Universität
Konstanz

Integrated Physics Course IV
Exp.- Section - Atomic Physics
SoSe 19

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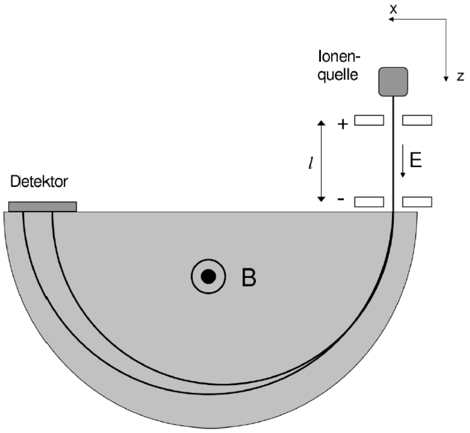
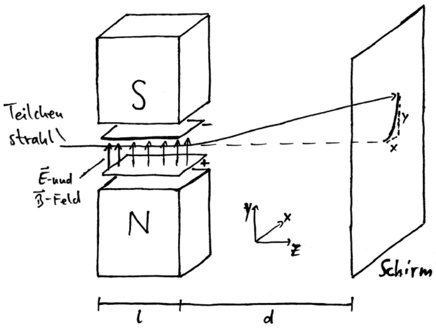
Problem Set 4

Posted: 06.05.2019, Due: Wednesday 13.05.2019

Exercise 9: Charged particles in EM fields (written exercise) **(8 Points)**



1. As depicted in the figure above, some charged ions are first accelerated in an electric field. Right before the E-field, the ions have zero speed. After the E-field region, they pass through a vertical magnetic field, describing semicircular paths. Derive the radius of those paths.
$$r = \sqrt{\frac{2mEl}{e}}\frac{1}{B}$$
where m is the mass of the ion and e is the elementary charge.



2. Now consider a situation in which the particles (also ionized) run through a length l under the influence of an electric field $\vec{E}$ and magnetic field $\vec{B}$ which are parallel to each other as depicted in the drawing (**Kaufmann Spectrometer**) . As a result, they are both accelerated and deviated.The particles are detected on a screen placed at a distance d and they enter the field area with an initial velocity v. The initial speed can take any value, but by no means must be the same for all particles. Show that a parabola $y = Ax^2$ is mapped onto the screen by a beam of polyenergetic particles of identical mass m and charge e. In the field, take a velocity $v \approx v_z = const$. Calculate the deviations $x(v)$ and $y(v)$ on the screen, and eliminate v. The result is:

$$y(x) = \frac{m}{e}\frac{E}{B^2}(ld + \frac{l^2}{2})^{-1}x^2$$

3. For the situation described in a), calculate the distance at which hydrogen ions H^+ and H_2^+ ions hit the detector. Calculate as well the distance at which ions of the isotopes $^{16}O^+$ and $^{18}O^+$ arrive at the detector. Here: $|\vec{E}| = 5000\,\mathrm{V/m}$, $l = 40\,\mathrm{cm}$ and $|\vec{B}| = 1\,\mathrm{T}$. (The ions' mass can be estimated as the mass number times the proton mass.)
4. Calculate for the same four types of ions the positions x and y where they hit the screen in the structure of b). Assume $|\vec{E}| = 5000\,\mathrm{V/m}$, $l = 4\,\mathrm{cm}$, $|\vec{B}| = 0.01\,\mathrm{T}$ and $d = 20\,\mathrm{cm}$, and that the ions already enter the field area with a kinetic energy of $1000\,\mathrm{eV}$.
5. Investigate how the Wien speed filter works and demonstrate the derivation of the passage speed.

Exercise 10: macroscopic harmonic oscillator (**1 Cross**)

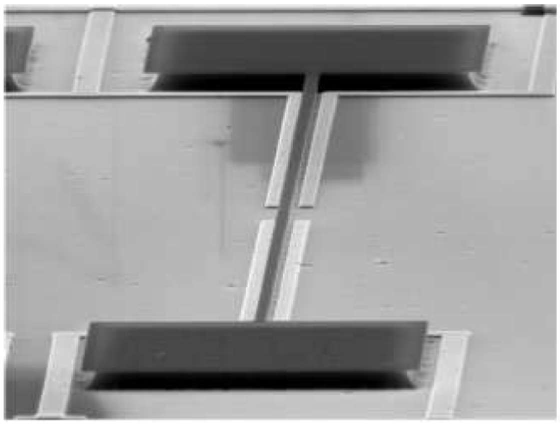
The frequency of the m-th flexural mode of a very long and thin high stress beam can be approximated as:

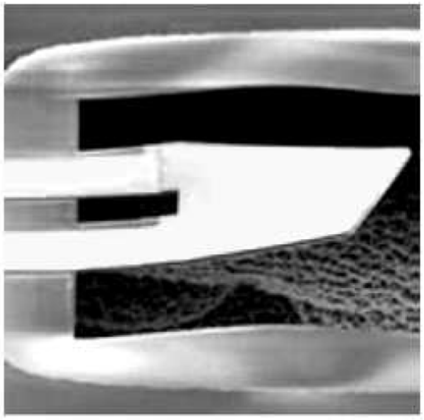
$$f_m \approx \frac{m}{2L}\sqrt{\frac{\sigma}{\rho}} \tag{1}$$

with mode index m, length L, tensile stress σ and density ρ.

1. First, consider the string of an ukulele with scale length (i.e. oscillating length between saddle and bridge of the instrument) of 38 cm, diameter 0.7 mm, density (nylon) 1.15 $\mathrm{g}/\mathrm{cm}^{3}$ and a tension of 72.2 MPa.

   Consider the fundamental mode of the string as a quantum mechanical harmonic oscillator. What are the energy eigenvalues, what is the zero point energy and what is the temperature? What are the zero-point fluctuations of the flexural mode? Can this mode be considered as a quantum mechanical harmonic oscillator at room temperature (if not, how far would it have to cool the string)?





2. The left figure shows a nanomechanical resonator. The string is made of SiN on a quartz substrate. The dimensions are: 45 *mu* m long, 100 nm thick and 260 nm wide. The density of SiN is 2.8 *cdot* 10^{3} $\mathrm{kg}/\mathrm{m}^{3}$, the tension is 1.4 GPa. If you answer the questions from a), can a flexural mode of this structure be seen as a quantum mechanical harmonic oscillator?

3. The resonator in the right image is a so-called FBAR (thin film bulk acoustic resonator). For example, FBARs are included in every cell phone as a frequency filter. An FBAR consists of a piezoelectric material between two electrodes. This FBAR is made of aluminum nitride; the upper and lower electrodes are made of aluminum. When an electrical voltage is applied, the thickness of the FBAR changes. By an AC voltage with the appropriate frequency, it can be excited to thickness vibrations. The frequency of the lowest thickness mode is

   $$f \approx \frac{c}{2d}, \tag{2}$$

   with speed of sound c and thickness d. Dimensions: Thickness of Aluminum nitride layer 330 nm, thickness of aluminum electrodes each unit [130] nm (lateral dimensions approx. 50 μ m, almost visible to the naked eye). The speed of sound is 9100 m/s.

   Compare the energy eigenvalues of this resonator with those of the nanomechanical resonator from a). Solid state structures can be cooled to temperatures of up to

10 mK in so-called 3 He - ^{4} He demixing cryostats. Which energy eigenstate n of the FBAR (also called phonon occupation number) corresponds to this temperature?

Exercise 11: Rutherford-scattering (**1 Cross**)

A collimated beam of $\alpha-$particles of kinetic energy unit [4.8] MeV is perpendicular to an aluminum foil of thickness unit $[2 \cdot 10^{-5}]$ m. The beam intensity is 10^{6} particles/s.

1. How many particles are counted per minute in a detector constructed under 30° to the direction of incidence, if the detector iI n θ and ϕ ±2° covered?
2. How many particles are counted in this detector if it is equally spaced below 60°? $\left(\rho = 2.7 \cdot 10^3 \ \mathrm{kg/m}^3, Z_{Al} = 13, A_{Al} = 27\right)$