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Quantum Optics
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Problem 1: Beam splitter and Hong-Ou-Mandel effect.

A beam splitter transforms the incoming mode operators $\hat{a}_{\text{in}}$ and $\hat{b}_{\text{in}}$ into the outgoing mode operators $\hat{a}_{\text{out}}$ and $\hat{b}_{\text{out}}$ given by:

$$\hat{a}_{\text{out}} = \sqrt{\eta}\hat{a}_{\text{in}} - i\sqrt{1-\eta}\hat{b}_{\text{in}}, \qquad \hat{b}_{\text{out}} = \sqrt{\eta}\hat{b}_{\text{in}} - i\sqrt{1-\eta}\hat{a}_{\text{in}}.$$

a) Show that such a transformation may be generated by the unitary operator

$$\hat{T} = \exp[-i\theta(\hat{a}_{\text{in}}^\dagger \hat{b}_{\text{in}} + \hat{a}_{\text{in}}\hat{b}_{\text{in}}^\dagger)],$$

where $\eta = \cos^2(\theta)$.
b) Show that if the incoming state is a two-mode coherent state $|\alpha_{\text{in}}\rangle \otimes |\beta_{\text{in}}\rangle$, the outgoing state will be given by $|\alpha_{\text{out}}\rangle \otimes |\beta_{\text{out}}\rangle$ with

$$\alpha_{\text{out}} = \sqrt{\eta}\alpha_{\text{in}} - i\sqrt{1-\eta}\beta_{\text{in}}, \qquad \beta_{\text{out}} = \sqrt{\eta}\beta_{\text{in}} - i\sqrt{1-\eta}\alpha_{\text{in}}.$$

c) Show that if the incoming state is a two-mode Fock state $|1\rangle \otimes |1\rangle$, the outgoing state will be given by

$$(2\eta - 1)|1\rangle|1\rangle - i\sqrt{2\eta(1-\eta)}(|0\rangle|2\rangle + |2\rangle|0\rangle).$$

d) What does the result of the point (c) for the particular case of $\eta = 1/2$ physically mean (in terms of an experiment)?

Problem 2: Cat state: phase distribution and phase space representations.

Consider the superposition state

$$|\alpha_+\rangle = N(|\alpha\rangle + |-\alpha\rangle),$$

where N is a normalization constant.
a) Obtain the continuous phase distribution for this state.
b) Obtain the Husimi function for this state and plot it.
c) Using the characteristic function $\chi(\lambda) = \text{tr}\{\hat{\rho}\hat{D}(\lambda)\}$, where $\hat{\rho}$ is the density operator for this state and $\hat{D}(\lambda)$ is the displacement operator, obtain the Wigner function through convolution, $W(\beta) = (1/\pi^2)\int d^2\lambda \exp(\lambda^*\beta - \lambda\beta^*)\chi(\lambda)$, and plot it using an appropriate software (Wolfram Mathematica or similar). Is this a classical state?