Assignment 3

Discussion on Friday, 10.2.

9) It is possible to understand the phenomenon of FPU recurrence (cf. Chapter 3 of Lecture notes, new version) in terms of soliton formation.

- Consult section 4.2 (new version!) to understand why the KdV equation can be used to describe solitons in the FPU Hamiltonian (3.21).
- Use periodic boundary conditions in a lattice of $2N$ sites; the initial condition is then

$$q_n = B \sin \frac{\pi n}{N} \quad n = 1, \cdots, 2N \quad . \tag{1}$$

  They are simpler to work with, and contain the same physics as the original FPU fixed boundary condition at a lattice with N sites (assuming perfectly reflecting boundaries). Why?
- Find an appropriate rescaling that will make the bottom of the cosine well "mimic" (i.e. match value and local curvature) the $-M(M+1)\text{sech}^2\xi$ form of the M-soliton solution (check that $M \gg 1$ under reasonable conditions).
- Assuming that the recurrence is controlled by the few deepest bound states of the -sech^2 well (the few largest solitons) derive a condition for soliton recurrence, i.e. the time it takes for the faster solitons, as they go around the lattice, to reach the slower ones and find themselves at the same point in space (assume constant speed of propagation and $M \gg 1$).
- Rescale to obtain the recurrence time in the original units.

Result:

$$T_r = \frac{3\sqrt{2}}{\pi^{3/2}} \frac{N^{5/2}}{\sqrt{\alpha B}} \tag{2}$$

It is usual to divide this result by the longest period of linear oscillations, $T_1 = 2N$; then

$$\frac{T_r}{T_1} = \frac{3}{\sqrt{2}\pi^{3/2}} \frac{N^{3/2}}{\sqrt{\alpha B}} \quad . \tag{3}$$