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Quantum Optics
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Problem 1: Simplified photon counting theory.

Assume that the probability for a photon to induce a detector response (e.g. photoelectron ejection) over a certain period of time T is given by η.
a) Find the probability $P_m^{(n)}$ for the detection of m photons from a Fock state $|n\rangle$ during the interval T.
b) Considering the expansion of the density matrix of a general state in terms of Fock states and the probabilities for this state to bear given numbers of photons, find the probability P_m for the detection of m photons from a general state during the time T.
c) Show that $P_m = \rho_{mm}$ (the density matrix element of the chosen general state) if $\eta = 1$.

Problem 2: Second-order correlation function and non-classicality.

For the squeezed coherent state $\hat{D}(\alpha)\hat{S}(\xi)|0\rangle = |\alpha, \xi\rangle$, show that $g^{(2)}(\tau)$ is given by

$$g^{(2)}(\tau) = 1 + \frac{|\alpha|^2[\cosh(2|\xi|) - \sinh(2|\xi|)\cos(2\theta - \phi) - 1] + \sinh^2(|\xi|)\cosh(2|\xi|)}{(|\alpha|^2 + \sinh^2(|\xi|))^2},$$

where $\alpha = |\alpha|e^{i\theta}$ and $\xi = e^{i\phi}|\xi|$.

b) Show which conditions $|\alpha|$ should fulfill (for fixed values of $|\xi|$, θ and ϕ) for this state to have sub-Poissonian statistics. How does the phase $\Omega = 2\theta - \phi$ influence $g^{(2)}(\tau)$?
c) Estimate the minimum value of $g^{(2)}(\tau)$ in the space of possible values of $|\xi|$, $|\alpha|$ and Ω.

Problem 3: Photon counting statistics.

Compute the second order correlation function $\left[g^{(2)}(\tau)\right]$ for a 2-mode Fock state $|n_{\omega_1}, n_{\omega_2}\rangle$, where $\omega_1 \neq \omega_2$ and $\mathbf{k}_1 \| \mathbf{k}_2$. Show that when $n_{\omega_1} = n_{\omega_2} = \frac{1}{2}n$, one gets

$$g^{(2)}(\tau) - 1 = \frac{1}{2}\cos(\omega_1 - \omega_2)\tau - \frac{1}{n}.$$

From there study the photon counting statistics of this state in a time interval T using

$$\mathrm{Var}\,\hat{N} - \langle\hat{N}\rangle = \frac{\langle\hat{N}\rangle^2}{T^2}\int_{-T}^{+T} \mathrm{d}\tau\ (T - |\tau|)\left[g^{(2)}(\tau) - 1\right].$$

How does it compare to the Poissonian photon counting statistics in dependence on the value of T? Are the photons bunched or anti-bunched? HINT: consider that the field amplitudes are equal.