UNIVERSITY OF KONSTANZ
Department of Physics
Thiago Lucena, Dr. Andrey Moskalenko

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Quantum Optics
Winter semester 2018/2019 - Exercise sheet 11
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Problem 1: Beam splitter and Hong-Ou-Mandel effect.

A beam splitter transforms the incoming mode operators $\hat{a}_{\text{in}}$ and $\hat{b}_{\text{in}}$ into the outgoing mode operators $\hat{a}_{\text{out}}$ and $\hat{b}_{\text{out}}$ given by:

$$\hat{a}_{\text{out}} = \sqrt{\eta}\hat{a}_{\text{in}} - i\sqrt{1-\eta}\hat{b}_{\text{in}}, \qquad \hat{b}_{\text{out}} = \sqrt{\eta}\hat{b}_{\text{in}} - i\sqrt{1-\eta}\hat{a}_{\text{in}}.$$

a) Show that such a transformation may be generated by the unitary operator

$$\hat{T} = \exp[-i\theta(\hat{a}_{\text{in}}^\dagger \hat{b}_{\text{in}} + \hat{a}_{\text{in}}\hat{b}_{\text{in}}^\dagger)],$$

where $\eta = \cos^2(\theta)$.

b) Show that if the incoming state is a two-mode coherent state $|\alpha_{\text{in}}\rangle \otimes |\beta_{\text{in}}\rangle$, the outgoing state will be given by $|\alpha_{\text{out}}\rangle \otimes |\beta_{\text{out}}\rangle$ with

$$\alpha_{\text{out}} = \sqrt{\eta}\alpha_{\text{in}} - i\sqrt{1-\eta}\beta_{\text{in}}, \qquad \beta_{\text{out}} = \sqrt{\eta}\beta_{\text{in}} - i\sqrt{1-\eta}\alpha_{\text{in}}.$$

c) Show that if the incoming state is a two-mode Fock state $|1\rangle \otimes |1\rangle$, the outgoing state will be given by

$$(2\eta - 1)|1\rangle|1\rangle - i\sqrt{2\eta(1-\eta)}(|0\rangle|2\rangle + |2\rangle|0\rangle).$$

d) What does the result of the point (c) for the particular case of $\eta = 1/2$ physically mean (in terms of an experiment)?

Problem 2: Quasi-probability distributions.

Considering the operator-valued Dirac δ-distribution,

$$\hat{\delta}^{(2)}(\alpha - \hat{a}) = \frac{1}{\pi^2}\int d^2\beta e^{\beta(\hat{a}^\dagger - \alpha^*) - \beta^*(\hat{a} - \alpha)} = \frac{1}{\pi^2}\int d^2\beta \hat{D}(\beta) e^{\beta^*\alpha - \beta\alpha^*},$$

show that:

a) The Husimi function, $Q(\alpha, \alpha^*) = \frac{1}{\pi}\langle\alpha|\hat{\rho}|\alpha\rangle$, is given by $\langle \ddagger \hat{\delta}^{(2)}(\alpha - \hat{a}) \ddagger \rangle$.

b) The Glauber-Sudarshan function, $P(\alpha, \alpha^*) = \frac{e^{|\alpha|^2}}{\pi^2}\int d^2\beta \langle -\beta|\hat{\rho}|\beta\rangle e^{|\beta|^2} e^{-(\beta\alpha^* + \beta^*\alpha)}$, is given by $\langle : \hat{\delta}^{(2)}(\alpha - \hat{a}) : \rangle$. Do not forget to derive the given expression for $P(\alpha, \alpha^*)$ from the coherent representation of the density matrix: $\hat{\rho} = \int d^2\alpha |\alpha\rangle P(\alpha, \alpha^*) \langle\alpha|$.