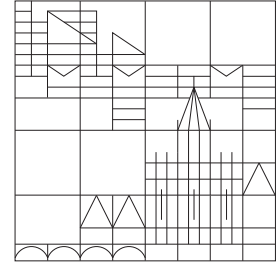


<http://tinyurl.com/qo2018>



## Quantum Optics

### Winter semester 2017/2018 - Exercise sheet 1

Distributed: 07.11.2017, Discussion: 09.11.2017

#### Problem 1: Quantum harmonic oscillator.

- Using the uncertainty principle, show that the lowest energy of an oscillator is  $\hbar\omega/2$ .
- Find the energy levels and the ground state wave function of a system composed of two harmonic oscillators of masses  $m_1$  and  $m_2$ , having identical frequencies and coupled by the interaction  $\frac{1}{2}k(\hat{x}_1 - \hat{x}_2)^2$ .

#### Problem 2: Quantization of the electric field.

- Calculate the vacuum expectation values for the electric field operator and its variance. Explain the results.
- By applying a unitary transformation to the photon annihilation and creation operators corresponding to monochromatic modes, we may introduce new operators, associated with non-monochromatic modes. Prove that the non-monochromatic mode coefficients  $\mathbf{E}'_\lambda(\mathbf{r})$  of the field operator  $\hat{\mathbf{E}} = \sum_\lambda \mathbf{E}_\lambda \hat{a}_\lambda + \text{H.c.}$  mode expansion are not orthogonal ( $\int d^3r \mathbf{E}'_\lambda(\mathbf{r}) \mathbf{E}'_{\lambda'}(\mathbf{r}) \neq \delta_{\lambda,\lambda'}$ ) and find the corresponding form of the Hamiltonian operator. Hint:  $\hat{a}'_\nu = \sum_\lambda U_{\nu\lambda} \hat{a}_\lambda$ .

#### Problem 3: Casimir effect.

Model the electron as two parallel plates of area  $L^2$ , separated by distance  $L$  and carrying charge  $q = e/2$ . Balance the Casimir and electrostatic forces and from this determine a value for the fine-structure constant  $\alpha$ .