Universität
Konstanz

Integrated Physics Course IV
Exp.- Section - Atomic Physics
SoSe 19

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Problem set 6

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Exercise 16: Rutherford model **(1 Cross)**

Ernest Rutherford introduced a new atomic model owing to the unexpected results obtained in his scattering experiments. In this new model, much of the atom's positive charge and mass is concentrated in a tiny volume in the center of the atom and the electrons move in orbits of fixed radious R around it. In the following, we assume $R = 0.53 \cdot 10^{-10}\, m$.

(a) Calculate the electron's orbital period T.

(b) What is the total energy of the electron [eV]?

(c) Calculate the energy emission rate of the electron.

(d) Estimate the atom's life time.

(e) Expose briefly the contradictions arising form this model and how are they resolved.

Exercise 17: Bohr model (written exercise) **(10 Points)**

In Bohr's model, electrons move in circular orbits around the nucleus - similar to Rutherford's model - but in this case the one assumes discrete stable circular orbits. The latter are determined by the fact that the circumference must be a multiple of the de Broglie wavelength of the electron.

(a) Niels Bohr established that the electron's angular momentum $L = rp$ is quantized with $L = n\hbar$. Show that this is equivalent to his previously published assumption that the circumference of the orbit must be an integer n multiple of the de Broglie wavelength.

(b) For a stable orbit, the centripetal force must be equal to the Coulomb force. Write down the corresponding equation for an atomic nucleus with charge number Z.

(c) Select your favorite version of the quantization from a). From that equation plus the one in b), obtain the quantized orbits r_n and velocities v_n of the electrons.

(d) Calculate the expression for the quantized kinetic energy $E_{\mathrm{kin},n}$ and total energy of the electron E_n. Is E_n positive or negative? Why?

(e) The Bohr model can explain the appearance of spectral lines by assuming that electrons can jump between orbits while absorbing or emitting a photon. The energy of the photons $\hbar\omega$ is given by the difference $E_{n_2} - E_{n_1}$. Emissions lines that end at the same lower energy level are grouped together. Write down the names of these hydrogen series. Calculate the wavelength of the lowest line of the four series, i.e. for the transitions $n = 2 \to n = 1$, $n = 3 \to n = 2$, $n = 4 \to n = 3$ and $n = 5 \to n = 4$. Which of them lie in the visible range?

(f) Calculate the wavelengths of the corresponding spectral lines for single ionized helium 4. Which ones lie in this case in the visible range? Write the wavelength differences for the above four lines between hydrogen and deuterium (isotope shift).

(g) Calculate the radius of the orbit of the electron of a hydrogen atom in the state $n = 30$. What is the wavelength for the transition $n = 30 \to n = 29$? To which spectral range does it belong? What is a Rydberg atom? Explain where does it appear naturally and how can they be generated in the lab and be used to selectively measure very dense energy levels for high n.

Exercise 18: Bohr model's limits **(1 Cross for a-e and f-g)**

(a) Why do we refer in this problem set to Bohr model as such and not as Bohr atomic model?

(b) The picture of electrons orbiting the nucleus like plantes do to the sun contradicts a famous quantum mechanical feature - which one?

(c) As you will see later in the semester (and we briefly introduced in a previous problem set), the angular momentum $\vec{L}$ is subject to the uncertainty relation such that only its magnitude and one of its components can be fully determined, but not all its three components simultaneously (i.e, one can't know at the same time the magnitude and direction of the vector). What does the Bohr model say in this regard?

(d) Later in this semester you will calculate the wave functions of the Hydrogen atom. You will see that the total angular momentum of the ground state is $L = 0$. What does the Bohr model say in this respect?

(e) Calculate what speed an electron has in Bohr's ground state. Does it make sense?

(f) In exercise 17 you calculated the total energy of the electron according to Bohr model:

$$E_{\mathrm{tot}} = -\frac{Ze^2}{8\pi\varepsilon_0 r}.$$

From IK2 you know that accelerated charges radiate. What does this means in the context of energy conservation?

(g) With help of Heisenberg's uncertainty principle one can save the matter stability. Show that the total energy of the electron is in the Bohr model:

$$E_{\text{tot}} \gtrsim \frac{\hbar^2}{2m_e \langle r \rangle^2} - \frac{Ze^2}{4\pi\varepsilon_0 \langle r \rangle}$$

From this, find the lower bound to E_{tot} and determine E_{min}.

Finally, you should note that in Bohr's model the line strengths in the spectra were not predicted, as well as the transition probabilities between states, the splitting of lines with or without magnetic field (see sodium D-line), and the Pauli principle was not yet known (but also was not needed for a one-electron system).