UNIVERSITY OF KONSTANZ
Department of Physics
Thiago Lucena, Dr. Andrey Moskalenko

http://tinyurl.com/qo2018

Quantum Optics
Winter semester 2017/2018 - Exercise sheet 1
Distributed: 07.11.2017, Discussion: 09.11.2017

Problem 1: Quantum harmonic oscillator.

a) Using the uncertainty principle, show that the lowest energy of an oscillator is $\hbar\omega/2$.

b) Find the energy levels and the ground state wave function of a system composed of two harmonic oscillators of masses m_1 and m_2, having identical frequencies and coupled by the interaction $\frac{1}{2}k(\hat{x}_1 - \hat{x}_2)^2$.

Problem 2: Quantization of the electric field.

a) Calculate the vacuum expectation values for the electric field operator and its variance. Explain the results.

b) By applying a unitary transformation to the photon annihilation and creation operators corresponding to monochromatic modes, we may introduce new operators, associated with non-monochromatic modes. Prove that the non-monochromatic mode coefficients $\mathbf{E}'_\lambda(\mathbf{r})$ of the field operator $\hat{\mathbf{E}} = \sum_\lambda \mathbf{E}_\lambda \hat{a}_\lambda + \text{H.c.}$ mode expansion are not orthogonal ($\int d^3r \mathbf{E}'^*_\lambda(\mathbf{r})\mathbf{E}'_{\lambda'}(\mathbf{r}) \neq \delta_{\lambda,\lambda'}$) and find the corresponding form of the Hamiltonian operator. Hint: $\hat{a}'_\nu = \sum_\lambda U_{\nu\lambda}\hat{a}_\lambda$.

Problem 3: Casimir effect.

Model the electron as two parallel plates of area L^2, separated by distance L and carrying charge $q = e/2$. Balance the Casimir and electrostatic forces and from this determine a value for the fine-structure constant α.