



**Integrierter Kurs Physik IV**  
**Exp.-Teil – Atomphysik**  
**SoSe 19**

Prof. E. Weig, Anh-Tuan Le, Felix Rochau

**Problem set 9**

Posted: 10.06.2019, Due: 17.06.2019

Exercise 25: Doppler Broadening of Spectral Lines (written) (8 Points)

A Sodium-vapour lamp basically emits because of a transition we will discuss in the following. At the moment we ignore the fact, that the line is in reality a double line. The lamp is operated at 500K. A spectroscopic measurement yields a Gaussian profile with a linewidth of  $\delta\omega_D = 1.07 \times 10^{10} \text{ s}^{-1}$

1. The Doppler broadening leads to a considerable broadening of the spectral line. Consider an atom moving with velocity  $\vec{v}$  and emitting a photon with frequency  $\omega_0$  in direction  $\vec{k}$ . What's the frequency that the observer "sees"? What is the frequency the light wave (moving in z-direction) should have to be able to be absorbed by the moving atom?
2. Now consider a gas in thermal equilibrium at 500K. Calculate the number of atoms that emit/absorb in the frequency interval between  $\omega$  and  $\omega + d\omega$ . Extract the radiation intensity  $P(\omega)d\omega$ .

*Hint: Use Maxwell velocity distribution*

$$n_i(v_z)dv_z = \frac{N_i}{v_\omega\sqrt{\pi}} \exp\left[-\left(\frac{v_z}{v_\omega}\right)^2\right] dv_z$$

with  $v_\omega = \sqrt{\frac{2k_B T}{m}}$   
and  $N_i = \int_{-\infty}^{\infty} n_i(v_z)dv_z$ .

Solution:  $P(\omega)d\omega = P(\omega_0) \cdot \exp\left(-\left[\frac{c(\omega-\omega_0)}{\omega_0 v_\omega}\right]^2\right) d\omega$

3. Calculate the linewidth  $\delta\omega_D(\omega_0, T, m) = |\omega_1 - \omega_2|$  with  $P(\omega_1) = P(\omega_2) = P(\omega_0)/2$ .

solution:  $\delta\omega_D = \frac{\omega_0}{c} \sqrt{\frac{8k_B T \cdot \ln 2}{m}}$

4. What is the linewidth of the Na-D line ( $\Delta E = 2,11 \text{ eV}$ )? What is the colour of the lamp? Compare the Doppler broadening to the natural linewidth of the NA-D line (lifetime  $\tau = 16 \text{ ns}$ ; Molecular mass  $M_{Na} = 0.023 \text{ kg/mol}$ ).
5. In fact we have to consider a double line. Look in the literature for the distance of the two line. Can they be resolved?

Exercise 26: Stern-Gerlach-Experiment (1 Cross)

During a Stern-Gerlach experiment, H atoms in the ground state exit an oven with a mean velocity of  $v_x = 14,5 \text{ km/s}$ . The magnetic field  $B$  is oriented in  $z$  direction and has a maximal gradient of  $dB_z/dz = 600 \text{ T/m}$ .

1. Calculate the maximal acceleration of the H atoms.
2. The  $B$ -field covers a distance of  $\Delta x_1 = 0.75 \text{ m}$  and the detector has a distance of  $\Delta x_2 = 1.75 \text{ m}$  to the "end" of the magnetic field. Calculate the distance of the two spots on the detector. For simplicity calculate with the maximale acceleration inside the field and with no acceleration after the end of the field.
3. What happens if you do the experiment with charged particles (e.g. free electrons)?

Exercise 27: Einstein-de-Haas Effect (1 Cross)

1. A lead cylinder is suspended in a way that it can rotate around it's symmetry axis without friction. With a coil it is magnetized to saturation. After changing the polarity of the coil current one can observe that the cylinder ( $M_{Fe}$  and  $\Theta_{Fe}$ ) is rotating with frequency  $\omega$ . Explain this observation and calculate  $\omega$  as a function of the atomic angular momentum. In which direction is the cylinder rotating?
2. Calculate the gyromagnetic ratio of a lead atom from the magnetic moment of the cylinder and the measured frequency.
3. Calculate  $\omega$ . Suppose that the angular momentum of each lead atom is the angular momentum of a electron on the first Bohr orbital. The length of the cylinder is 1 cm, it's weight is 1 g.

*Hint:*  $\rho_{Fe} = 7.87 \text{ g/cm}^3$