Universität
Konstanz

Integrierter Kurs Physik IV
Exp.-Teil – Atomphysik
SoSe 19

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Problem set 13

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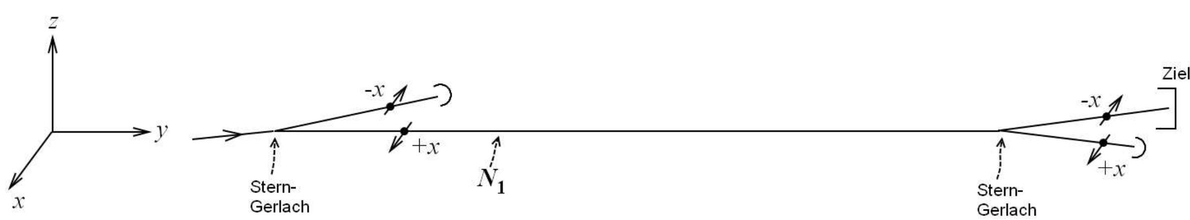
<u>Exercise 36:</u> Series of Stern-Gerlach experiments (1 cross)

A Stern-Gerlach setup (SG) is used to filter a beam of particles with spin $S = 1/2$. The SG is positioned in a way that it splits the particles by the $x-$component of their magnetic moment μ_x. Only particles with magnetic moment in positive $x-$direction (*state* $|x+\rangle$) are retained. These are N_1 per second. As shown in the sketch, the stream ends up in a second SG which filters only particles with magnetic moment in negative $x-$direction (*state* $|x-\rangle$). The SGs are much smaller than the overall instruction itself. From the outside of the SGs $\vec{B} = 0$.



1. How many particles arrive at the target?
2. Now use a third Stern-Gerlach filter to maximize the number of particles reaching the target. Place it in the sketch and show the direction of the magnetic moment of the particles, both in the rejected and retained beam.
   Notes: the SGs do not have to be drawn in full details. Use the straight line as the retaining beam and ignore the fact the the beam is bent by the SG. Also consider the orientation of the SG.
3. How many particles arrive now at the target (as a function of N_1)?
4. Give the state of the particle after the filter as the function of $|x-\rangle$ and $|x-\rangle$ (the

absolute phase is not important here).

5. Give an example of an atom or a particle that could be used for this experiment.

6. Sketch a comparable setup of three linear polarizers for visible light.

Exercise 37: Lamb-Shift (1 cross)

In the years 1947-1952 Lamb and Retherford were able to show that even the relativistic Dirac theory does not yet fully describe the hydrogen atom. They observed a small difference in energy between the terms $2^2S_{1/2}$ and $2^2P_{1/2}$ (Nobel-Prize 1955).

1. Explain qualitatively the Lamb Shift effect and its causes. For which states does it become pronounced? Sketch the scheme for the states $n = 1; 2$ of hydrogen atom while considering relativistic effects, spin-orbit coupling and Lamb shift. (There is no external magnetic field and the hyperfine structure can be neglected).

2. Calculate the transition series $2^2P_{3/2} \to 1^2S_{1/2}$, $2^2P_{1/2} \to 1^2S_{1/2}$ and $2^2S_{1/2} \to 1^2S_{1/2}$ for hydrogen atom taking the fine structure and the Lamb shift into account.

3. Compare the magnitude of the calculated Lamb shift with the Doppler broadening calculated in exercise 25. Consider why the small energy shift of the Lamb-Shift was not observable at that time by using optical spectroscopy.

Exercise 38: Hydrogen atom in an electric field (linear Stark effect) (written) **(10 points)**

Consider an electron inside the Coulomb potential $V(r)$ of an proton with the Hamilton operator $H_0 = \frac{\mathbf{p}^2}{2m} + V(r)$. Now consider only the wave function with quantum number $n = 2$.

1. Which of these operators $\left[\hat{H},\ \hat{p}_i,\ \hat{L}^2 \text{ und } \hat{L}_z\right]$ have common eigenstates? How many eigenstates belong to the wave functions, i. what is the degeneracy?

2. Characterize the states according to the inversion $\mathbf{r} \to \mathbf{r}' = -\mathbf{r}$, which is defined in spherical coordinates as $(r, \vartheta, \varphi) \to (r' = r, \vartheta' = \pi - \vartheta, \varphi' = \pi + \varphi)$.

3. A constant electric field of strength $\approx 10^4 - 10^5$ V/cm in **z**-direction is applied to the system. Show that this field can be considered as a small perturbation to the inner atomic field. The resulting potential energy is given by $V = -e\vec{r} \cdot \vec{E}$. Give the Hamilton operator in stationary perturbation theory of first order.

4. Calculate the matrix elements $V_{\Psi_{2\ell m}, \Psi_{2\ell' m'}}$ based on the associated eigenfunctions of the unperturbed system.
Note: some of the matrix elements disappear by symmetry reason.

5. Calculate the eigenvalues of the perturbed system and the corresponding wave functions.

6. Sketch the splitting of the levels and consider (without calculation) how the energy splitting is related to the direction of the electric dipol moment.