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Quantum Optics
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Problem 1: Squeezed coherent states of light.

a) For the squeezed coherent state $\hat{D}(\alpha)\hat{S}(\xi)|0\rangle = |\alpha, \xi\rangle$, show that the variation of the photon number operator is given by

$$\langle \alpha, \xi|\hat{N}^2 - \langle \hat{N}\rangle^2|\alpha, \xi\rangle = |\alpha\mu - \alpha^*\nu|^2 + 2|\mu\nu|^2,$$

where $\mu = \cosh(|\xi|)$, $\nu = e^{i\phi}\sinh(|\xi|)$ and $\xi = e^{i\phi}|\xi|$.

b) Prove that the squeezed coherent states are overcomplete by showing that

$$\frac{1}{\pi}\int d^2\alpha |\alpha, \xi\rangle\langle\alpha, \xi| = 1.$$

Problem 2: Displaced number states of light.

a) Given the displaced number states $\hat{D}(\alpha)|n\rangle = |n, \alpha\rangle$, show that

$$\sum_{n=0}^{\infty} |n, \alpha\rangle\langle n, \alpha| = \frac{1}{\pi}\int d^2\alpha |n, \alpha\rangle\langle n, \alpha| = 1.$$

b) For the case when $m \geq n$, show that

$$\langle n|m, \alpha\rangle = e^{-|\alpha|^2/2}\sqrt{\frac{n!}{m!}}(-\alpha^*)^{m-n}\mathrm{L}_n^{(m-n)}(|\alpha|^2),$$

where $\mathrm{L}_k^{(\alpha)}(x) = x^{-\alpha}\frac{1}{k!}(\frac{d}{dx} - 1)^k x^{k+\alpha} = \sum_{i=0}^{k}(-1)^i \frac{(k+\alpha)!x^i}{(\alpha+i)!(k-i)!i!}$ is the generalized Laguerre polynomial.

Problem 3: Higher-order squeezing.

The state $|\psi\rangle$ is said to be squeezed to the $2n$-th order in some quadrature component $\hat{x}(\theta) = (\hat{a}e^{i\theta} + \hat{a}^\dagger e^{-i\theta})/\sqrt{2}$ if the mean value $\langle\psi|(\hat{x}(\theta) - \langle\hat{x}(\theta)\rangle)^{2n}|\psi\rangle$ is lower than $\langle\alpha|(\hat{x}(\theta) - \langle\hat{x}(\theta)\rangle)^{2n}|\alpha\rangle$ for a coherent state $|\alpha\rangle$. It can be proven that $\langle\psi|(\hat{x}(0) - \langle\hat{x}(0)\rangle)^{2n}|\psi\rangle < 2^{-n}(2n-1)!!$ and that usual squeezed states are squeezed to any even order $2n$. To show that this is true, calculate the $n = 2$ variance for a squeezed state $|\xi\rangle$ and a coherent state $|\alpha\rangle$ and compare the results.