Universität
Konstanz

Integrierter Kurs Physik IV
Exp.-Teil – Atomphysik
SoSe 19

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Problem set 11

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Exercise 30: LS coupling and relativistic correction (written) **(8 points)**

To calculate level splittings because of the relativistic correction and LS coupling, the mean values $< 1/r >$, $< 1/r^2 >$ and $< 1/r^3 >$ are needed for states described by the H atom wave functions.
Hint:$\langle \vec{r} \rangle \equiv \int d^3r \vec{r} \left| \Psi \left(\vec{r}, t \right) \right|^2$

1. Calculate $< 1/r >$ and $< 1/r^2 >$ for the 2s state. $R_{2,0}(r) = \frac{2}{(2a)^{3/2}}(1 - \frac{r}{2a})e^{-r/2a}$. The first relativistic correction is given by

$$E = \sqrt{p^2c^2 + m^2c^4} = mc^2 + \frac{p^2}{2m} - \frac{p^4}{8m^3c^2} + \ldots.$$

   We have to consider the increase in mass of the electron due to it's velocity, which leads to a correction of the corresponding energies.
   $H = \dfrac{p^2}{2m} + V$ leads to $p^4 = 4m^2[H - V]^2$, with $V = \dfrac{e^2}{4\pi\varepsilon_0 r}$ the Coulomb energy the energy of a eigenstate $< H >= E_n$ as known from the Bohr model, or $H\Psi = E_n\Psi$.

   Calculate $< W_{mv} >=< \dfrac{p^4}{8m^3c^2} >$ for the 2s state.

2. Calculate $< 1/r^3 >$ for the 2p state. $R_{2,1}(r) = \frac{1}{\sqrt{3}(2a)^{3/2}} \frac{r}{a} e^{-r/2a}$. Calculate the LS coupling correction of the energy

$$< E_{ls} >=< \frac{e^2}{4\pi\varepsilon_0} \cdot \frac{1}{2m^2c^2} \cdot \frac{1}{r^3} \vec{L} \cdot \vec{S} >$$

   for the states ${}^2P_{3/2}$ and ${}^2P_{1/2}$. First you have to calculate the angle between $\vec{L}$ and $\vec{S}$ for each case respectively (the vector addition leads to $\vec{J}$ with length $3/2\ \hbar$ or $1/2\ \hbar$).

<u>Exercise 31:</u> Landé-Factor (1 Cross)

In a weak magnetic field $\vec{L}$ and $\vec{S}$ couple to form the total angular momentum $\vec{J} = \vec{L} + \vec{S}$.

1. With a drawing, starting from $\vec{\mu}_L = -\frac{e}{2m}\vec{L}$ and $\vec{\mu}_S = -\frac{e}{m}\vec{S}$, show that the magnetic moment $\vec{\mu}_J = \vec{\mu}_L + \vec{\mu}_S$ is *not* parallel to $\vec{J}$. Use the intercept theorem.

2. Project $\vec{\mu}_J$ on $\vec{J}$, to get the Landé-Factor

$$g_j = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}$$

   l, s and j are defined with $|\vec{L}| = \hbar\sqrt{l(l+1)}$, $|\vec{S}| = \hbar\sqrt{s(s+1)}$ and $|\vec{J}| = \hbar\sqrt{j(j+1)}$, and $\vec{\mu}_J \cdot \frac{\vec{J}}{|\vec{J}|} = -\frac{e}{2m} g_j |\vec{J}|$.

<u>Exercise 32:</u> Electron Spin classical analogy? (1 Cross)

In the following we will try to do a classical treatment of the spin of a electron. Consider the electron to be a ball of $r_e < 10^{-16}$ m and $m_e = 9.109 \cdot 10^{-31}$ kg.

1. What is the rotation frequency we need, to explain the spin obtained from measurements?

2. What rotaion frequency does the electron need to have to explain the experimentally measured magnetic moment of an electron? Consider the charge to be concentrated on the equatorial line of the ball. The resulting circulating current leads to a magnetic dipole moment. Check your units!

3. What's the resulting rotation speed along the equatorial line of the electron. What is the result for a classical explaination of the electron spin?