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http://tinyurl.com/qo2019

Quantum Optics
Winter semester 2018/2019 - Exercise sheet 1
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Problem 1: Quantum harmonic oscillator.

a) Using the uncertainty principle, show that the lowest energy of an oscillator is $\hbar\omega/2$.

b) Find the energy levels and the ground state wave function of a system composed of two harmonic oscillators of masses m_1 and m_2, having identical frequencies and coupled by the interaction $\frac{1}{2}k(\hat{x}_1 - \hat{x}_2)^2$.

Problem 2: Quantization of the electric field.

a) Calculate the vacuum expectation values for the electric field operator and its variance. Explain the results.

b) By applying a unitary transformation to the photon annihilation and creation operators corresponding to monochromatic modes, we may introduce new operators, associated with non-monochromatic modes. Prove that the non-monochromatic mode coefficients $\mathbf{E}'_\lambda(\mathbf{r})$ of the field operator $\hat{\mathbf{E}} = \sum_\lambda \mathbf{E}_\lambda \hat{a}_\lambda + \text{H.c.}$ are not orthogonal ($\int d^3r \mathbf{E}'^*_\lambda(\mathbf{r})\mathbf{E}'_{\lambda'}(\mathbf{r}) \neq \delta_{\lambda,\lambda'}$) and find the corresponding form of the Hamiltonian operator. Hint: $\hat{a}'_\nu = \sum_\lambda U_{\nu\lambda}\hat{a}_\lambda$.

Problem 3: Casimir effect.

Model the electron as two parallel plates of area L^2, separated by distance L and carrying charge $q = e/2$. Balance the Casimir and electrostatic forces and from this determine a value for the fine-structure constant α.

Problem 4: Fock states of light.

a) Considering the multimode Fock state $|\{n\}\rangle = \Pi_k |n_k\rangle$, evaluate $\langle \hat{a}_i^\dagger \hat{a}_j\rangle$,$\langle \hat{a}_i^\dagger \hat{a}_j^\dagger\rangle$ and $\langle \hat{a}_i \hat{a}_j\rangle$.

b) Show for the operator $\hat{d} = \sum_s [\alpha_s \hat{a}_s + \beta_s \hat{a}_s^\dagger]$ that $\langle \hat{d}^\dagger \hat{d}\rangle \geq 0$ for any state vector by proving that

$$\langle\{n\}|\hat{d}^\dagger \hat{d}|\{n\}\rangle = \sum_s \left[|\alpha_s|^2 n_s + |\beta_s|^2 (n_s + 1)\right] \quad .$$