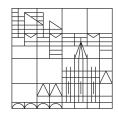
Universität Konstanz



Integrierter Kurs Physik IV Exp.-Teil – Atomphysik SoSe 19

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Problem set 10

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Exercise 28: Pauli matrices (written) (8 points)

The two possible states of a particle with spin s = 1/2 can be expressed as

$$\left|s = \frac{1}{2}, m_s = \frac{1}{2}\right\rangle = \begin{pmatrix} 1\\0 \end{pmatrix} = \left|\uparrow\right\rangle \text{ and } \left|s = \frac{1}{2}, m_s = -\frac{1}{2}\right\rangle = \begin{pmatrix} 0\\1 \end{pmatrix} = \left|\downarrow\right\rangle$$

With quantization in z direction, the matrix representation of the spin operator is $\hat{\mathbf{s}} = \frac{\hbar}{2}\hat{\boldsymbol{\sigma}}$ with the Pauli spin matrices

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

1. Show

$$\hat{\sigma}_i^2 = 1$$
, $\hat{\sigma}_x \hat{\sigma}_y = i\hat{\sigma}_z$, $[\hat{\sigma}_x, \hat{\sigma}_y] = 2i\hat{\sigma}_z$, $\hat{\boldsymbol{\sigma}} \times \hat{\boldsymbol{\sigma}} = 2i\hat{\boldsymbol{\sigma}}$

2. Calculate the anti commutators $(\{\hat{A}, \hat{B}\}) := \hat{A}\hat{B} + \hat{B}\hat{A}$

$$\{\hat{\sigma}_x, \hat{\sigma}_y\}, \{\hat{\sigma}_y, \hat{\sigma}_z\}, \{\hat{\sigma}_x, \hat{\sigma}_z\}$$

- 3. The ladder operators are given by $\hat{\sigma}_{\pm} := \frac{1}{2} \left(\hat{\sigma}_x \pm i \hat{\sigma}_y \right)$. Show the effect of $\hat{\sigma}_{\pm}$ on the states $|\uparrow\rangle$ und $|\downarrow\rangle$
- 4. Calculate the eigenstates and eigenvectors of \hat{s}_z and $\hat{\mathbf{s}}^2$. Show the following equalities

$$\langle sm'_s | \hat{s}_z | sm_s \rangle = \hbar m_s \delta_{m_s, m'_s}$$
$$\langle sm'_s | \hat{\mathbf{s}}^2 | sm_s \rangle = \hbar^2 s(s+1) \delta_{m_s, m'_s}$$
$$\langle sm'_s | \hat{s}_{\pm} | sm_s \rangle = \hbar \sqrt{(s \mp m_s)(s \pm m_s + 1)} \delta_{m_s \pm 1, m'_s}.$$

5. Obtain the following relation which is important for the time evolution of states

$$e^{i\alpha\hat{\sigma}_i} = 1\cos\alpha + i\hat{\sigma}_i\sin\alpha$$

Exercise 29: The Zeeman splitting for transitions identification (1 cross for a-c and d-f)

Inside a weak magnetic field of 2 T, the spectral line of an atom ($\lambda_0 = 766, 7012nm$) splits into six components

line	wavelength [nm]
A1	766,6097
A2	766,6463
B1	766,6829
B2	766,7195
C1	766,7561
C2	766,7927

Looking in a plane perpendicular to the B field, the lines are linearly polarized (A and C in the plane, B parallel to the B field). Measuring in the plane parallel to the field, only A and C can be observed with opposing circular polarizations.

- 1. Is it due to the normal or the anomalous Zeeman effect? Why?
- 2. We have LS coupling and we consider a transition inside the fine structure of the main level, s = 1/2. To find out which orbitals are involved in the transitions, first calculate all occurring $\Delta(gm_j) = g_2m_{j2} g_1m_{j1}$.
- 3. You want to find out into how many sub levels the two levels split up. Then you know j. For one level two lines along the B field are missing. What is the conclusion from that?
- 4. What do we know about the other level from the fact that there are six lines?
- 5. What are the two possible transitions? Use spectroscopic nomenclature $^{2s+1}X_j$ with X = S, P, D, ... for l = 0, 1, 2, ...
- 6. Calculate the Landé g factor that is involved. Identify the transition by comparing with b).