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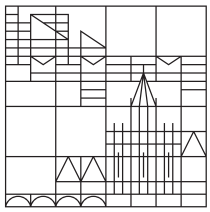

Integrated Physics Course IV
Exp.-Section – Atomic Physics
SoSe 19

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Problem Set 2

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Exercise 4: Wave function of an electron (written exercise) **(8 points)**

a) Consider an electron moving with momentum $p = \hbar k$ in x-direction . What is the corresponding wave function $\psi(x,t)$?
Note: $\omega = E/\hbar$ and $E = p^2/2m$.

b) Determine the phase velocity of the electron wave from a) by tracing a fixed phase over time. How does the phase velocity of the wave relate to the velocity $v = p/m$ of the electron?

c) Now consider an electron whose wave function is given by a continuous superposition of plane waves of the form

$$\psi(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk e^{-i\left(\omega(k)t - kx\right)} a(k) \tag{1}$$

('wave packet'). Assuming all wavenumbers belong to the interval $[k_0 - \delta k, k_0 + \delta k]$ and contribute equally strong, i.e. $a(k) = const.$, compute the wave function at $t = 0$.

d) When calculating the wave function of the electron from c) in a general point in time, the nonlinear term $\omega(k)$ causes problems in the exponent. To approximate $\psi(x,t)$ develop $\omega(k)$ around k_0 to the linear order. Under which condition on Δk is this approximation allowed? Show the history of $a(k)$ and $\omega(k)$ in a single sketch. Calculate the approximated integral for $\psi(x,t)$.

e) What is the velocity of the maximum of the electron wave from d)? How does this velocity relate to the velocity of an electron with the momentum $\hbar k_0$?

Exercise 5: Wave function and probability (written exercise) **(4 points)**

At time $t = 0$ a particle's wave function is defined by:

$$\psi(x,0) = \begin{cases} A\frac{x}{a}, & \text{for } 0 \leq x \leq a \\ A\frac{b-x}{b-a}, & \text{for } a \leq x \leq b \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

where A, a and b are constants.

a) Normalize ψ (i.e. write A as a function of a and b).

b) Sketch $\psi(x,0)$ as a function of x.

c) What is the position with highest probability to find the particle at $t = 0$?

d) Calculate the probability to find the particle to the left of a. Check your result with the case $b = a$ and $b = 2a$.