UNIVERSITY OF KONSTANZ
Department of Physics
Thiago Lucena, Dr. Andrey Moskalenko

http://tinyurl.com/qo2019

Quantum Optics
Winter semester 2018/2019 - Exercise sheet 5
Distributed: 26.11.2018, Discussion: 03.12.2018

Problem 1: Squeezed coherent states of light.

For the squeezed coherent state $\hat{D}(\alpha)\hat{S}(\xi)|0\rangle = |\alpha, \xi\rangle$, show that the variance of the photon number operator is given by

$$\langle \alpha, \xi | \hat{N}^2 - \langle \hat{N} \rangle^2 | \alpha, \xi \rangle = |\alpha\mu - \alpha^* \nu|^2 + 2|\mu\nu|^2,$$

where $\mu = \cosh(|\xi|)$, $\nu = e^{i\phi}\sinh(|\xi|)$ and $\xi = e^{i\phi}|\xi|$.

Problem 2: Displaced number states of light.

a) Given the displaced number states $\hat{D}(\alpha)|n\rangle = |n, \alpha\rangle$, show that

$$\sum_{n=0}^{\infty} |n, \alpha\rangle\langle n, \alpha| = \frac{1}{\pi}\int d^2\alpha |n, \alpha\rangle\langle n, \alpha| = 1.$$

b) For the case when $m \geq n$, show that

$$\langle n | m, \alpha \rangle = e^{-|\alpha|^2/2}\sqrt{\frac{n!}{m!}}(-\alpha^*)^{m-n}\mathrm{L}_n^{(m-n)}(|\alpha|^2),$$

where $\mathrm{L}_k^{(\alpha)}(x) = x^{-\alpha}\frac{1}{k!}(\frac{d}{dx} - 1)^k x^{k+\alpha} = \sum_{i=0}^{k}(-1)^i \frac{(k+\alpha)! x^i}{(\alpha+i)!(k-i)! i!}$ is the generalized Laguerre polynomial.

Problem 3: Squeezed states in analogue quantum gravity.

The intersection between general relativity and quantum physics is one of the biggest puzzles of modern physics. While quantum field theory managed to unify the strong, weak and electromagnetic forces, gravity remains as the only fundamental force to be described by its own (classical) theory. There are, however, predictions of effects that lay exactly in the boundary between these fields: quantum gravitational effects, to which both the Hawking and Unruh-Davies effects belong. Direct detection of any of these two effects is currently technically infeasible (one needs to detect quantum particles either by having highly accelerated detectors in vacuum or detectors placed at the horizon of black holes). In view of this, quantum opticians proposed the use of squeezed states to study and test the validity of these effects. How? It happens that two-mode squeezed states are composed of pairs of entangled photons, and as one cuts the connection between these photons (which in mathematics corresponds to using a partial trace and in physics corresponds to introducing a horizon), all that is left is a single-mode thermal state. This on the one hand means that information has been lost (in the sense that part of the two-photon wave function has crossed the horizon), while on the other hand means that thermal photons have been created by the presence of the horizon! Show that from a two-mode squeezed state one can obtain a thermal state after tracing over any of the modes of the total density matrix of the state.
Hint: $\hat{S} = \exp\{-e^{i\theta}\tanh|\xi|\hat{a}_1^\dagger\hat{a}_2^\dagger\}\exp\{-\ln[\cosh|\xi|](\hat{n}_1 + \hat{n}_2 + 1)\}\exp\{e^{-i\theta}\tanh|\xi|\hat{a}_1\hat{a}_2\}$.