
Assignment 2

Discussion on Friday, 9.12.

8) Show that m iterations of the Arnold "Cat-map" $x \to Tx$ with $T = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$ and $x \in (\mathbb{R} \mod 1)^2$ can be expressed in terms of the Fibonacci numbers $F_n = F_{n-2} + F_{n-1}$ with $F_1 = F_2 = 1$.

9) Find the eigenvalues and eigenvectors of the tangent map of T. (Note the usefulness of the golden-mean numbers $\phi_\pm = \frac{1 \pm \sqrt{5}}{2}$ with the property $\phi_+ \phi_- = -1$.) Identify and try to sketch the stable and unstable manifolds of the fixed point of T. Comment on their structure in the mod-1 phase space in 2-d.

10) [Numerical] The phase space of the map can be partitioned into M boxes of size ϵ^2. A set of states can then be characterized by the the probabilities p_i for finding a state in box i and the information entropy is defined as $I = -\sum_i^M p_i \ln p_i$.

- Investigate the information loss for a set of N initial states randomly distributed in one of the M boxes under successive applications of the map.
- How large should N be chosen for a given ϵ (starting point: $N = 100$ and $\epsilon = 1/10$)?
- Relate the result to the Kolmogorov entropy $K = \lambda_+$ (in this special case) with λ_+ the positive Lyapunov exponent derived from exercise 9).