
Assignment 4

Discussion on Friday, 24.2. in P 805

10) **Locating the Sievers-Takeno breather using nonlinear dynamics techniques**

Consider the second-order recurrence relation

$$2\xi_n - \xi_{n+1} - \xi_{n-1} + \lambda\left[(\xi_n - \xi_{n+1})^3 + (\xi_n - \xi_{n-1})^3\right] - \omega^2\xi_n = 0$$

obeyed by the amplitudes of the lattice discplacements in the rotating wave approximation (cf. Section 16.1 of Lecture notes). Set $\lambda = 0.5$.

(a) Rewrite the recurrence relation as a two-dimensional map. (Hint: use the difference variable $p_{n+1} = \xi_{n+1} - \xi_n$; note that it will be necessary to invert a cubic equation in order to obtain p_{n+1} from p_n and ξ_n.)

(b) Determine the fixed point(s) of the map and look at the local stability properties of the point $(\xi_0, p_0) = (0, 0)$. Write down eigenvalues μ_i and eigenvectors $\vec{v}_i$ of the tangent map.

(c) (numerical). Draw the unstable manifold of the fixed point for $\omega^2 = 4.5$. (Hint: run about 30 iterations of the map using initial conditions very close to the fixed point. A good recipe is to start from points along the eigenvector of the tangent map which corresponds to the unstable direction, i.e. $(\xi_0, p_0) = e^{-x}(\vec{v}_u)$, with x uniformly spaced between 5 and 8).

(d) Note the maximal amplitude $\alpha = \max\{\xi_n\}$ of the oscillation by inspecting the manifold. Get a plot of the actual orbit ξ_n vs n for a typical initial condition.

(e) Repeat steps c) and d) for $\omega^2 = 4.3, 4.1, 4.05$. What do you observe? Interpret the result in terms of linear lattice dynamics. Draw a rough plot of $\lambda\alpha^2$ vs. ω^2.

Good luck!