Universität
Konstanz

Integrierter Kurs Physik IV
Exp.-Teil – Atomphysik
SoSe 19

Prof. E. Weig, Anh-Tuan Le, Felix Rochau

Problem set 10

Posted: 17.06.2019, Due: 24.06.2019

Exercise 28: Pauli matrices (written) **(8 points)**

The two possible states of a particle with spin $s = 1/2$ can be expressed as

$$\left| s = \frac{1}{2}, m_s = \frac{1}{2} \right\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |\uparrow\rangle \text{ and } \left| s = \frac{1}{2}, m_s = -\frac{1}{2} \right\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |\downarrow\rangle$$

With quantization in z direction, the matrix representation of the spin operator is $\hat{\mathbf{s}} = \frac{\hbar}{2}\hat{\boldsymbol{\sigma}}$ with the Pauli spin matrices

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \; \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \; \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

1. Show
$$\hat{\sigma}_i^2 = \mathbb{1}, \; \hat{\sigma}_x \hat{\sigma}_y = i\hat{\sigma}_z, \; [\hat{\sigma}_x, \hat{\sigma}_y] = 2i\hat{\sigma}_z, \; \hat{\boldsymbol{\sigma}} \times \hat{\boldsymbol{\sigma}} = 2i\hat{\boldsymbol{\sigma}}$$

2. Calculate the *anti commutators* ($\{\hat{A}, \hat{B}\} := \hat{A}\hat{B} + \hat{B}\hat{A}$)
$$\{\hat{\sigma}_x, \hat{\sigma}_y\}, \; \{\hat{\sigma}_y, \hat{\sigma}_z\}, \; \{\hat{\sigma}_x, \hat{\sigma}_z\}$$

3. The ladder operators are given by $\hat{\sigma}_\pm := \frac{1}{2}(\hat{\sigma}_x \pm i\hat{\sigma}_y)$. Show the effect of $\hat{\sigma}_\pm$ on the states $|\uparrow\rangle$ und $|\downarrow\rangle$

4. Calculate the eigenstates and eigenvectors of $\hat{s}_z$ and $\hat{\mathbf{s}}^2$. Show the following equalities
$$\langle sm_s' | \hat{s}_z | sm_s \rangle = \hbar m_s \delta_{m_s, m_s'}$$
$$\left\langle sm_s' \left| \hat{\mathbf{s}}^2 \right| sm_s \right\rangle = \hbar^2 s(s+1) \delta_{m_s, m_s'}$$
$$\langle sm_s' | \hat{s}_\pm | sm_s \rangle = \hbar \sqrt{(s \mp m_s)(s \pm m_s + 1)} \delta_{m_s \pm 1, m_s'}.$$

5. Obtain the following relation which is important for the time evolution of states

$$e^{i\alpha\hat{\sigma}_i} = \mathbb{1}\cos\alpha + i\hat{\sigma}_i \sin\alpha$$

Exercise 29: The Zeeman splitting for transitions identification (1 cross for a-c and d-f)

Inside a weak magnetic field of 2 T, the spectral line of an atom ($\lambda_0 = 766,7012nm$) splits into six components

line	wavelength [nm]
A1	766,6097
A2	766,6463
B1	766,6829
B2	766,7195
C1	766,7561
C2	766,7927

Looking in a plane perpendicular to the B field, the lines are linearly polarized (A and C in the plane, B parallel to the B field). Measuring in the plane parallel to the field, only A and C can be observed with opposing circular polarizations.

1. Is it due to the normal or the anomalous Zeeman effect? Why?
2. We have LS coupling and we consider a transition inside the fine structure of the main level, $s = 1/2$. To find out which orbitals are involved in the transitions, first calculate all occurring $\Delta(gm_j) = g_2 m_{j2} - g_1 m_{j1}$.
3. You want to find out into how many sub levels the two levels split up. Then you know j. For one level two lines along the B field are missing. What is the conclusion from that?
4. What do we know about the other level from the fact that there are six lines?
5. What are the two possible transitions? Use spectroscopic nomenclature $^{2s+1}X_j$ with $X = S, P, D, ...$ for $l = 0, 1, 2, ...$
6. Calculate the Landé g factor that is involved. Identify the transition by comparing with b).