Universität Konstanz

Integrierter Kurs Physik IV
Exp.-Teil – Atomphysik
SoSe 19

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Problem set 8

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Exercise 22: Dipole matrix elements (written) **(7 Points)**

The simplest Hydrogen wave functions $\Psi_{n,l,m}(r,\vartheta,\varphi)$ are:

$$\Psi_{1,0,0} = \frac{1}{\sqrt{\pi}a^{3/2}}e^{-r/a}, \quad \Psi_{2,0,0} = \frac{1}{4\sqrt{2\pi}a^{3/2}}(2-\frac{r}{a})e^{-r/(2a)} \quad \text{und} \quad \Psi_{2,1,0} = \frac{1}{4\sqrt{2\pi}a^{3/2}}\frac{r}{a}e^{-r/(2a)}\cos\vartheta.$$

a is the Bohr radius.

1. Calculate the dipole matrix element

$$\vec{D} = \int d^3\vec{r}\,\Psi_A^*\,e\vec{r}\,\Psi_B = \int_0^\infty r^2\,dr\,\int_0^{2\pi}d\varphi\int_0^\pi \sin\vartheta\,d\vartheta\,\Psi_A^*(r,\vartheta,\varphi)\,e\begin{pmatrix} r\sin\vartheta\cos\varphi \\ r\sin\vartheta\sin\varphi \\ r\cos\vartheta \end{pmatrix}\Psi_B(r,\vartheta,\varphi)$$

   for the cases:

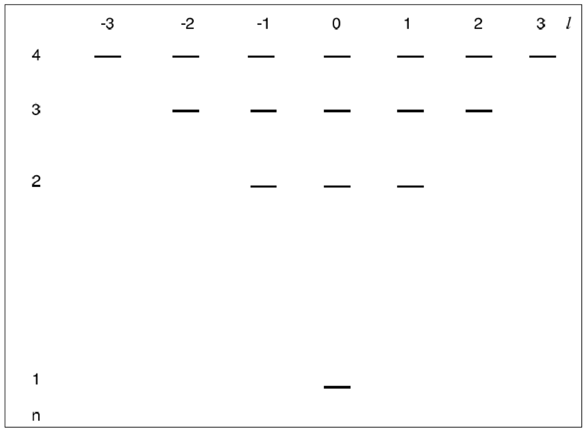
   i) $\Psi_A = \Psi_{1,0,0}$ and $\Psi_B = \Psi_{1,0,0}$,

   ii) $\Psi_A = \Psi_{1,0,0}$ and $\Psi_B = \Psi_{2,0,0}$,

   iii) $\Psi_A = \Psi_{1,0,0}$ and $\Psi_B = \Psi_{2,1,0}$.

   (Symbol e in the formula for $\vec{D}$ depicts the elementary charge.)

2. Take the results of a) as confirmation that dipole transitions between levels of arbitrary (here different) n are allowed, but the l must differ exactly by one, and it does not matter if the initial or the final level has the higher l. In our hydrogen model (so far without relativistic correction and spin), for every n (starting with 1) there are n degenerate states with $l = 0,\ldots,n-1$. A splitting or degeneracy with respect to m is not considered in this subtask, i. In a) we took only Ψ with $m = 0$; so we conclude



that there are allowed dipole transitions when δm and m are zero, but we have not yet checked other cases. The former figure shows the states up to $n = 4$. Enter all allowed dipole transitions by joining the corresponding bars, considering only states with $m = 0$.)

Exercise 23: Normal Zeeman effect I **(1 Cross)**

If one places a hypothetical hydrogen atom (without electron spin) in a time-independent magnetic field, then the Schrödinger energy levels $n\ell$ split into groups of $2\ell + 1$ sub-levels, since the magnetic field removes the energy degeneracy for m. The energy shift is proportional to m, with the same proportionality factor for different $n\ell$.

1. Sketch the splitting of a s, a p and a d level.
2. How many different lines can be observed in the transition from a $p-$ level to a s level, taking into account the selection rule $\delta m = 0, \pm 1$? How many for a transition from d to a p level? How many at any transition?
3. Calculate the energy difference between two adjacent m values of the $p-$level for $B = 1\,T$. What value do you get in this case for the Lamor frequency ω_L?

Exercise 24: Normal Zeeman effect II **(1 Cross)**

Consider a d state with quantum angular momentum $\ell = 2$ and a f state with $\ell = 3$ (no spin) in a magnetic field $B = 1$ T along the z axis.

1. Draw and calculate the possible orientation angles α of $\vec{L}$ with respect to the z axis. Also, determine the associated precession frequencies $\omega_\mathrm{L} = \frac{|\vec{M}|}{|\vec{L}| \sin \alpha}$, where the torque $\vec{M}$ is given by $\vec{B}$ exerting the magnetic moment $\vec{\mu} = -\mu_\mathrm{B} \vec{L}/\hbar$ (see Lecture).
2. What is the energy difference for $\ell = 2$ and $\ell = 3$? neighboring levels from the splitting in the magnetic field, i.e. those that differ by $\Delta m = 1$? Sketch the optically allowed transitions from d to f; $\Delta m = 0, \pm\ 1$. How many different wavelengths of light does one need to excite all these transitions?