UNIVERSITÄT KONSTANZ
Fachbereich Physik
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Nonlinear Physics (Solitons, Chaos, Localization)
WS 2005/2006
Exercises

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Assignment 1

*Friday 11.11.: Introduction to calculation of Poincare cuts (preparation for 7.),*
*Friday, 18.11.: Discussion of exercises 1.–6.*
*Friday, 25.11.: Discussion of exercise 7.*

1. Using the approach of Section 1.4, show that the isotropy of space (invariance of the Lagrangian with respect to infinitesimal rotations) implies the conservation of angular momentum of a particle system, i.e.
$$\vec{L} = \sum_{j=1}^{N} m_j \vec{q}_j \times \dot{\vec{q}}_j = const. \tag{1}$$
2. Verify that the solitary wave (4.46) is an acceptable solution to the original problem of long waves, i.e. that it is a low amplitude, long wave solution. (Hint: Go back to the original units). Is there a condition on the parameter $\lambda$ of (4.46) for this to occur?
3. Show that the KdV soliton satisfies the particle property $P = MV$. (Hint: use the definitions of "momentum"- and "mass"given in the KdV Lagrangian theory to derive an expression for the momentum $P$ and the mass $M$ of the solitary wave (4.46). )
4. Verify that the operators $B$ (Eq. 5.12) and $L$ (5.2) form a Lax pair, i.e. they satisfy (5.6).
5. Find an asymptotic solution (as $t \to \infty$) to the KdV equation (5.1) with the initial condition $u(x, 0) = -\mu\delta(x)$, where $\mu > 0$ is a constant.
6. Find an asymptotic solution (as $t \to \infty$) to the KdV equation (5.1) with the initial condition $u(x, 0) = -N(N + 1)/\cosh^2 x$, where $N > 0$ is an integer.
7. (Numeric application): Generate projections of orbits in phase space (Poincare cuts at $x = 0, p_x > 0$ ) for the Henon-Heiles Hamiltonian
$$H = \frac{1}{2}(p_x^2 + p_y^2) + \frac{1}{2}(x^2 + y^2) + x^2 y - \frac{1}{3}y^3 \tag{2}$$
at values $E = 1/12, 1/8, 0.16666$. For each energy use more than one initial condition.

Additional course material will be provided at
`http://theo.physik.uni-konstanz.de/theodorakopoulos`