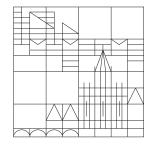
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http://tinyurl.com/qo2018



Quantum Optics

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Problem 1: Coherent states of light.

- a) Compute the photon number fluctuations $\langle (\hat{a}^{\dagger}\hat{a})^2 \langle \hat{a}^{\dagger}\hat{a} \rangle^2 \rangle$ for a coherent state $|\alpha\rangle$. Compare this value with the photon number expectation value for this same state.
- b) Given the displacement operator $D(\alpha) = \exp(\alpha \hat{a}^{\dagger} \alpha^* \hat{a})$, for which $|\alpha\rangle = D(\alpha)|0\rangle$, use the Baker-Campbell-Hausdorff formula to show that:

$$\hat{a}^{\dagger}|\alpha\rangle = \left(\frac{\partial}{\partial\alpha} + \frac{\alpha^*}{2}\right)|\alpha\rangle$$
 .

c) Show that:

$$\hat{D}(\alpha)\hat{D}(\beta) = \hat{D}(\alpha + \beta) \exp(i\operatorname{Im}\{\alpha\beta^*\})$$

d) Expressing the coherent state $|\alpha\rangle$ in terms of number states, show that:

$$|\langle \alpha' | \alpha \rangle|^2 = \exp(-|\alpha - \alpha'|^2)$$
.

Problem 2: Algebra of bosonic operators.

- a) Given two operator-valued functions $\hat{F}(\hat{a}, \hat{a}^{\dagger})$ and $\hat{G}(z; \hat{a}, \hat{a}^{\dagger}) = e^{\hat{a}z} \hat{F}(\hat{a}, \hat{a}^{\dagger}) e^{-\hat{a}z} = \hat{F}(\hat{a}, e^{\hat{a}z} \hat{a}^{\dagger} e^{-\hat{a}z})$, where $z \in \mathbb{C}$ and \hat{F} is an arbitrary power series in \hat{a} and \hat{a}^{\dagger} , show that from the derivative of \hat{G} relative to z for $\hat{F} = \hat{a}^{\dagger}$ one can conclude that $d\hat{G}(z; \hat{a}, \hat{a}^{\dagger})/dz = [\hat{a}, \hat{F}(\hat{a}, \hat{a}^{\dagger} + z)]$ and $\hat{G}(z; \hat{a}, \hat{a}^{\dagger}) = \hat{F}(\hat{a}, \hat{a}^{\dagger} + z)$ for any \hat{F} . Using these relations, show that $\partial \hat{F}(\hat{a}, \hat{a}^{\dagger})/\partial \hat{a}^{\dagger} = [\hat{a}, \hat{F}(\hat{a}, \hat{a}^{\dagger})]$.
- b) Consider the operator-valued functions $\hat{G}(\hat{a},\hat{a}^{\dagger})$ and $\hat{F}(\hat{a},\hat{a}^{\dagger})=\hat{a}\hat{G}(\hat{a},\hat{a}^{\dagger})$. Knowing that such functions can be rearranged in any desired way with respect to the order of \hat{a} and \hat{a}^{\dagger} by means of the commutation relations, consider the function obtained from \hat{G} by moving all anihilation operators to the right and the creation operators to the left, which is usually denoted by $\hat{G}^{(N)}(\hat{G}^{(N)})=\hat{G}$ and called "normal ordered form" of the given function. Use the relation obtained in section (a) to show that $\hat{F}(\hat{a},\hat{a}^{\dagger})=\partial\hat{G}^{(N)}(\hat{a},\hat{a}^{\dagger})/\partial\hat{a}^{\dagger}+\hat{G}^{(N)}(\hat{a},\hat{a}^{\dagger})\hat{a}$. Considering the "normal order operator" $\hat{N}\hat{A}=:\hat{A}:$, which when applied on a given operator function $\hat{A}(\hat{a},\hat{a}^{\dagger})$ forcibly arranges all annihilation operators to the right and the creation operators to the left without accounting for the commutation relations (i.e., $:\hat{a}\hat{a}^{\dagger}:=\hat{a}^{\dagger}\hat{a}$ and $\hat{G}^{(N)}\neq:\hat{G}:$), and replacing \hat{a} for $\hat{a}+\partial/\partial\hat{a}^{\dagger}$ within the normal order operator, show that $\hat{F}^{(N)}(\hat{a},\hat{a}^{\dagger})=:\hat{F}(\hat{a}+\partial/\partial\hat{a}^{\dagger},\hat{a}^{\dagger}):$