Universität Konstanz

Integrierter Kurs Physik IV
Exp.-Teil – Atomphysik
SoSe 19

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Problem set 7

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Exercise 19: Franck-Hertz Experiment **(1 Cross)**

In a Frank-Hertz experiment electrons are accelerated with a variable voltage and then they collide with atoms in a Hg vapor at a pressure of about 10^{-2} mbar. The ionization energy of Hg is 4.9 eV.

(a) What is the wavelength of the radiation released by the recombination of electrons and Hg ions?

(b) Let's assume now that we have Hydrogen atoms instead of Hg. At which acceleration voltage do you expect for the first time ionization events and hence recombination radiation?

(c) Not only ionization of the Hg atoms takes place, but also excitation of electrons form the ground state to a higher one. Let's assume we want to use a spectral grating to separate two hydrogen excited states, $E_1 = 12.09$ eV and $E_2 = 12.75$ eV respectively. The atomic spectra of hydrogen is subdivided into series, whose transition energies are:

$$E_{n,n'} = 13.6 \text{ eV} \left(\frac{1}{n^2} - \frac{1}{n'^2}\right) \tag{1}$$

What values of n and n' correspond to the above transitions? Which spectral resolution $\frac{\Delta\lambda}{\lambda}$ must our grating have in order to be able to measure them?

Exercise 20: Eigenfunction of the angular momentum operator (written exercise) **(7 Points)**

(a) Start by computing all the elements of the matrix

$$\begin{pmatrix} \partial r/\partial x & \partial r/\partial y & \partial r/\partial z \\ \partial\theta/\partial x & \partial\theta/\partial y & \partial\theta/\partial z \\ \partial\varphi/\partial x & \partial\varphi/\partial y & \partial\varphi/\partial z \end{pmatrix} \tag{2}$$

(r, θ, φ) means spherical coordinates and (x, y, z) Cartesian coordinates.

(b) Express now $L_x = -i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)$, $L_y = -i\hbar\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right)$ and $L_z = -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$ in spherical coordinates.

Use $\frac{\partial}{\partial z} = \frac{\partial r}{\partial z}\frac{\partial}{\partial r} + \frac{\partial \theta}{\partial z}\frac{\partial}{\partial \theta} + \frac{\partial \varphi}{\partial z}\frac{\partial}{\partial \varphi}$ etc. Continue to show that with the definitions $L_+ = L_x + iL_y$ and $L_- = L_x - iL_y$ one obtains:

$$L_+ = \hbar e^{i\varphi}\left(\frac{\partial}{\partial\theta} + i\cot\theta\frac{\partial}{\partial\varphi}\right) \quad \text{und} \quad L_- = -\hbar e^{-i\varphi}\left(\frac{\partial}{\partial\theta} - i\cot\theta\frac{\partial}{\partial\varphi}\right) \tag{3}$$

$L_z = -i\hbar\frac{\partial}{\partial\varphi}$ should have been obtained before.

(c) Check that $\frac{1}{2}(L_+L_- + L_-L_+) + {L_z}^2$ is equal to the norm of the angular momentum $L^2 = L_x^2 + L_y^2 + L_z^2$ by inserting L_+ and L_-. Do *not* use the representation of the products L_+L_- and L_-L_+ with L^2 and L_z that you know from the lecture, but only the definitions from b). With the explicit formulas from b) show the following:

$$\frac{L^2}{\hbar^2} = -\frac{\partial^2}{\partial\theta^2} - \cot\theta\frac{\partial}{\partial\theta} - \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2} \tag{4}$$

$$= -\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) - \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}. \tag{5}$$

Exercise 21: The electron in the hydrogen atom **(1 Cross)**

In exercise 17 you calculated the quantized orbits r_n and velocities v_n of the electron according to Bohr model. Now consider a hydrogen atom an show that the orbital frequency of the $n-th$ orbit is:

$$f_n = \frac{e^4 m}{32\pi^3\epsilon_0^2\hbar^3}\frac{1}{n^3}$$

Prove that in the limit of large n the frequency of the photon emitted in the transition $n \to n-1$ coincides with the orbital frequency of the electron in the $n-th$ orbit.