Universität
Konstanz

Integrated Physics Course IV
Exp.- Section - Atomic Physics
SoSe 19

Prof. E. Weig, Anh-Tuan Le, Felix Rochau

Problem set 5

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Exercise 12: How fast does the coffee get cold?? **(1 Cross)**

Consider a thermo bottle (or thermo flask) as a cylinder with a diameter of 10 cm and height of 25 cm. You can neglect the bottom and lid of the bottle, and just assume the bottle to be a simple cylinder. At a first approximation, we will also neglect the difference in area between the inner and outer cylinder - the space that holds the vacuum responsible of the thermal isolation of the bottle. Let's assume we fill the bottle with a warm liquid (e.g. water, heat capacity $0.18\ \mathrm{cal\,mol^{-1}\ K^{-1}}$) at a temperature of 350 K. How long does it take the water inside the bottle to reach a temperature of 310 K? Note: 1 cal = 4.184 J.

First, calculate the energy radiated by the surface of the bottle into the vacuum using Stefan-Bolzmann law.

How much does the result change if we consider now a vacuum with limited dimensions (i.e., we now take into account the finite space between the two cylinders)? The outer wall of the bottle (at 300 K) also radiates towards the inner part of the bottle. The liquid volume sourrounded by the inner wall behaves as a black body (completely absorbs the ratiation).

Recall:

$$\int \frac{dx}{a^4 - x^4} = \frac{1}{4a^3} \ln \frac{a+x}{a-x} + \frac{1}{2a^3} \arctan \frac{x}{a}$$

Excercise 13: Wien's displacement law **(1 Cross)**

The Rayleigh-Jeans catastrophe (*ultraviolet catastrophe*) was the prediction of 19th-20th century classical physics that a black body at thermal equilibrium will emit radiation in all frequency ranges, emitting more energy as the frequency increases. To solve this, Max Planck introduced his famous quantum hypothesis: he assumed that the energy E of an oscillator of frequency ν can only exist in discrete values which are proportional to the frequency $E = h\nu$. This is nowadays known as *Planck's law* (see lecture).

$$u(\nu, T) = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/k_B T} - 1}.$$

1. Calculate the spectral energy density $\tilde{u}$ as a function of the wavelength λ starting from Planck's law:

$$u(\nu, T)\, \mathrm{d}\nu = \tilde{u}(\lambda, T)\, \mathrm{d}\lambda.$$

2. Determine the maximum of $u(\nu, T)$ and $\tilde{u}(\lambda, T)$ (*Wien's displacement law*) and show that $\nu_{\max}\lambda_{\max} \neq c$. What are the physical implications?

   Note: The maximum can only be computed numerically.

<u>Excercise 14: Photoelectric effect</u> **(1 Cross)**

1. Show that the photoelectric effect is not possible for free electrons by means of energy and momentum conservation.

2. When shining the surface of a metallic material with light of different wavelengths one will obtain the following stopping potentials (at which the electron current vanishes):

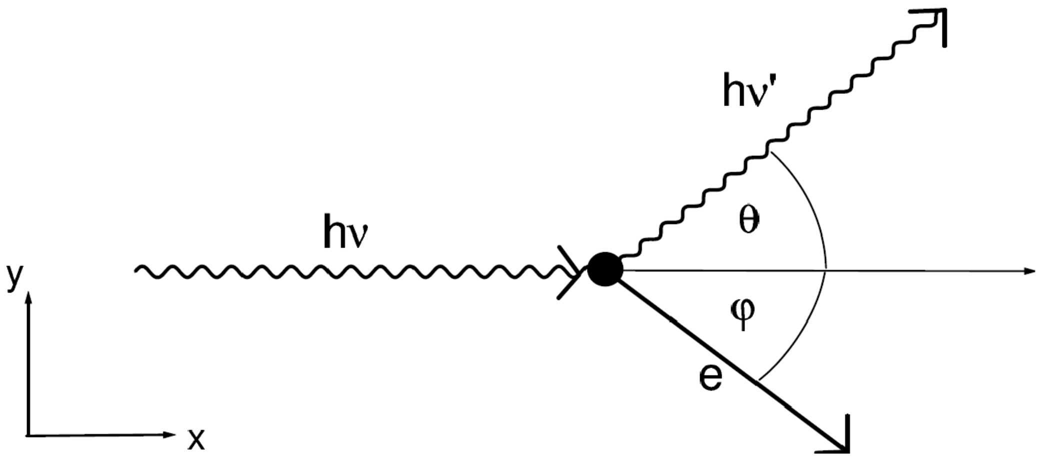
$U_s\,(V)$	1.48	1.15	0.93	0.62	036	0.24
$\lambda\,(nm)$	366	405	436	492	546	579

   Determine the cutoff frequency, the work function and the value of Planck's constant h. To do this, express the stopping potentials as a function of the light frequency. The elementary charge e and the speed of light c are assumed to be known.

3. A potassium electrode is irradiated with ultraviolet light of wavelength 250 nm. What is the kinetic energy of the emitted electrons when the work function is 2.21 eV? Which current can flow through this electron emission?

<u>Exercise 15: Compton scattering</u> (written) **(10 Points)**

The Compton effect describes the scattering of a photon with a charged particle (e.g. electron). The electron is at rest before the impact, and the angles at which the photon and electron are scattered after the impact are denoted by θ and φ, respectively. During the impact, the photon realeases part of its energy to the electron and consequently the wavelength of the scattered photon is larger than that of the incident.



The figure shows a sketch of the scattering process. λ' and ν' correspond to the photon after the scattering event. Derive the following equation:

$$\lambda' = \lambda + \lambda_C(1 - \cos\theta). \tag{1}$$

where the quantity $\lambda_C = \frac{h}{m_0 c}$ (m_0 is the rest mass of the electron) is known as the Compton wavelength. To carry this task, you should apply both energy and momentum conservation to the electron and photon. The vectorial characteristic of the momentum should not be omitted (separating the equations in x and y directions might be helpful) . We will treat the electron as a relativistic particle, with rest energy m_0c^2 before the scattering vent and with momentum $\vec{p}_e$ and energy $E_e = \sqrt{m_0^2c^4 + p_e^2c^2}$ after the event. A photon with frequency ν has an energy $h\nu$ and momentum (modulus) $h\nu/c$.

1. Write down the following three equations: energy conservation (2), momentum conservation in x direction (3) and momentum conservation in y direction (4). Only the variables λ, λ', θ, φ, p'_e should appear in the equations, where p'_e is the momentum of the electron after the collision.

2. Equation (3) contains the variable φ. Eliminate it using Equation (4) and the relationship $\cos^2(\varphi) = 1 - \sin^2(\varphi)$.

3. Equations (2) and (3) contain annoying roots. Get rid of them by clicking Move the roots to one side and square the equations.

4. Convert equation (3) to p'_e and then eliminate that variable.

5. Simplify Equation (2) until you obtain Equation (1), finally arriving the Compton relationship.

6. Show that it is not possible for the electron to absorb the photon.

7. A photon with the energy $1 \cdot 10^4$ eV makes a collision with a resting electron and is scattered at an angle of 60°. Indicate the wavelengths of the incoming and outgoing photons (consider four digits in the mantissa of the values). Calculate the kinetic energy of the electron after the impact and the angle at which it scatters away.