Universität
Konstanz

Integrierter Kurs Physik IV
Exp.-Teil – Atomphysik
SoSe 19

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Problem set 12

Posted: 01.07.2019, Due: 08.07.2019

<u>Exercise 33:</u> Direct measurement of wave functions (1 cross)

Download the following scientific article: J.S. Lundeen et al.: *Direct measurement of quantum wavefunction*, Nature, **474**, 188 (2011). You can access the article from inside the eduroam WLAN network or via VPN connection to the university network. Read the article and discuss it in the exercise group.

<u>Exercise 34:</u> Harmonic Oscillator: Position measurments (1 cross)

1. Calculate the quantum mechanical position uncertainty Δx of the 1D harmonic oscillator with energy eigenstate n *Hint:* Write the position operator $\hat{x}$ as linear combination of the annihilation and creation operators $\hat{a}$ and $\hat{a}^\dagger$. The same goes for $\hat{x}^2$.
2. Suppose one oscillator is in a certain energy eigenstate. Can you tell which one it is from a measurement of x? After the measurement, is the oscillator still in that certain energy state? What is the fundamental reason for that?

<u>Exercise 35:</u> 3D-Oscillator in Cartesian and spherical coordinates
(written) **(10 points)**

We consider the Hamilton function of an isotropic harmonic oscillator in 3D

$$\hat{H} = \frac{\hat{p}^2}{2m_\mathrm{e}} + \frac{1}{2}m_\mathrm{e}\omega^2\hat{r}^2,$$

with $\hat{r}^2 = \hat{x}^2 + \hat{y}^2 + \hat{z}^2$.

1. Rewrite the Hamilton function as a sum of three operators. Choose the three operators in a way that they commute pairwise and that we know their eigenvalues from the 1D problem.

2. Calculate the energy $E = \hbar\omega(n_1 + n_2 + n_3 + 3/2)$, with n_i positive integers.
3. What is the degeneration of the first four energy levels?

   To calculate this result with the results from the movement in a central potential, we use the dimensionless figures $\rho = r\sqrt{\frac{m_e\omega}{\hbar}}$ and $\epsilon = \frac{E}{\hbar\omega}$.
4. Rewrite the Schrödinger equation in the form

$$\left(-\frac{1}{\rho}\frac{\partial^2}{\partial\rho^2}\rho + \frac{\ell(\ell+1)}{\rho^2} + \rho^2 - 2\epsilon\right) R_\ell(\rho) = 0. \tag{1}$$

   One can show that the solutions of equation (1) depend on an integer n'

$$R_{n',\ell}(\rho) = \rho^\ell P_{n',\ell}(\rho) e^{-\rho^2/2}$$

   with $P_{n',\ell}(\rho)$ polynomial of degree $2n'$. The solutions correspond to certain values of ϵ

$$\epsilon = 2n' + \ell + 3/2.$$

   We use $n = 2n' + \ell$.
5. Calculate the energy levels. Show that you need three numbers n, ℓ and m to describe the states of the oscillator (see b)).
6. For the first four energy levels, calculate the possible quantum numbers. Count the degree of degeneracy of those four levels and compare with c).

   Extra: What is the degeneracy of the n-th level?
7. What is the relation between $|n_1; n_2; n_3\rangle$ and $|n; \ell; m\rangle$ for $n = 1$?

   Hint: For the 1D harmonic oscillator the first two eigenfunctions are $\phi_0(x) \propto e^{-\alpha x^2/2}$ *and* $\phi_1(x) \propto x e^{-\alpha x^2/2}$. *For* $\ell = 1$ *the spherical harmonics are given by*

$$Y_{1,1}(\theta,\varphi) = -\sqrt{\frac{3}{8\pi}}\sin\theta e^{i\varphi}$$
$$Y_{1,0}(\theta,\varphi) = \sqrt{\frac{3}{4\pi}}\cos\theta$$
$$Y_{1,-1}(\theta,\varphi) = \sqrt{\frac{3}{8\pi}}\sin\theta e^{-i\varphi}$$