}_{k'\sigma'}\gamma^{L}_{k\sigma}|\{\eta\}_{qp}\rangle|^2 = \sum_{\{\lambda\}_{qp}} \langle\{\eta\}_{qp}|\gamma^{L\dagger}_{k\sigma}\gamma^{R}_{k'\sigma'} + \gamma^{R\dagger}_{k'\sigma'}\gamma^{L}_{k\sigma}|\{\lambda\}_{qp}\rangle\langle\{\lambda\}_{qp}|\gamma^{L\dagger}_{k\sigma}\gamma^{R}_{k'\sigma'} + \gamma^{R\dagger}_{k'\sigma'}\gamma^{L}_{k\sigma}|\{\eta\}_{qp}\rangle$$

$$\sum_{\{\lambda\}_{qp}} |\{\lambda_{qp}\}\rangle\langle\{\lambda_{qp}\}| = \mathbb{1}$$

$$= \langle\{\eta_{qp}\}|(\gamma^{L\dagger}_{k\sigma}\gamma^{R}_{k'\sigma'} + \gamma^{R\dagger}_{k'\sigma'}\gamma^{L}_{k\sigma})(\gamma^{L\dagger}_{k\sigma}\gamma^{R}_{k'\sigma'} + \gamma^{R\dagger}_{k'\sigma'}\gamma^{L}_{k\sigma})|\{\eta_{qp}\}\rangle$$

$$= \langle\{\eta_{qp}\}|\gamma^{L\dagger}_{k\sigma}\gamma^{L}_{k\sigma}\gamma^{R}_{k'\sigma'}\gamma^{R\dagger}_{k'\sigma'} + \gamma^{R\dagger}_{k'\sigma'}\gamma^{R}_{k'\sigma'}\gamma^{L}_{k\sigma}\gamma^{L\dagger}_{k\sigma}|\{\eta_{qp}\}\rangle$$

$$\Rightarrow \quad S_{\mathrm{qp}}(\omega) = 4\pi\tilde{t}^2 \sum_{k,k',\sigma} \langle\!\langle \langle\{\eta\}_{\mathrm{qp}}|\gamma^{R\dagger}_{k'\sigma}\gamma^{R}_{k'\sigma}|\{\eta\}_{\mathrm{qp}}\rangle\langle\{\eta\}_{\mathrm{qp}}|\gamma^{L}_{k\sigma}\gamma^{L\dagger}_{k\sigma}|\{\eta\}_{\mathrm{qp}}\rangle\delta(E_{\lambda,\mathrm{qp}} - E_{\eta,\mathrm{qp}} - \omega)\rangle\!\rangle$$

- Qubit relaxation induced by quasiparticle tunneling

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$$S_{\rm qp}(\omega) = 4\pi\tilde{t}^2 \sum_{k,k',\sigma} \langle\langle\langle\{\eta\}_{\rm qp}|\gamma^{R\dagger}_{k'\sigma}\gamma^{R}_{k'\sigma}|\{\eta\}_{\rm qp}\rangle\langle\{\eta\}_{\rm qp}|\gamma^{L}_{k\sigma}\gamma^{L\dagger}_{k\sigma}|\{\eta\}_{\rm qp}\rangle\delta(E_{\lambda,{\rm qp}} - E_{\eta,{\rm qp}} - \omega)\rangle\rangle$$

$$E_{\lambda,qp} - E_{\eta,qp} = E^L_{\lambda,qp} + E^R_{\lambda,qp} - E^L_{\eta,qp} - E^R_{\eta,qp} = \left(E^L_{\lambda,qp} - E^L_{\eta,qp}\right) - \left(E^R_{\eta,qp} - E^R_{\lambda,qp}\right) = \epsilon^L - \epsilon^R$$

$$\langle\langle\langle\{\eta\}_{\rm qp}|\gamma^{R\dagger}\gamma^{R}|\{\eta\}_{\rm qp}\rangle\rangle\rangle = f(\varepsilon^R) \qquad \langle\langle\langle\{\eta\}_{\rm qp}|\gamma^{L}\gamma^{L\dagger}|\{\eta\}_{\rm qp}\rangle\rangle\rangle = 1 - f(\varepsilon^L)$$

$$S_{\rm qp}(\omega) = \frac{32E_J}{\pi\Delta}\int_\Delta^\infty n(\varepsilon)n(\varepsilon+\omega)f(\varepsilon)[1-f(\varepsilon+\omega)]d\varepsilon.$$

$$E_J = \frac{1}{8}\frac{g_T}{g_K}\Delta$$

$$g_T = 4\pi e^2 N_0^L N_0^R \tilde{t}^2$$

$$g_K = e^2/2\pi$$

- Qubit relaxation induced by quasiparticle tunneling

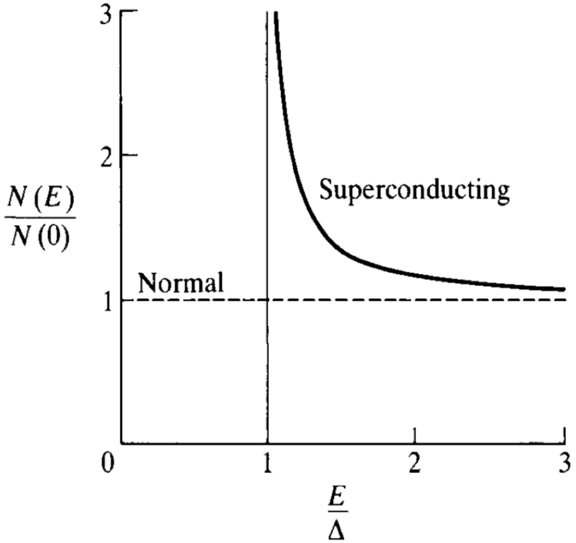
$$S_{\rm qp}(\omega) = \frac{32E_J}{\pi\Delta}\int_{\Delta}^{\infty} n(\varepsilon)n(\varepsilon+\omega)f(\varepsilon)[1-f(\varepsilon+\omega)]d\varepsilon.$$

$$n(E) = \frac{E}{\sqrt{E^2-\Delta^2}}$$

If, $E-\Delta=\delta E \ll \omega$ (i.e. we have cold QPs)

$$1-f(\varepsilon+\omega) \simeq 1$$

$$n(\varepsilon+\omega) = \sqrt{\frac{\Delta}{2\omega}}$$



$$S_{\rm qp}(\omega) = \frac{8E_J}{\pi}x_{\rm qp}\sqrt{\frac{2\Delta}{\omega}}$$

$$x_{\rm qp} = \frac{2}{\Delta}\int_{\Delta}^{\infty} n(\varepsilon)f(\varepsilon)d\varepsilon,$$

$$\Gamma_{10} = |\langle 0|\sin\frac{\hat{\phi}}{2}|1\rangle|^2 S_{\rm qp}(\omega_{10})$$

Normalized density of QPs

- Qubit relaxation induced by quasiparticle tunneling

$$S_{\mathrm{qp}}(\omega) = \frac{8E_J}{\pi} x_{\mathrm{qp}} \sqrt{\frac{2\Delta}{\omega}} \qquad x_{\mathrm{qp}} = \frac{2}{\Delta}\int_{\Delta}^{\infty} n(\varepsilon) f(\varepsilon) d\varepsilon, \qquad \Gamma_{10} = |\langle 0|\sin\frac{\hat{\phi}}{2}|1\rangle|^2 S_{\mathrm{qp}}(\omega_{10})$$

Normalized density of QPs

$$\frac{1}{T_1} = \Gamma_{10} + \Gamma_{01}$$

What about Γ_{01}?

$$\Gamma_{01} = \Gamma_{10} e^{-\hbar\omega_{10}/k_B T}$$

(assuming thermal equilibrium)

- How can we limit the qubit relaxation induced by QPs?

This is the subject of the next lecture!

- Further reading:

John M. Martinis and Kevin Osborne, *Superconducting Qubits and the Physics of Josephson Junctions*
arXiv:cond-mat/0402415

Gianluigi Catelani, et al, *Relaxation and frequency shifts induced by quasiparticles in superconducting qubits*
Phys Rev B **84**, 064517 (2011)

Leonid I. Glazman, Gianluigi Catelani, *Bogoliubov Quasiparticles in Superconducting Qubits*
arXiv:2003.04366