Understanding spin-qubit decoherence: From models to reality

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Collaborators:

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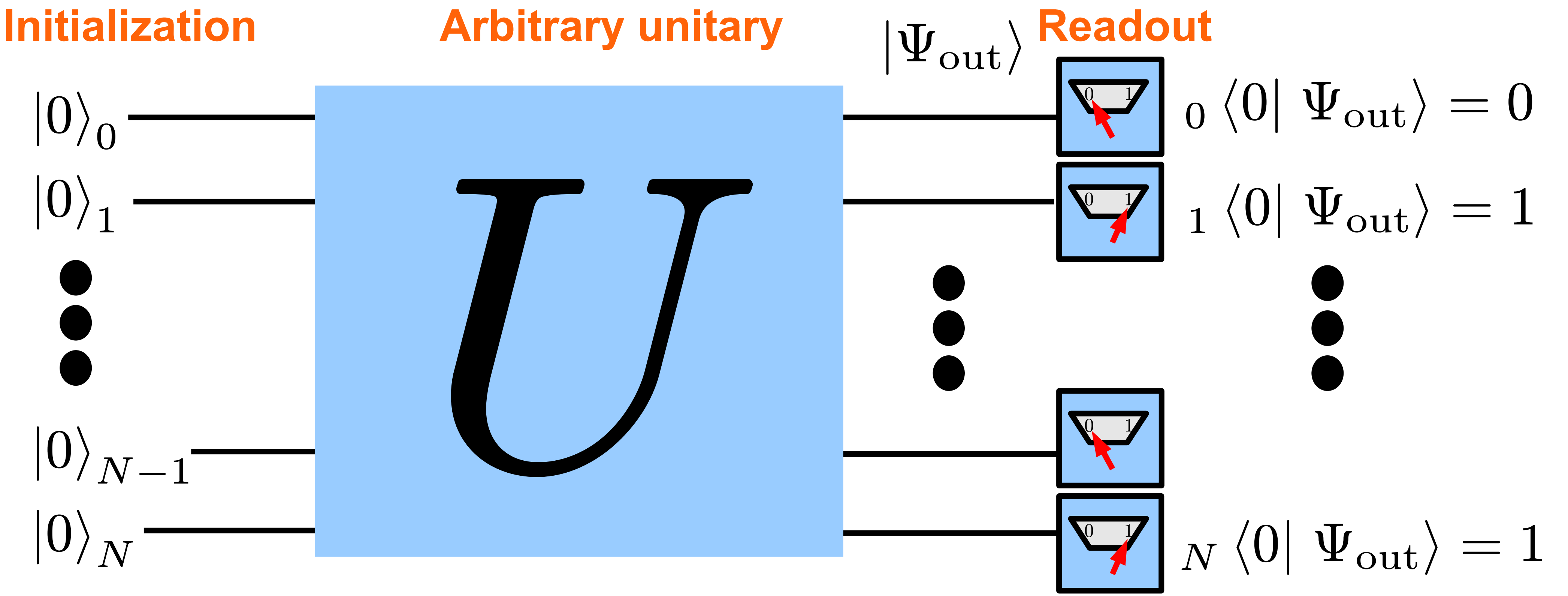
Waterloo: **Farzad Qassemi**, Frank Wilhelm, Jonathan Baugh
Oslo: Joakim Bergli

CIFAR CANADIAN INSTITUTE for ADVANCED RESEARCH

IQC Institute for Quantum Computing

NSERC CRSNG

Goal: Quantum Computation



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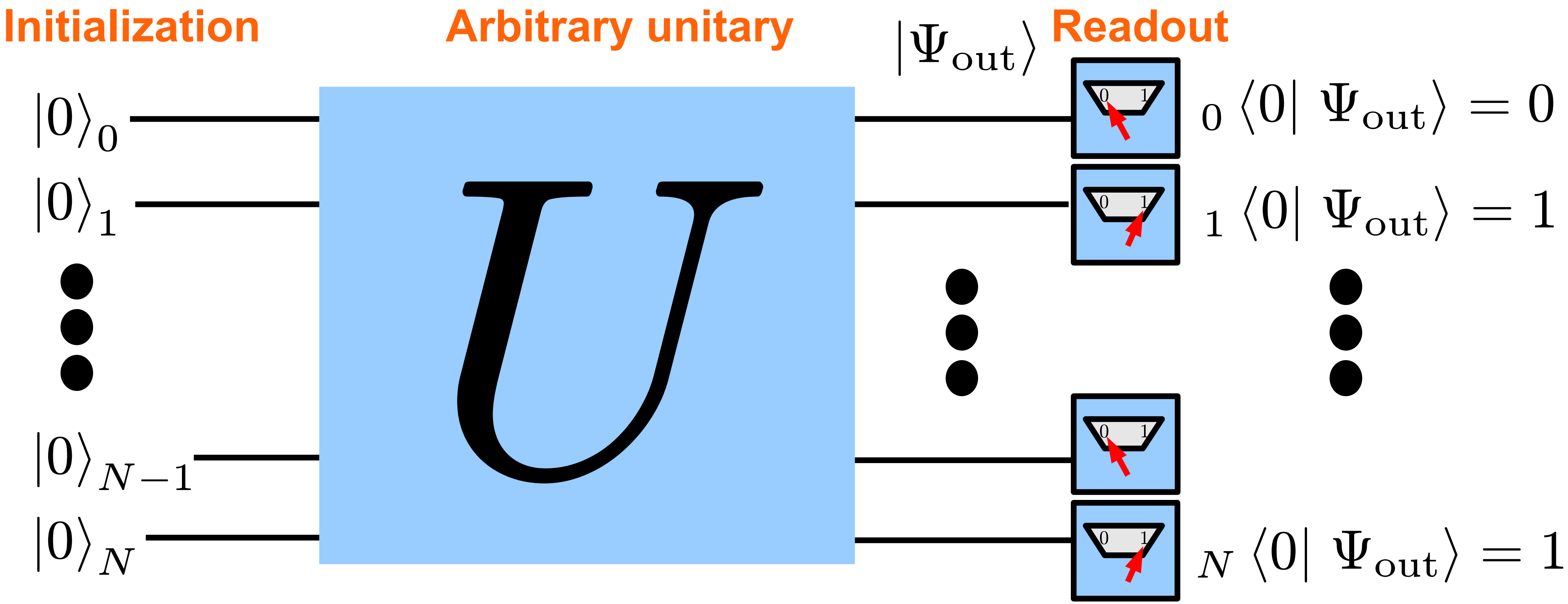
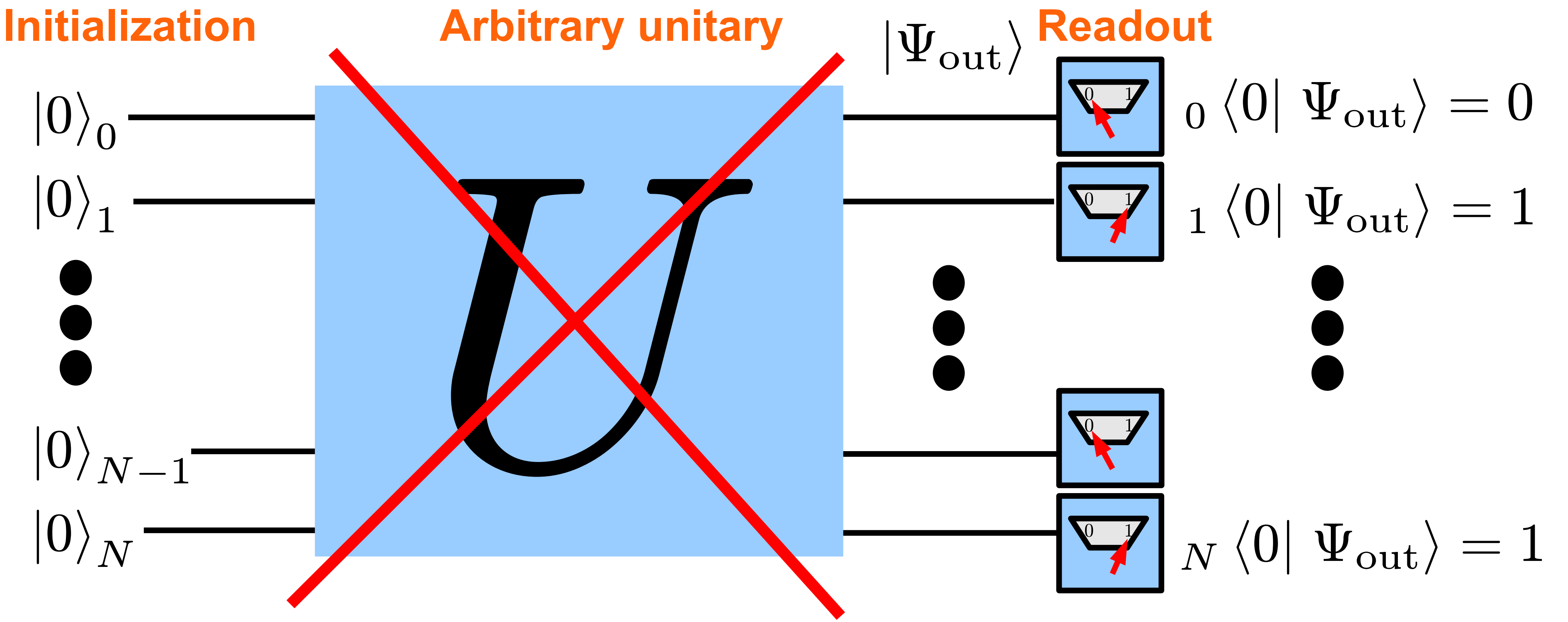


Physical Implementation:

$$U = \mathcal{T} \exp\left\{ -i \int_0^t dt' H(t') \right\} \qquad H \in \mathcal{H}_S$$

Reality: Imperfections

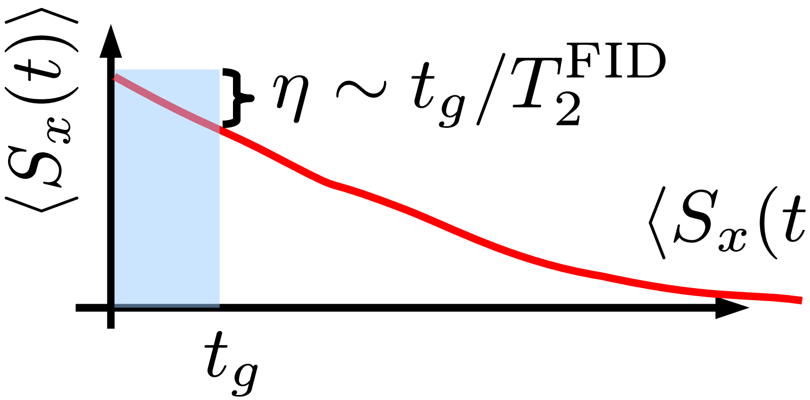


$$\tilde{U} = \mathcal{T} \exp\left\{-i \int_0^t dt' \left(H(t') + \delta H(t')\right)\right\} \quad \delta H \in \mathcal{H}_S \otimes \mathcal{H}_E$$

Types of error

In addition to initialization/readout error

Gate error (free-induction decay)



$$\langle S_x(t) \rangle \propto e^{-t/T_2^{\mathrm{FID}}}$$

Error-correction threshold

$$\eta < \eta_c \sim 10^{-6} - 10^{-2}$$

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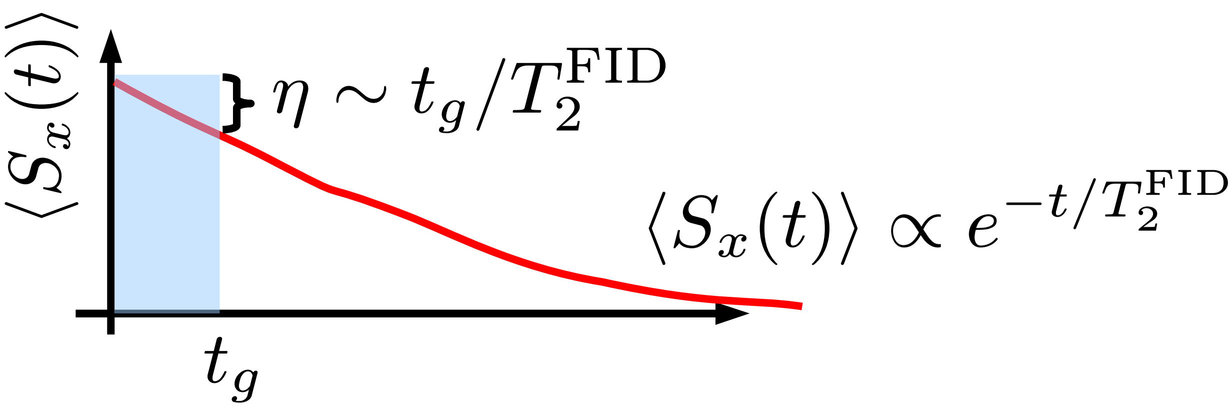

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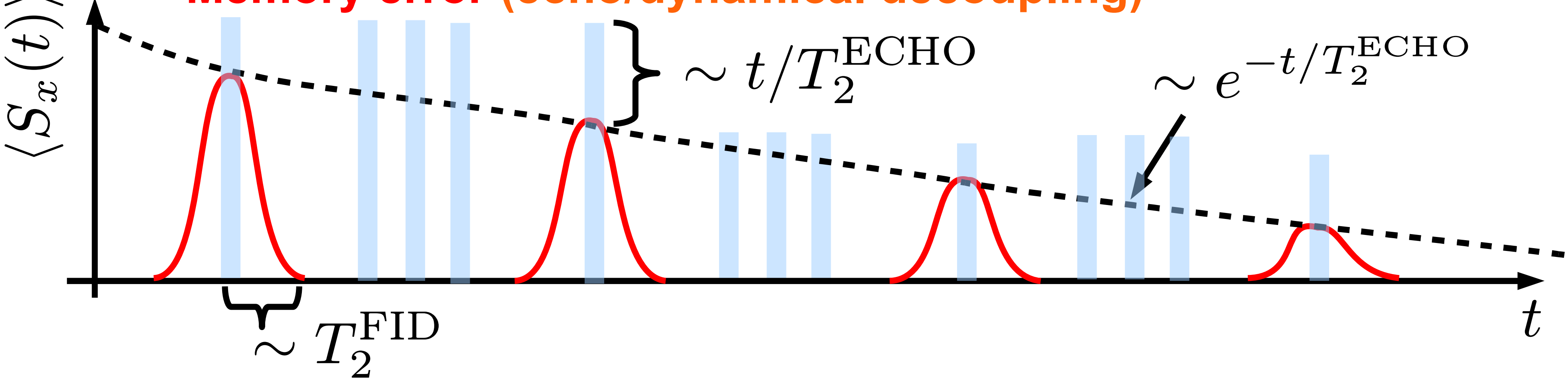
Memory error (echo/dynamical decoupling)



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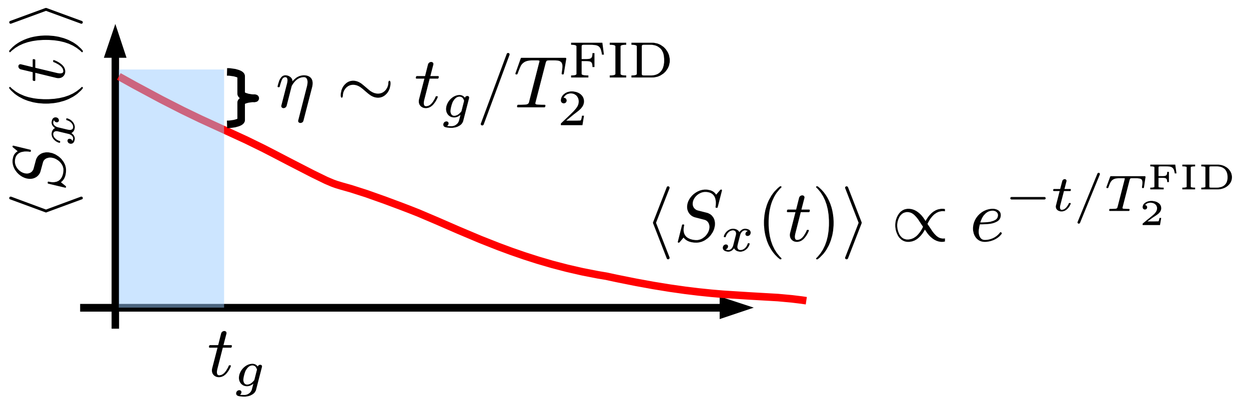

Error-correction threshold

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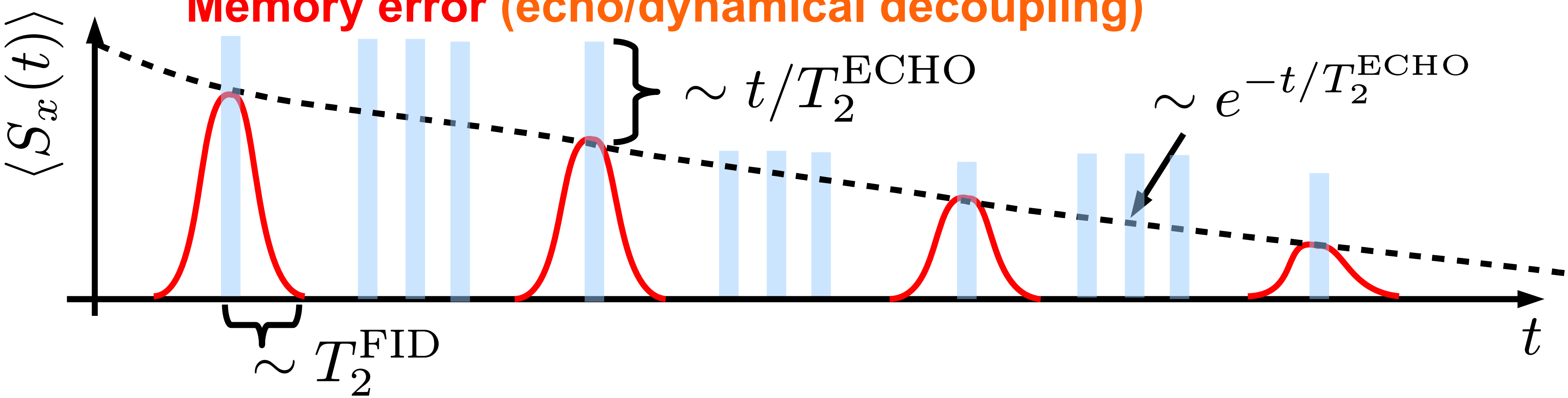
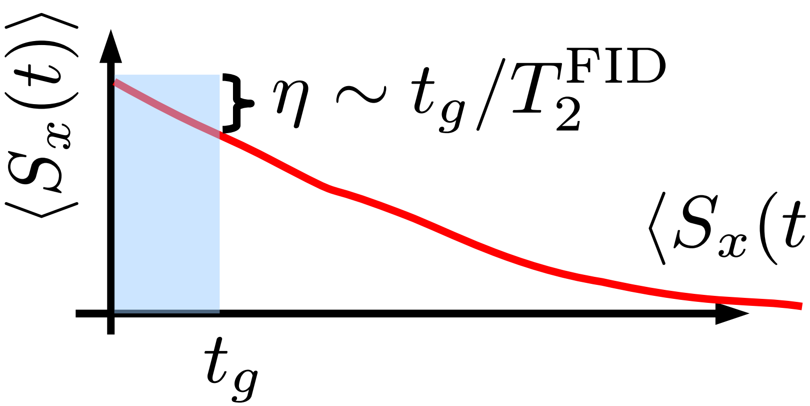


Even for single spin: $T_2^{\mathrm{ECHO}} \neq T_2^{\mathrm{FID}}$ **'Intrinsic' decay time is a myth!**

Types of error

In addition to initialization/readout error

Gate error (free-induction decay)



$$\langle S_x(t) \rangle \propto e^{-t/T_2^{\mathrm{FID}}}$$

Error-correction threshold

$$\eta < \eta_c \sim 10^{-6} - 10^{-2}$$

Focus on reducing gate error (increasing FID time): $T_2^{\mathrm{FID}} = T_2$

Caveat: Gating and decay not always independent (should really determine the gate fidelity).

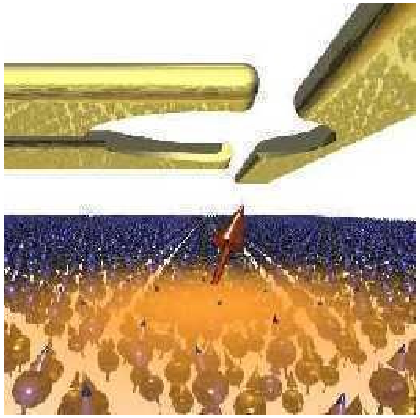
e.g.:

$$F = \mathrm{Tr}\left\{ U^\dagger \tilde{U} \right\}$$

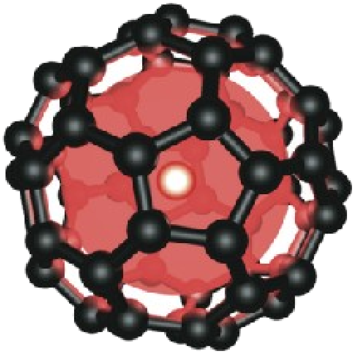
Nuclear spins are (almost) everywhere

NV centers in diamond

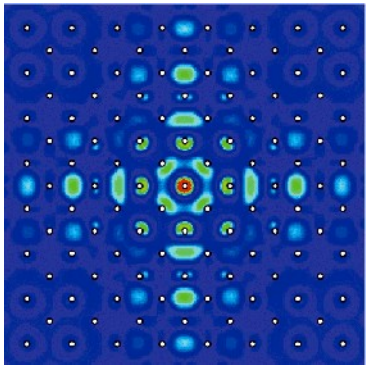
Quantum dots



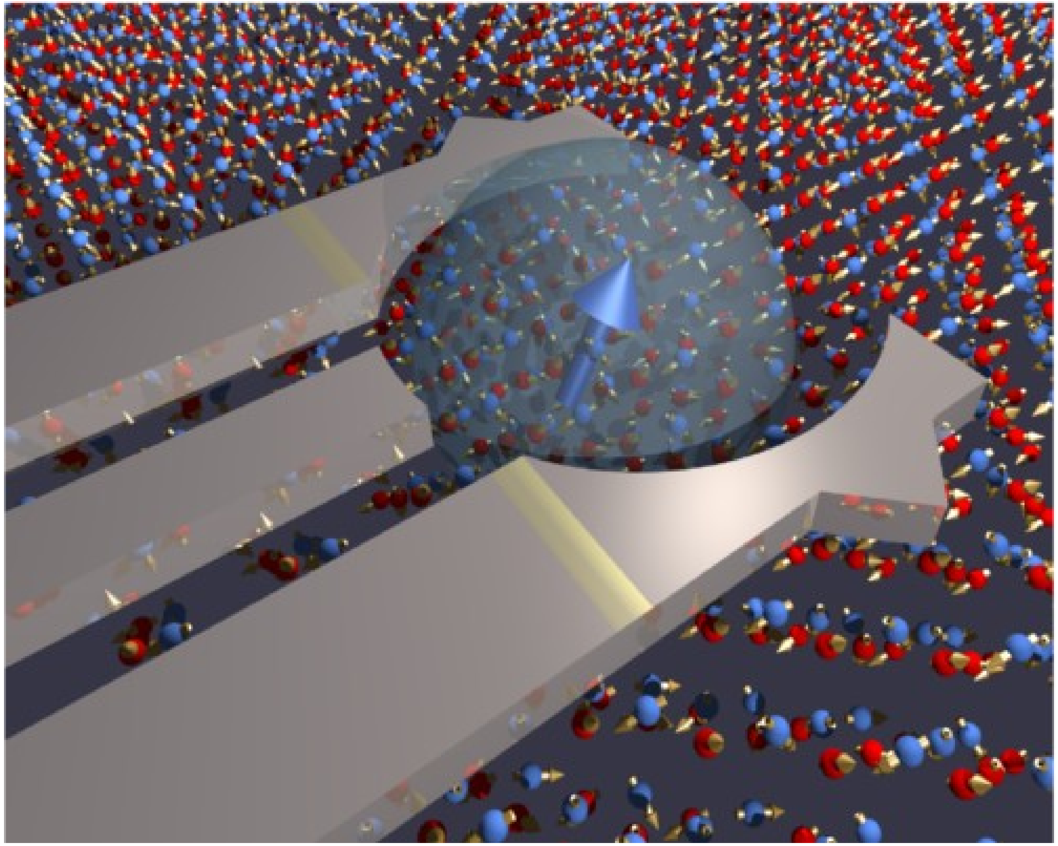
N@C_{60}



Phosphorus donors



Coherence
Problem: One spin sees many



$N \sim 10^6$
nuclei

WAC and J. Baugh, `Nuclear spins in nanostructures',
Phys. Stat. Solidi B (2009)

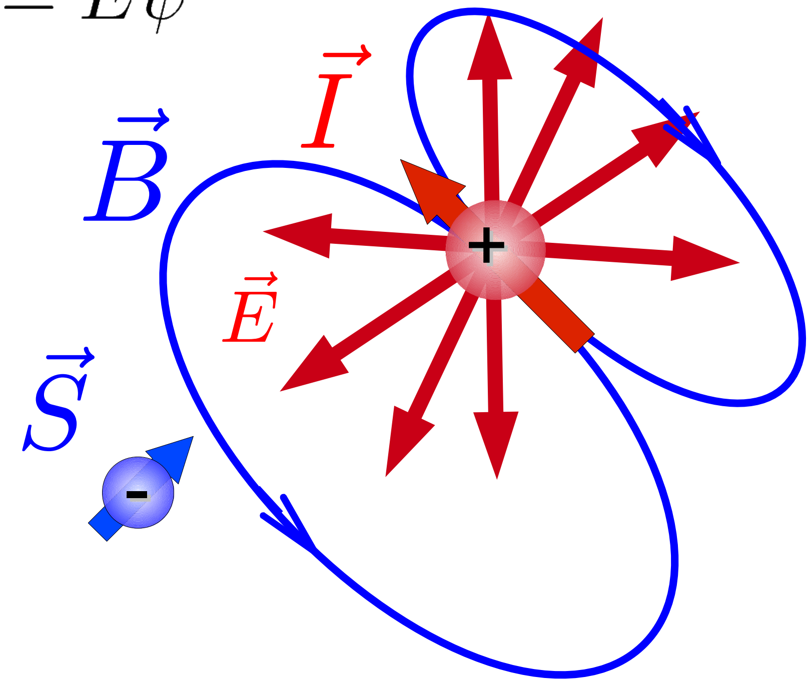
"Theory of everything" for spins in the solid state

$$\left(\boldsymbol{\alpha} \cdot \boldsymbol{\pi} + \beta mc^2 - |e|V(\mathbf{r})\right)\psi = E\psi$$

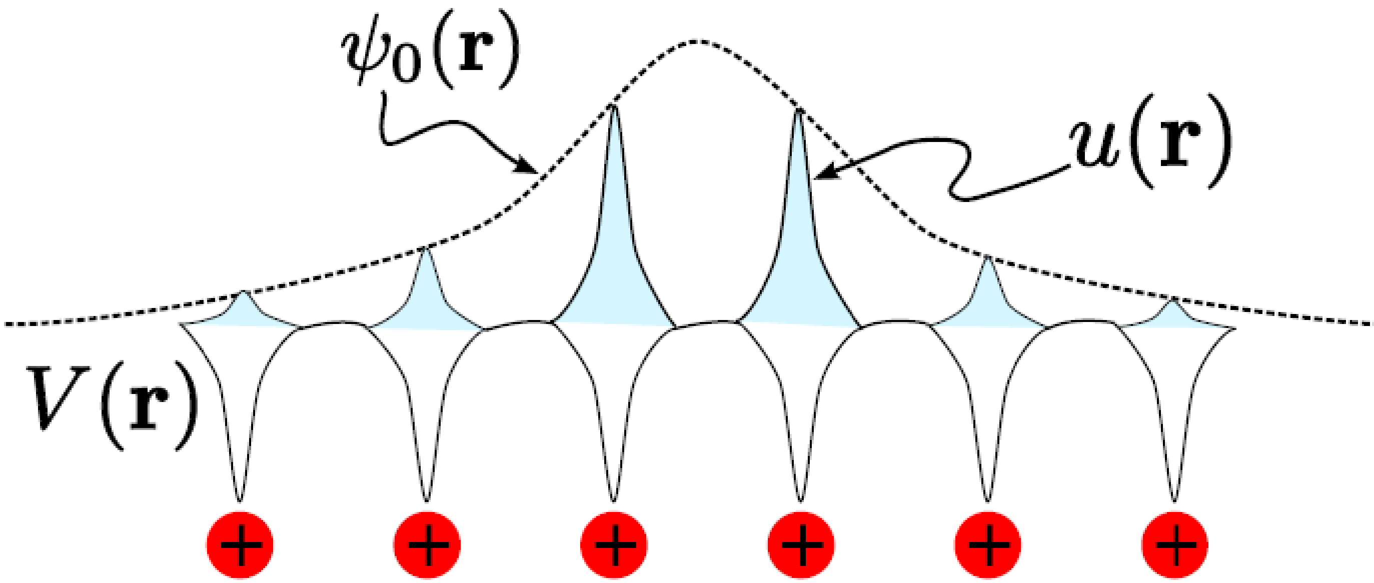
$$H_{\text{contact}} = \frac{8\pi}{3}\gamma_S\gamma_I\delta(\mathsf{r})\mathsf{S}\cdot\mathsf{I}$$

$$H_{\text{dip.}} = \gamma_S\gamma_I\frac{3(\mathsf{n}\cdot\mathsf{S})(\mathsf{n}\cdot\mathsf{I}) - \mathsf{S}\cdot\mathsf{I}}{r^3}$$

$$H_{\text{LI}} = \gamma_S\gamma_I\frac{\mathsf{L}\cdot\mathsf{I}}{r^3}$$



Confined electron

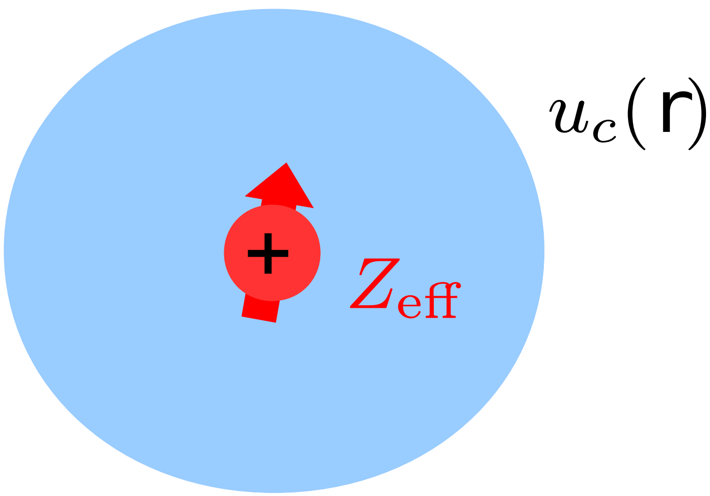


$$H_{\text{eff}} \simeq \langle \psi_0 | \, H \, | \psi_0 \rangle$$

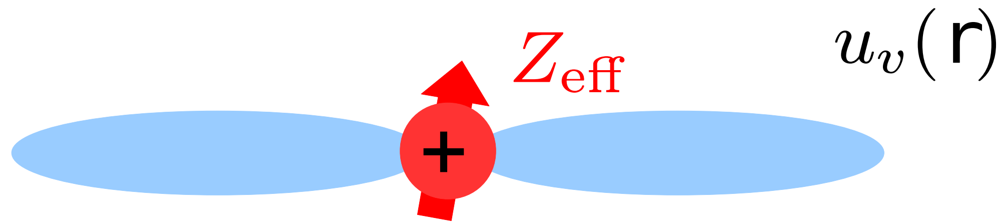
$$\langle \mathsf{r} \, | \psi_0 \rangle \simeq u(\mathsf{r}) \psi_0(\mathsf{r})$$

Interactions: s vs. p

s-state (electron)



p-state (hole)



$$\langle \psi^s_{\mathrm{orb}} | \, H_{\mathrm{contact}} \, | \psi^s_{\mathrm{orb}} \rangle \neq 0$$

$$\langle \psi^s_{\mathrm{orb}} | \, H_{\mathrm{dip.}} \, | \psi^s_{\mathrm{orb}} \rangle = 0$$

$$\langle \psi^s_{\mathrm{orb}} | \, H_{\mathrm{LI}} \, | \psi^s_{\mathrm{orb}} \rangle = 0$$

$$\langle \psi^p_{\mathrm{orb}} | \, H_{\mathrm{contact}} \, | \psi^p_{\mathrm{orb}} \rangle = 0$$

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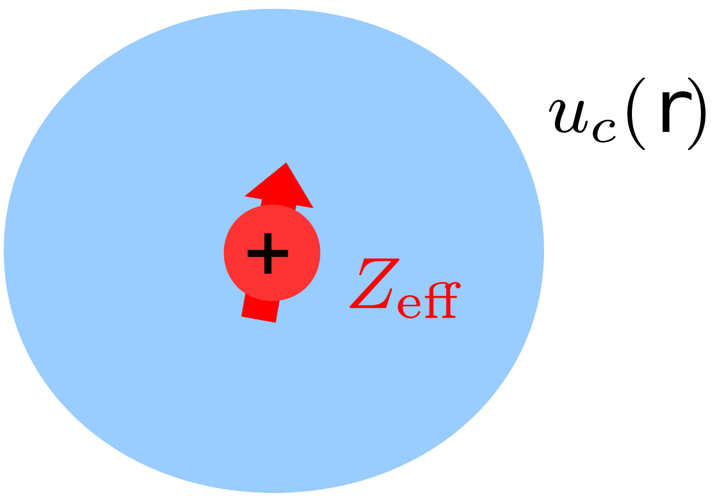
Interactions: s vs. p

s-state (electron)



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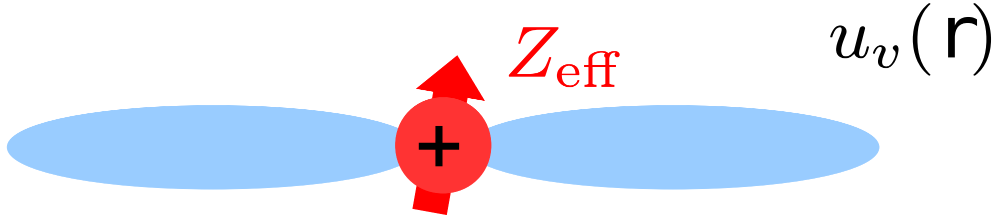

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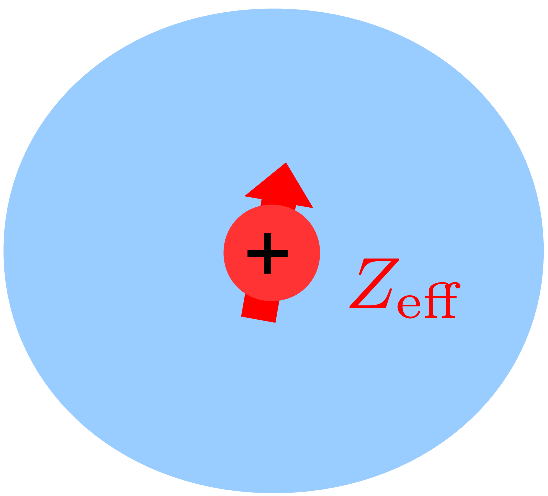
$$\langle \psi^p_{\mathrm{orb}} | H_{\mathrm{dip.}} | \psi^p_{\mathrm{orb}} \rangle \neq 0$$

$$\langle \psi^p_{\mathrm{orb}} | H_{\mathrm{LI}} | \psi^p_{\mathrm{orb}} \rangle \neq 0$$

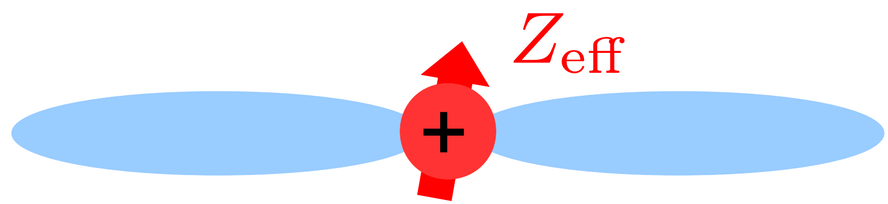
Anything else: NV Center, Nanotubes, graphene,...combination

Interactions: s vs. p

s-state



p-state



Project onto $m_J = \pm\frac{3}{2} \Rightarrow s_z = \pm\frac{1}{2}$

$$H_s^{\text{eff}} = A_s \mathbf{S} \cdot \mathbf{I}$$

$$H_p^{\text{eff}} = A_p s_z I_z$$

For 4s, 4p Hydrogen-like atomic orbitals (valence states of Ga, As):

$$\frac{A_p}{A_s} = \frac{1}{5}\left(\frac{Z_{\text{eff}}(4p)}{Z_{\text{eff}}(4s)}\right)^3$$

The two coupling strengths are comparable!

In a 2D GaAs system

$$\frac{A_h}{A_e} \approx 0.15$$

Large!

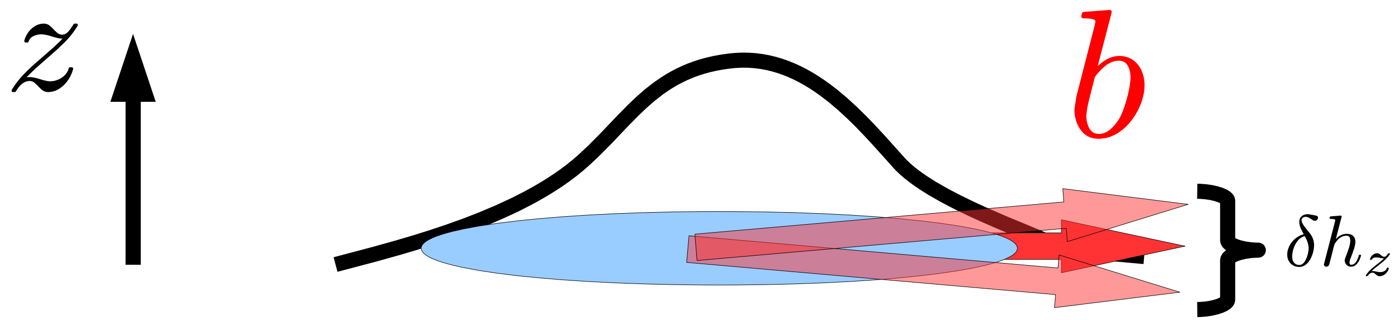
$$H = (b + h_z)s_z$$

Ising: no flip-flops

Fischer, WAC, Bulaev, Loss, PRB (2008)

Hole-spin Decoherence

$$H = h_z s_z + b s_x$$



Transverse fluctuations **suppressed** for large in-plane b

$$\langle s_z \rangle_t \simeq \frac{\cos\left(bt + \frac{1}{2}\arctan\left(t/\tau\right)\right)}{2\left[1+(t/\tau)^2\right]^{1/4}} \sim \frac{1}{\sqrt{t}} \qquad \tau = \frac{b}{(\delta h^z)^2}$$

Predicted: $\tau \simeq 1\,\mu s$

D. Brunner et al., Science (2009)

Measured: $\tau \gtrsim 1\,\mu s$

Electrons: Hyperfine Hamiltonian

$$H_{\mathrm{hf}} = bS^z + \mathbf{h} \cdot \mathbf{S} + \ldots$$

$$\mathbf{h} = \sum_k A_k \mathbf{I}_k$$

$$A = \sum_k A_k$$

Electron Zeeman energy

Coupling to nuclear field

Neglect:
Nuclear dipole-dipole,
Quadrupolar splitting,
Spin-orbit,
Phonons, ...

Hyperfine Hamiltonian

$$H_{\mathrm{hf}} = bS^z + \mathsf{h}\cdot\mathsf{S}$$

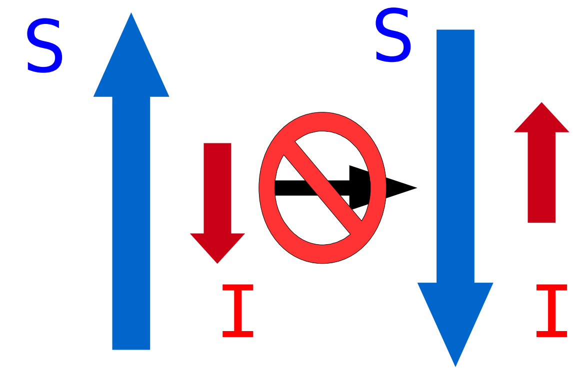
Electron Zeeman energy

Coupling to nuclear field

$$\mathsf{h} = \sum_k A_k \mathsf{I}_k$$

$$A = \sum_k A_k$$

$$\mathsf{h}\cdot\mathsf{S} = h^z S^z + \frac{1}{2}\left(h^+ S^- + h^- S^+\right)$$



V_{ff} does not conserve energy for large b

Hyperfine Hamiltonian

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Electron Zeeman energy

Coupling to nuclear field

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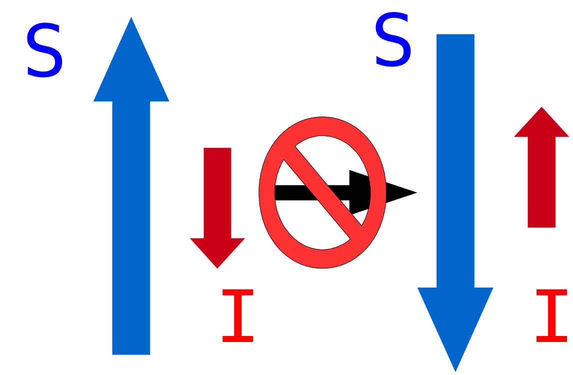
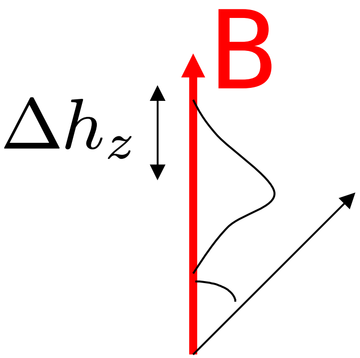


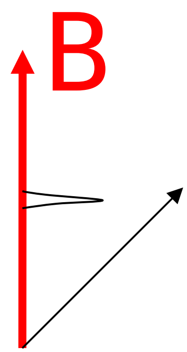
Perturbation theory in $\frac{A}{b} \ll 1$ $\quad b/g^*\mu_B \gtrsim 3.5\,\mathrm{T}\ (\mathrm{GaAs})$

Nuclear-spin bath preparation



$$h \Rightarrow \langle S_x \rangle_t \propto e^{-(t/\tau)^2} \quad \tau \sim \mathrm{ns}$$

measurement



$$h \Rightarrow \langle S_x \rangle_t \propto e^{i\omega t}$$

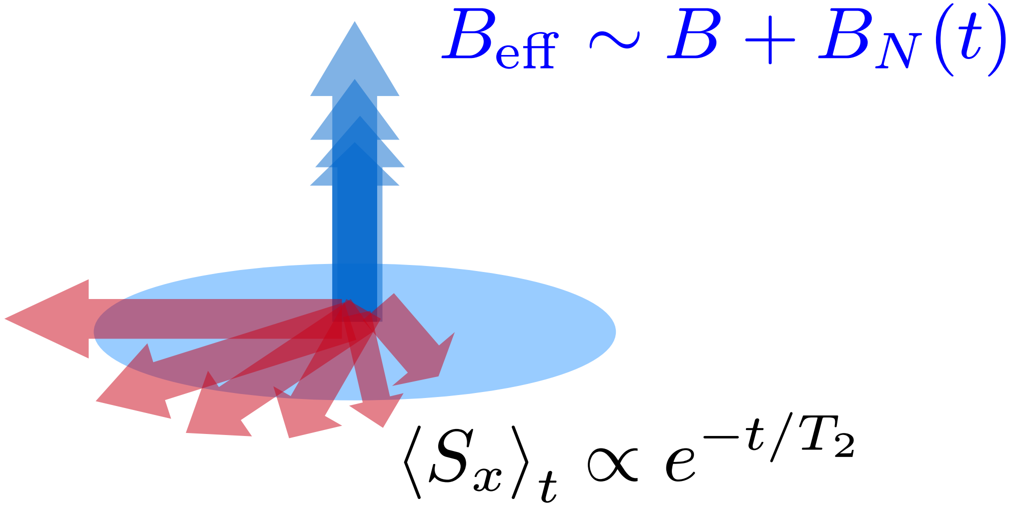
(narrowed state)

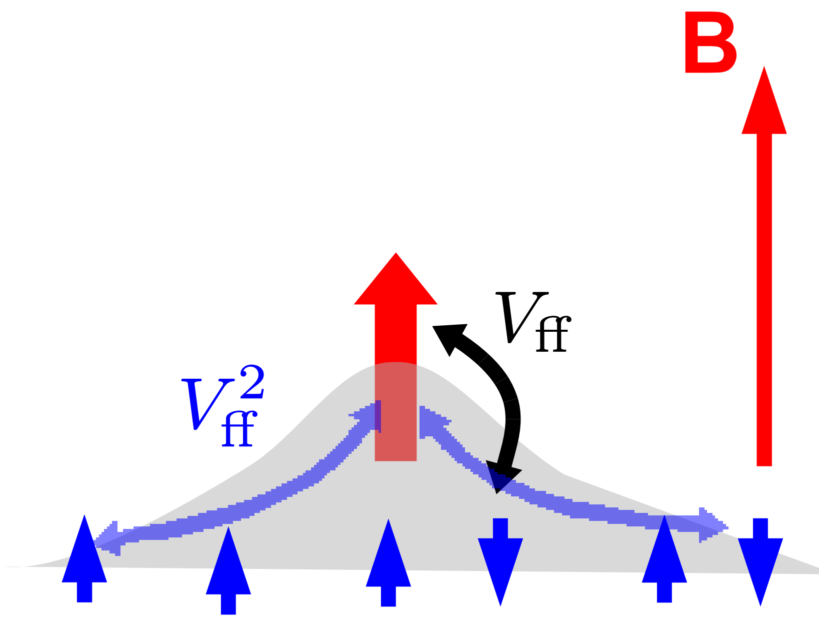
Theory: WAC and Loss, PRB (2004), Klauser, WAC and Loss, PRB (2006,2008), Stepanenko et al., PRL (2006), Giedke et al., PRA (2006), Ribeiro and Burkard, PRL (2009),

Expt.: Greilich et al., Science (2006), (2007), Reilly et al., Science (2008), Xu et al., Nature (2009), Vink et al., Nat. Phys. (2009), Latta et al., Nat. Phys. (2009)

After Narrowing...

Dynamics in nuclear-spin system lead to decay



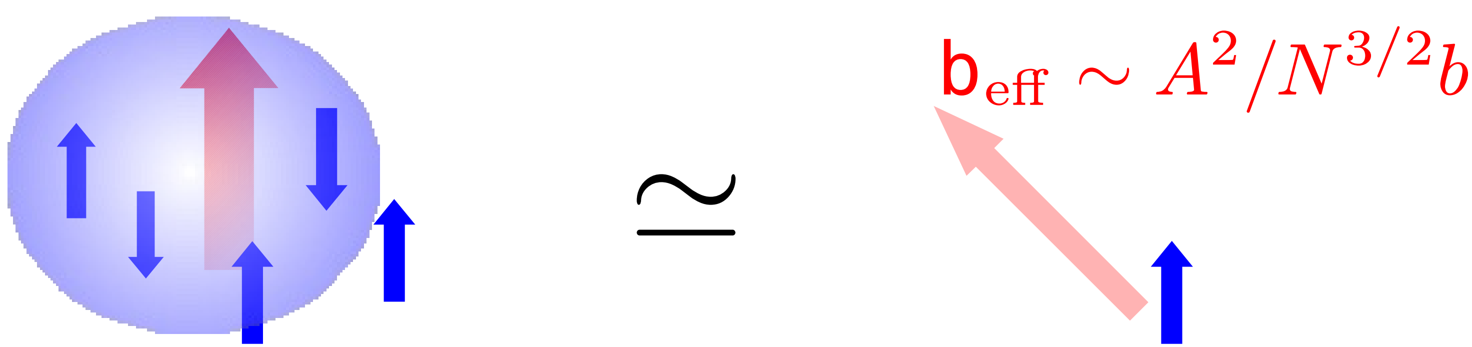


Nuclear-spin dynamics

D. Klauser, WAC, D. Loss, PRB (2008)

Short time:

$$\langle h_z(t)\rangle \simeq \langle h_z(0)\rangle \left(1 - \left(\frac{t}{\tau_n}\right)^2 + \mathcal{O}(t^3)\right) \qquad \tau_n \sim \frac{N^{3/2}b}{A^2} \sim 10^{-4}\,\mathrm{s}$$



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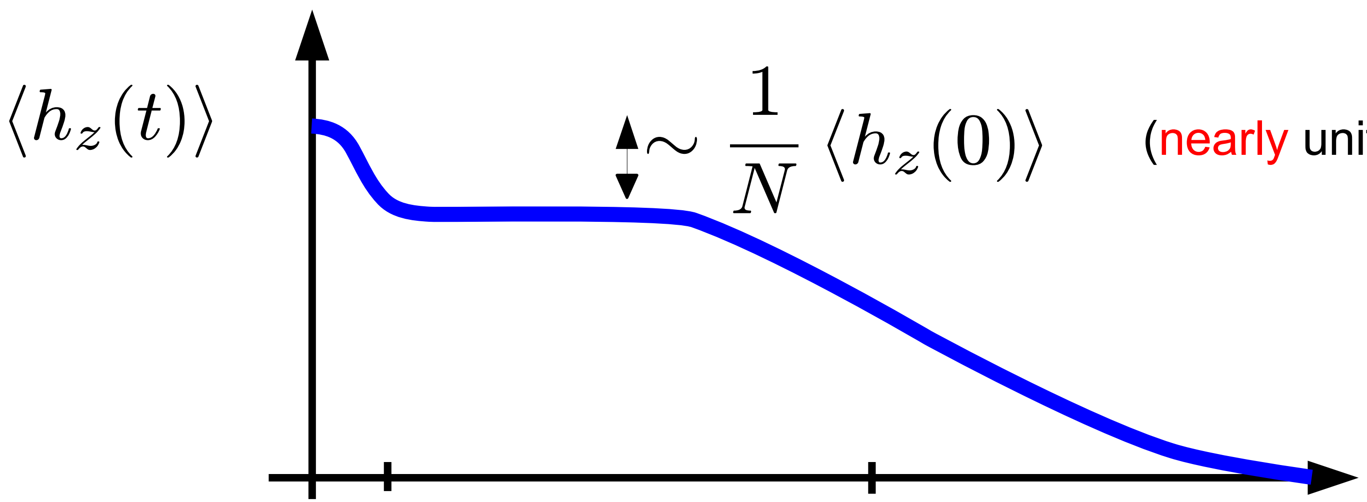
Beyond short time (generalized master equation):



(nearly uniform polarization)

$$\tau_c \sim \frac{N}{A} \sim 10^{-6}\,\mathrm{s} \qquad \tau_{\mathrm{diff}} \gtrsim 1 - 10\,\mathrm{s}$$

(Nuclear correlation time) (Dipolar spin diffusion)

PHYSICAL REVIEW VOLUME 125, NUMBER 3 FEBRUARY 1, 1962

Spectral Diffusion Decay in Spin Resonance Experiments

J. R. Klauder and P. W. Anderson
Bell Telephone Laboratories, Murray Hill, New Jersey

(Received September 1, 1961)

. While some progress has been made in solving, under rather restricted circumstances and with assumptions which are not by any means always valid, the exact quantum-mechanical equations of motion,[2] there is little hope of real progress in that direction on such immensely complicated questions as spectral diffusion.

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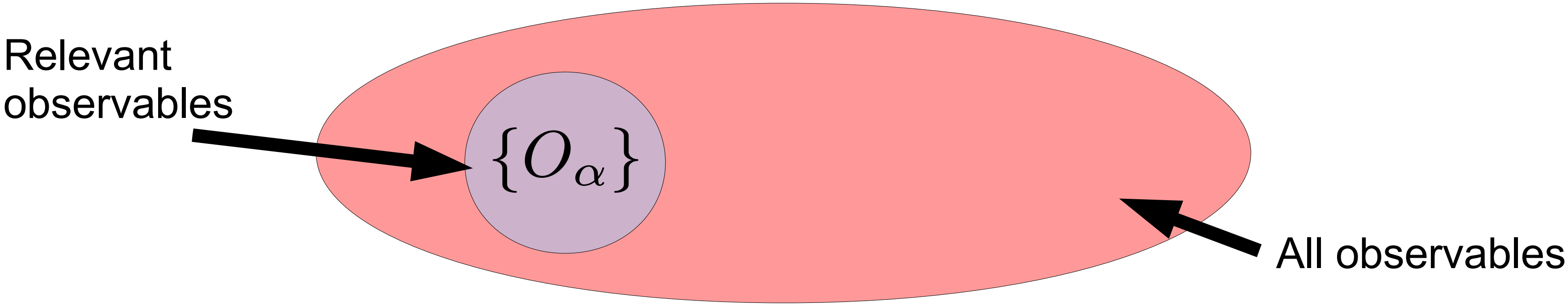
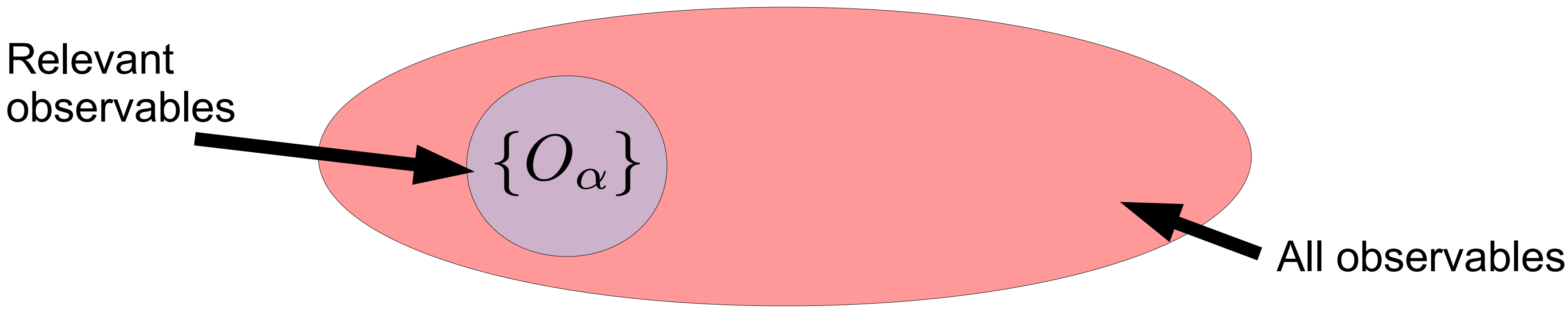
Not an 'easy' problem!

New approach: A general theory of coherent quantum dynamics



Von Neumann: $\dot{\rho} = -i\,[H, \rho]$ $\qquad \langle O_\alpha \rangle_t = \mathrm{Tr}\,\{\mathrm{O}\rho(\mathrm{t})\}$

New approach: A general theory of coherent quantum dynamics



Von Neumann: $\dot{\rho} = -i\,[H,\rho]$ $\qquad \langle O_\alpha \rangle_t = \mathrm{Tr}\,\{\mathrm{O}\rho(\mathrm{t})\}$

Nakajima-Zwanzig Generalized Master Equation

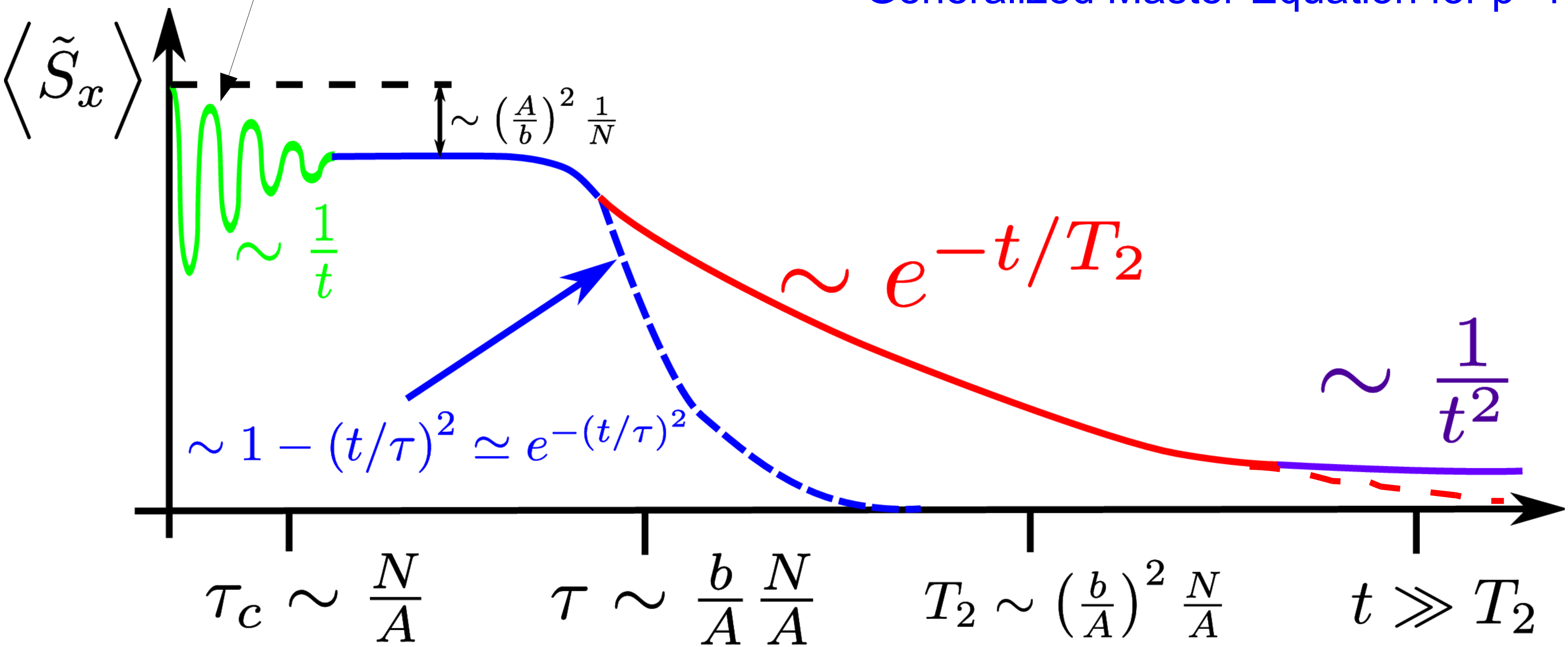
$$\left\langle \dot{O}_\alpha \right\rangle_t = -i\sum_\beta \omega_{\alpha\beta}\,\langle O_\beta\rangle_t - i\sum_\beta \int_0^t dt'\,\Sigma_{\alpha\beta}(t-t')\,\langle O_\beta\rangle_{t'}$$

$$H = H_0 + V \qquad \Sigma(t) = \sum_n \Sigma^{(n)}(t) \qquad \Sigma^{(n)}(t) = O\left(V^n\right)$$

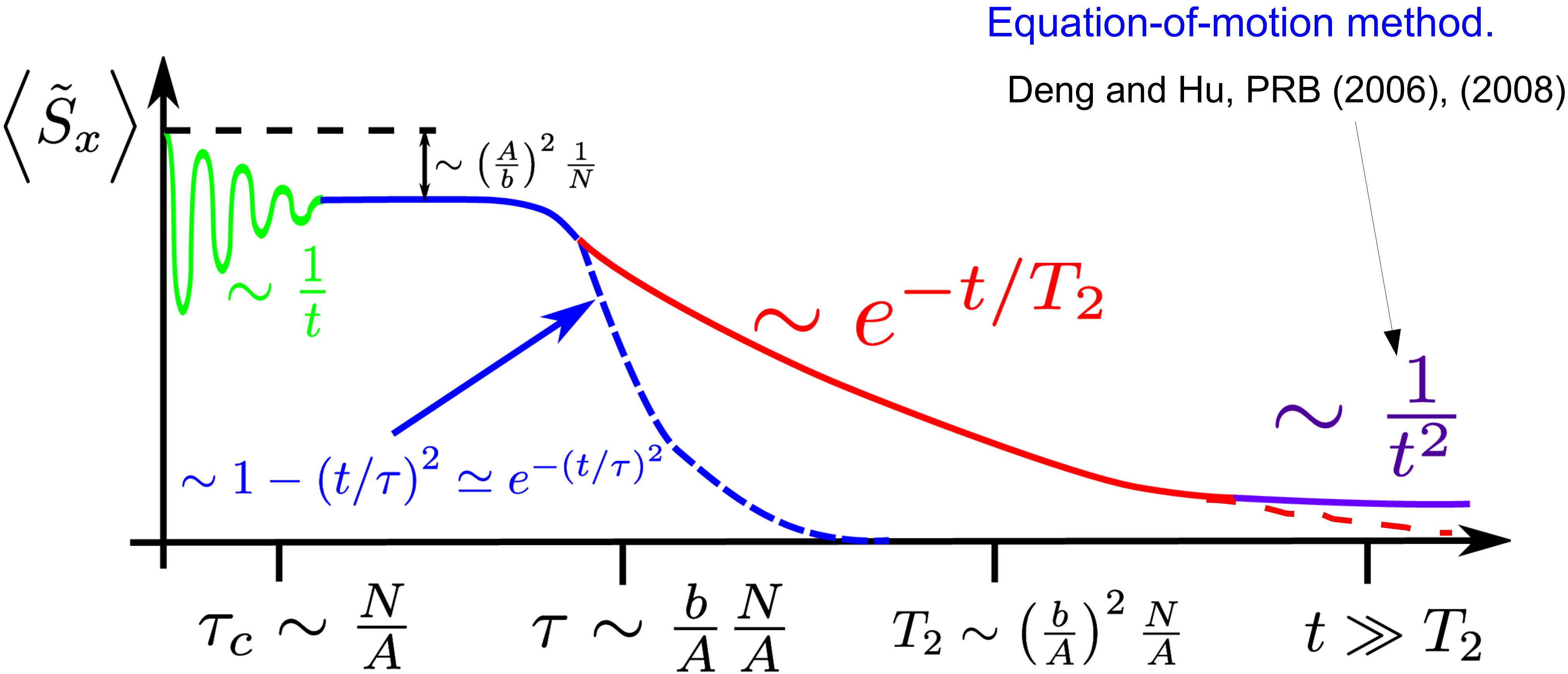
Free-induction decay: history

Khaetskii, Loss, Glazman, PRL (2002), PRB (2003)
WAC and Loss, PRB (2004)

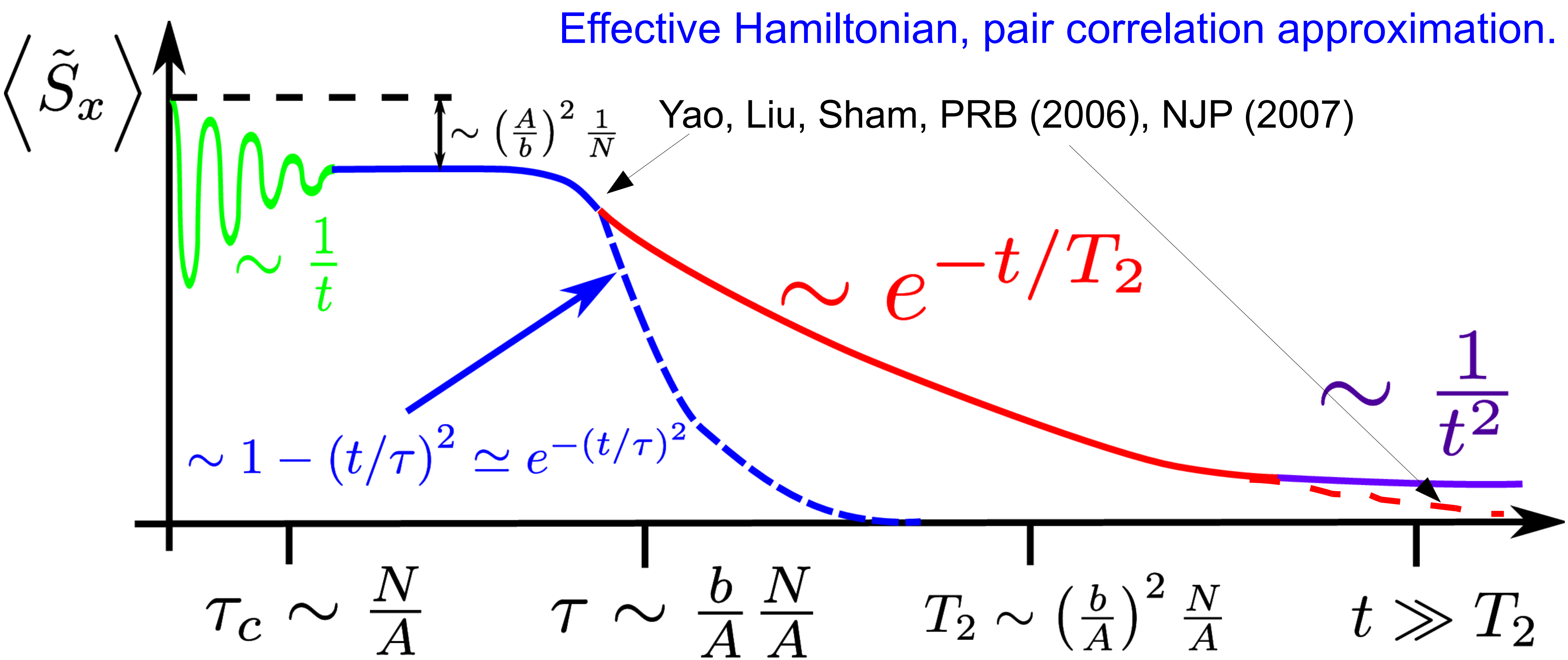
Exact solution for p=1,
Generalized Master Equation for p<1.



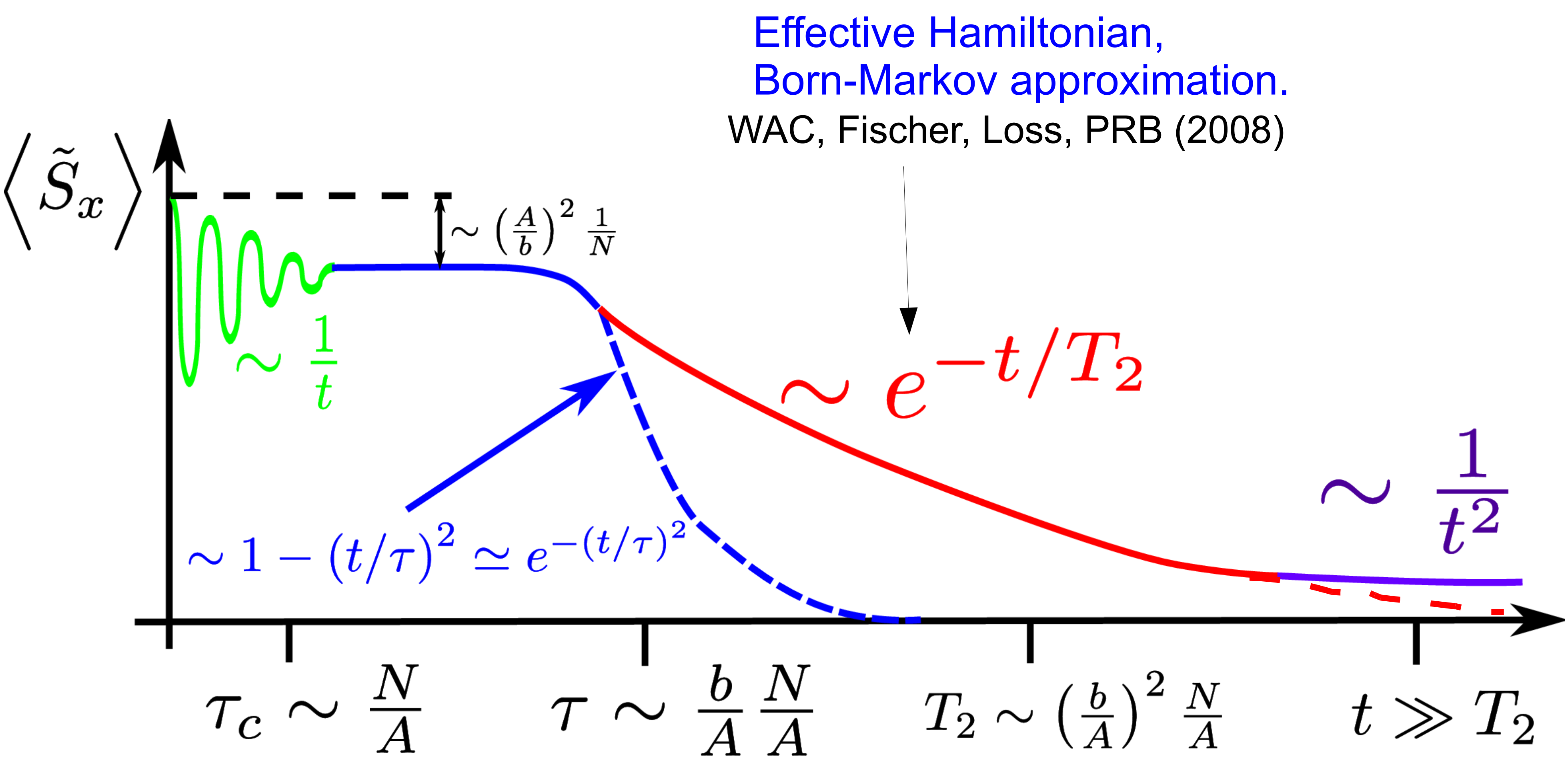
Free-induction decay: history



Free-induction decay: history



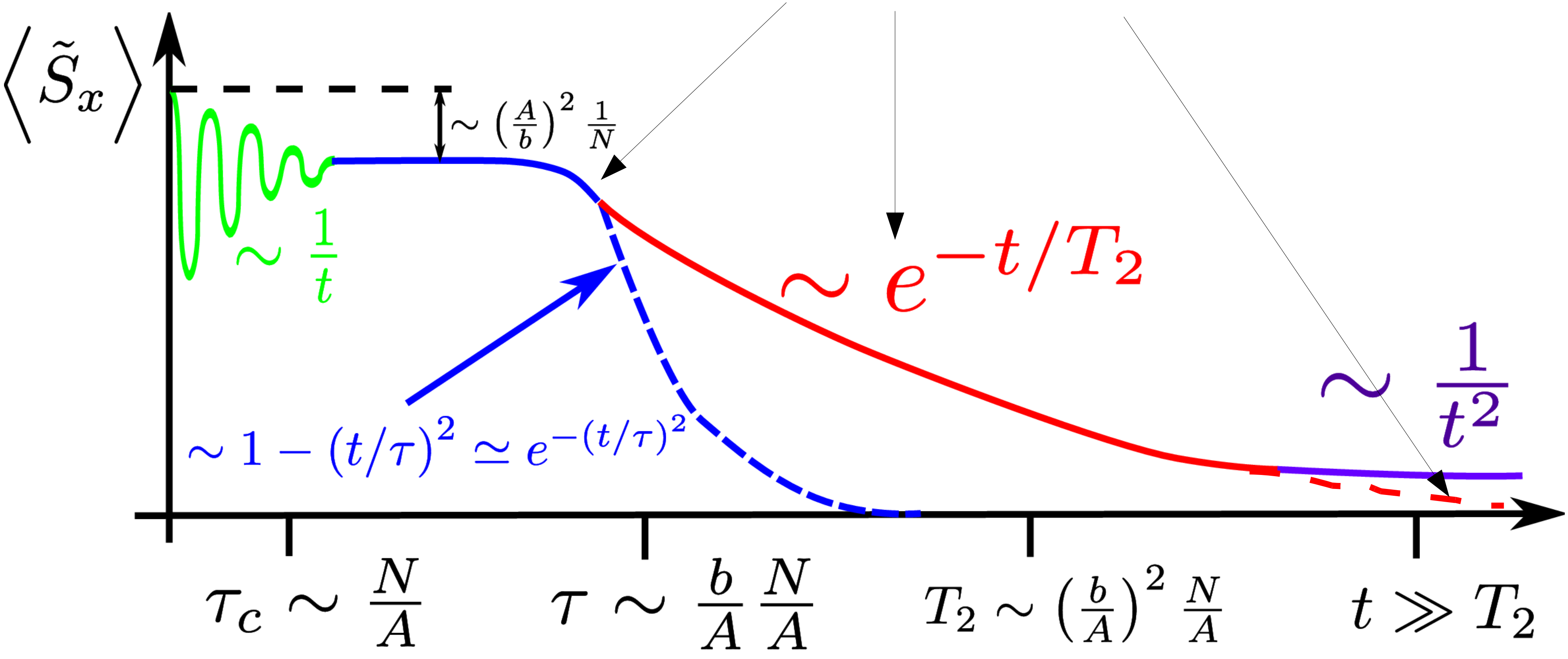
Free-induction decay: history



Free-induction decay: history

Effective Hamiltonian,
High-order resummation, low b-field.

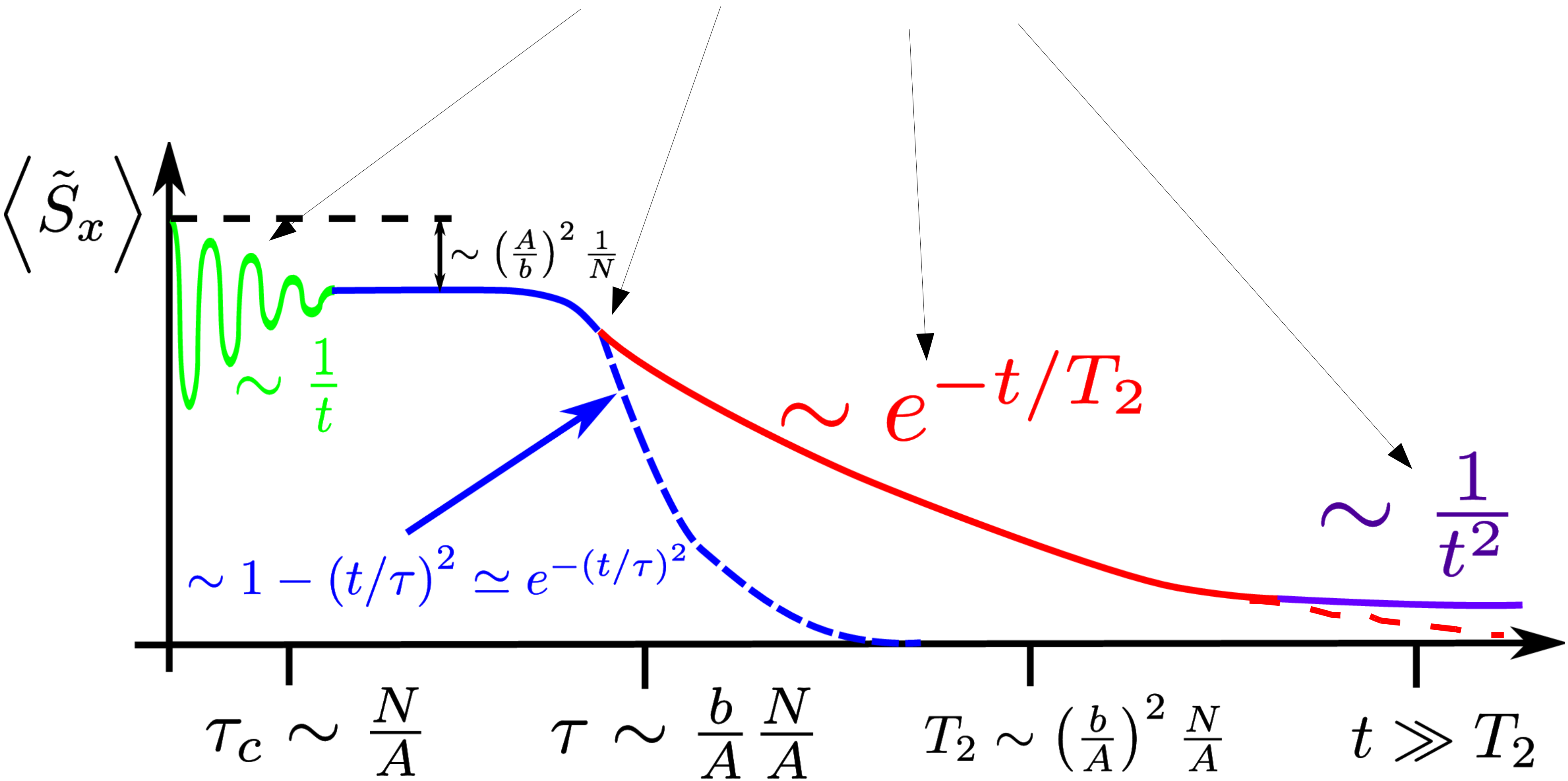
Cywinski, Witzel, Das Sarma, PRL (2009), PRB (2009)



Free-induction decay: history

Generalized Master Equation, Higher order.

WAC, Fischer, Loss, PRB (2010)



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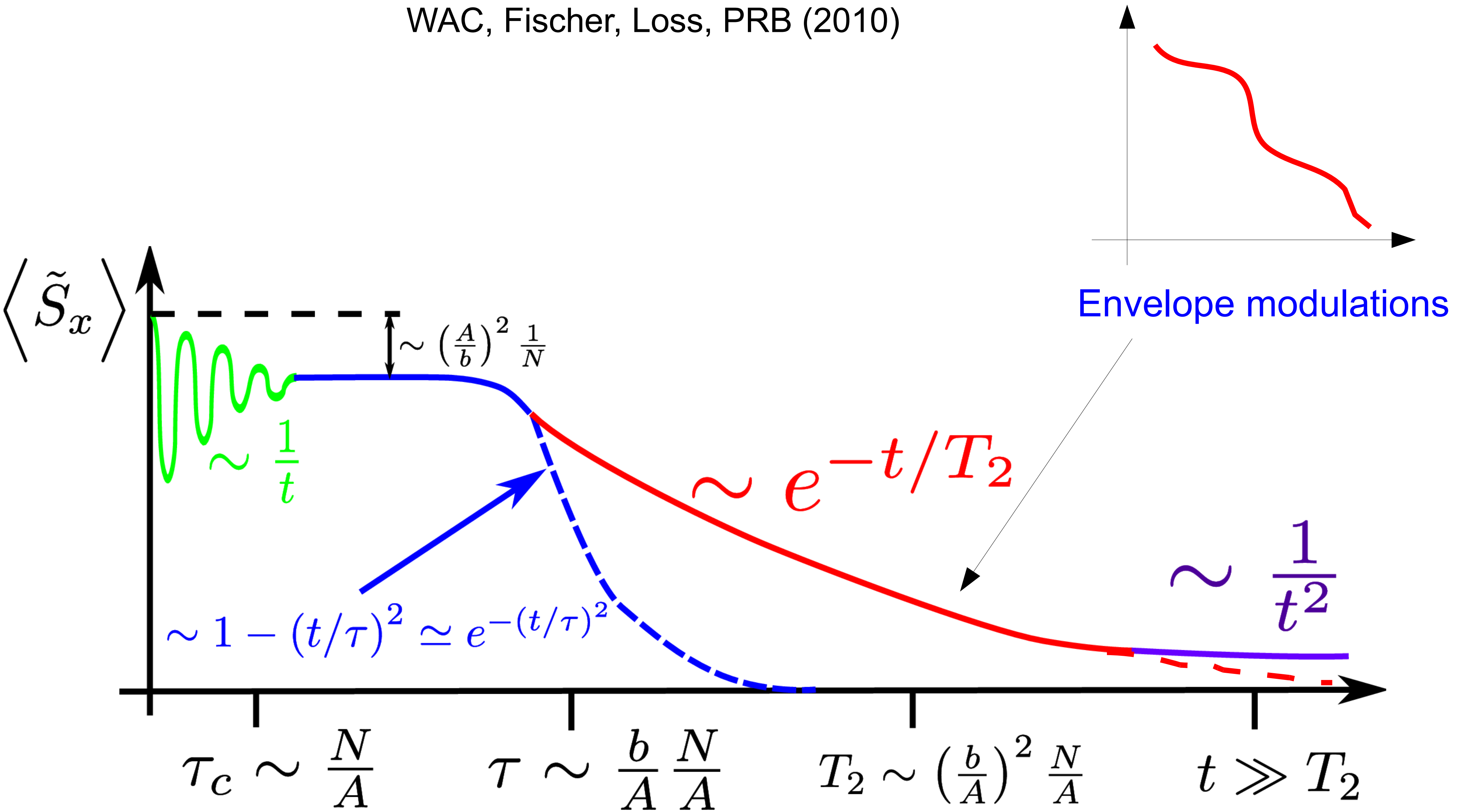

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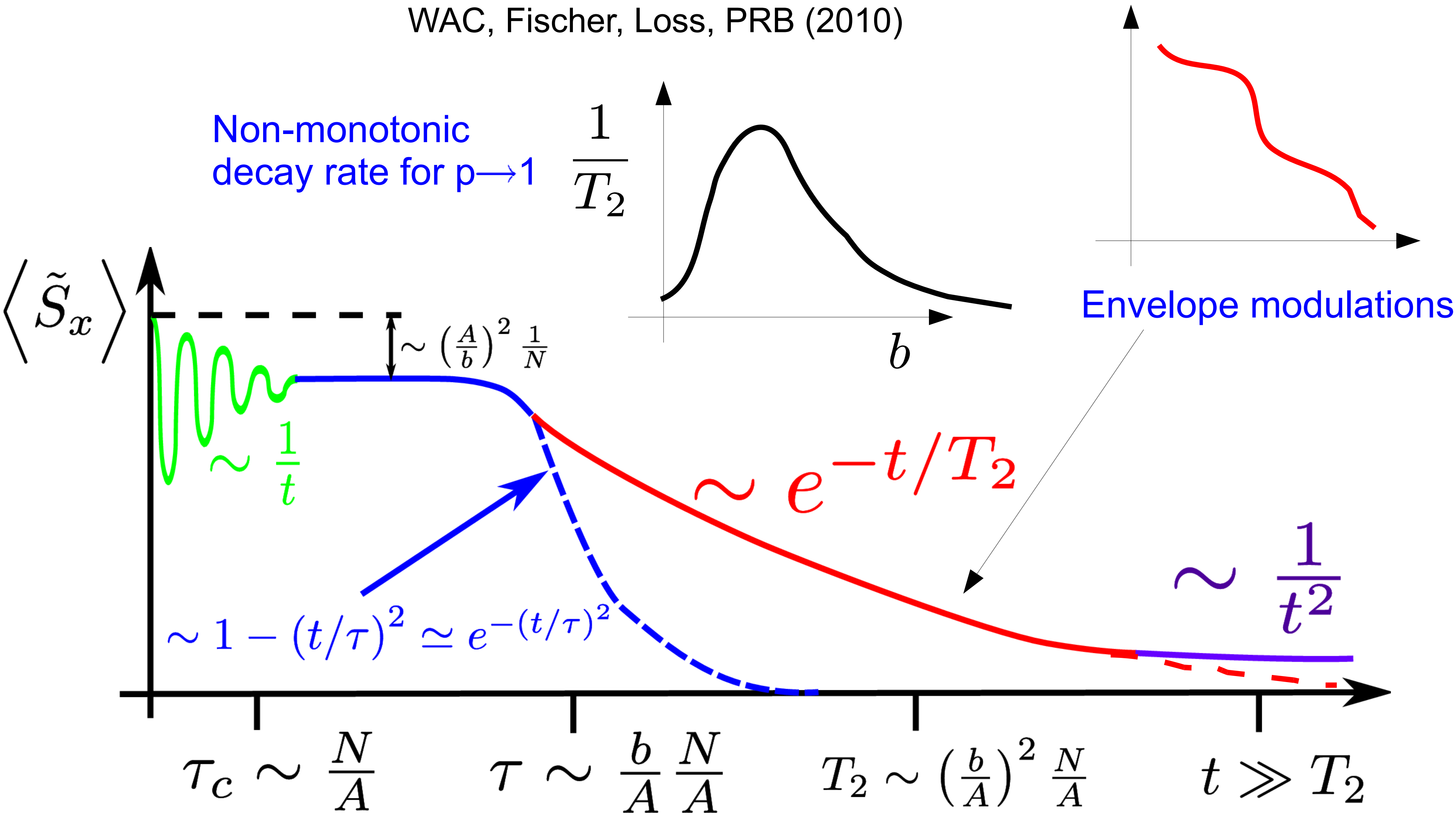


Solve the problem in two ways:

$$\langle \mathcal{O} \rangle_t = \langle \psi(0)| \, e^{iHt} \mathcal{O} e^{-iHt} \, |\psi(0)\rangle$$

$$H = H_0 + V_{\mathrm{ff}}$$

(1) Effective Hamiltonian

$$\tilde{H} = e^S H e^{-S} = H_0 + V_{\mathrm{eff}} + O\left(V_{\mathrm{ff}}^3\right)$$

neglected

$$\left|\tilde{\psi}(0)\right\rangle = e^S \, |\psi(0)\rangle = |\psi(0)\rangle + O\left(V_{\mathrm{ff}}\right)$$

Expand in powers of $V_{\mathrm{eff}} \sim O\left(V_{\mathrm{ff}}^2\right) \sim O\left(\frac{A}{b}\right)$

(2) Work directly with the 'real' Hamiltonian

Expand in powers of V_{ff}

Initial conditions

Fast initialization:

$$\rho(0) = \rho_S(0) \otimes \rho_I(0)$$

Sufficient condition: $\tau_{\text{init}} \lesssim 1/A \simeq 50\,\text{ps}$

Narrowed bath:

$$\rho_I(0) = \sum_i \rho_{ii} \left|n_i\right\rangle \left\langle n_i\right| \qquad \omega \left|n_i\right\rangle = \omega_n \left|n_i\right\rangle$$

Generalized Master Equation (GME)

Rotating frame: $$x_t = 2e^{-i(\omega_n + \Delta\omega)t}\,\langle S_+ \rangle_t$$

GME: $$\dot{x}_t = -i\Delta\omega x_t - i\int_0^t dt'\tilde{\Sigma}(t-t')x_{t'}$$

Lamb shift: $$\Delta\omega = -\mathrm{Re}\int_0^\infty dt\tilde{\Sigma}(t)$$

Markov: $$\frac{1}{T_2} = -\mathrm{Im}\int_0^\infty dt\tilde{\Sigma}(t) \qquad x_t \simeq x_0 e^{-t/T_2}$$

Direct expansion vs. effective H

$$\Sigma(s) = \int_0^\infty e^{-st}\Sigma(t)$$

Expanding in V_{ff}

$$\tilde{\Sigma} \simeq \tilde{\Sigma}^{(2)} + \tilde{\Sigma}^{(4)} + O\left(V_{\mathrm{ff}}^6\right)$$

$$\Delta\omega \simeq -\mathrm{Re}\tilde{\Sigma}^{(2)}(s=0^+) = O(V_{\mathrm{ff}}^2)$$

$$\frac{1}{T_2} \simeq -\mathrm{Im}\tilde{\Sigma}^{(4)}(s=0^+)$$

Expanding in $V_{\mathrm{eff}} \sim V_{\mathrm{ff}}^2$

$$\tilde{\Sigma}_{\mathrm{eff}} = \tilde{\Sigma}_{\mathrm{eff}}^{(2)} + O\left(V_{\mathrm{ff}}^8\right)$$

$$\Delta\omega_{\mathrm{eff}} \simeq -\mathrm{Re}\tilde{\Sigma}_{\mathrm{eff}}^{(2)}(s=0^+) = O(V_{\mathrm{ff}}^4)$$

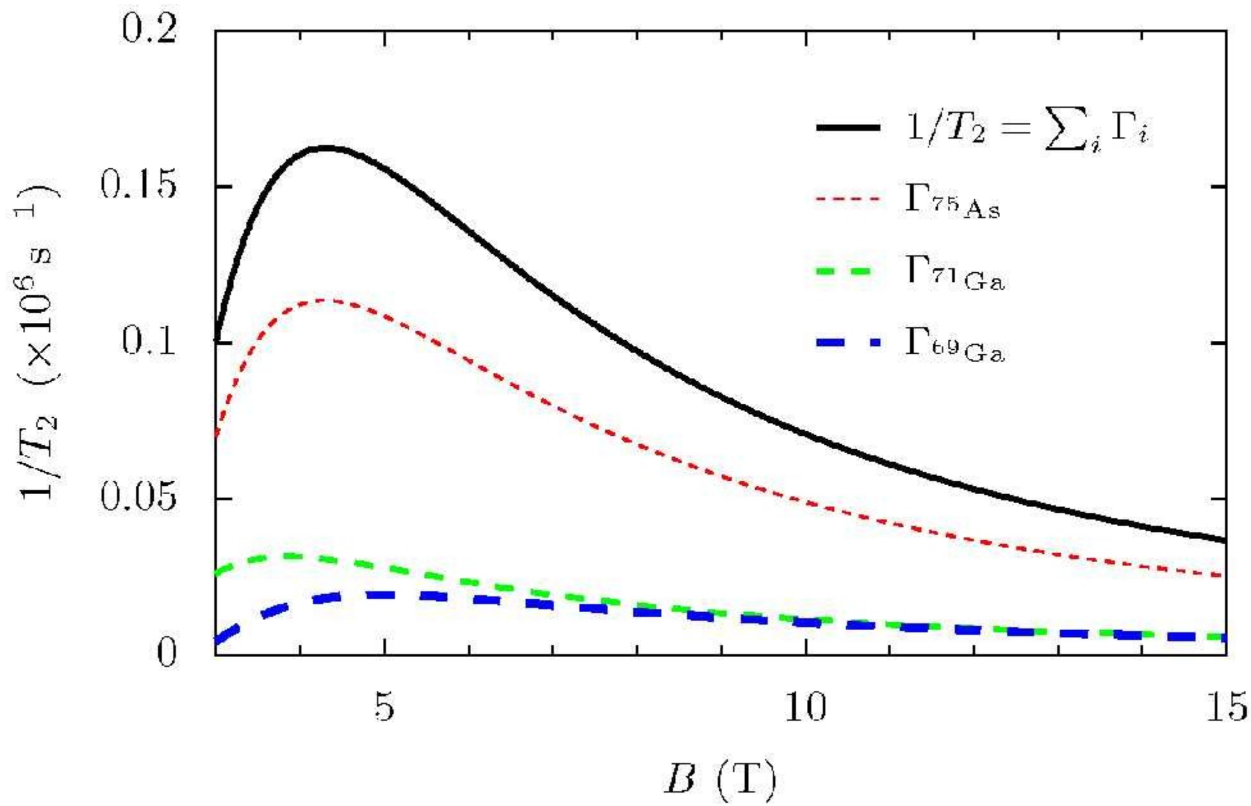
$$\frac{1}{T_2} \simeq -\mathrm{Im}\tilde{\Sigma}_{\mathrm{eff}}^{(2)}(s=0^+)$$

For one isotope: $\tilde{\Sigma}^{(4)} = \tilde{\Sigma}_{\mathrm{eff}}^{(2)}$ (with 1/N corrections)

Multiple isotopes: $\tilde{\Sigma}^{(4)} \neq \tilde{\Sigma}_{\mathrm{eff}}^{(2)}$

Non-monotonic decay rate

$$\frac{1}{T_2} \simeq -\mathrm{Im}\tilde{\Sigma}^{(4)}(s=0^+) \propto \frac{1}{b^2}\sum_{k,k'} A_k^2 A_{k'}^2 \delta(A_k - A_{k'} - \Delta\omega)$$



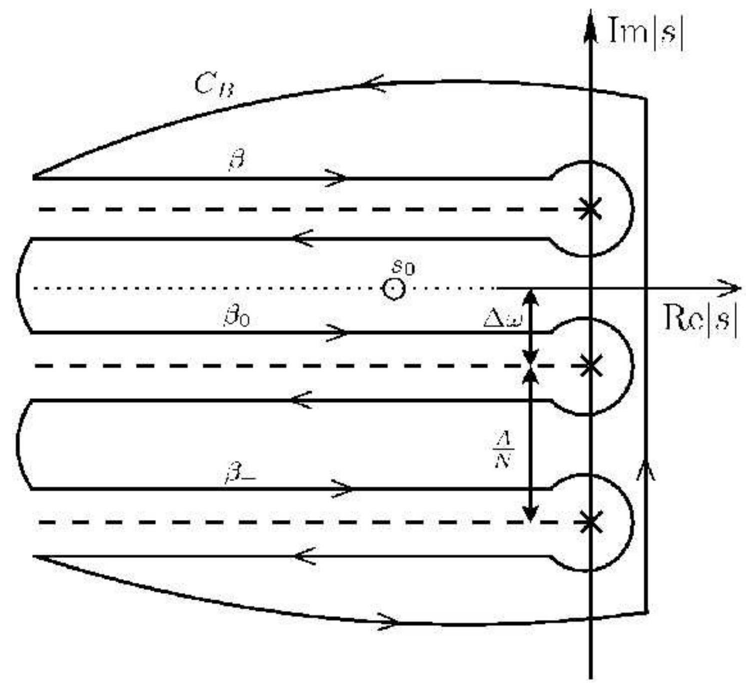
$$\Delta\omega \propto \frac{1}{b}$$

$$A_k \leq A/N$$

Qualitative behavior (maximum) is controlled by $\left(1-p^2\right)\dfrac{A}{b} < 1$

Full Non-Markovian time dependence

$$x(s) = \frac{x_0}{s - i\Delta\omega - i\tilde{\Sigma}(s)}$$



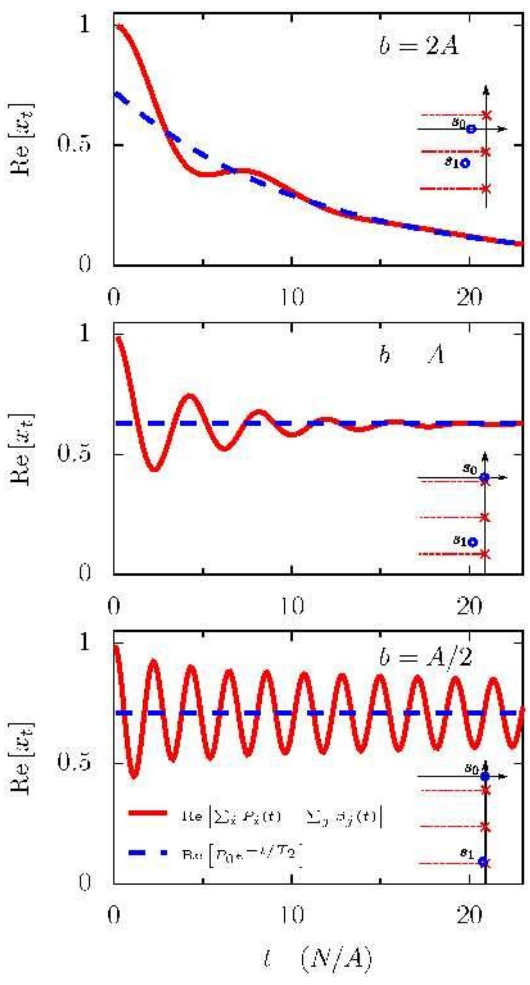
$$x_t = \lim_{\gamma \to 0} \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} e^{st} x(s)$$

$$= \sum_i \mathrm{Res}[e^{st} x(s), x = s_i] - \sum_\alpha \beta_\alpha(t)$$

Exponential decay or sustained oscillations

Power-law decay

Envelope Modulations



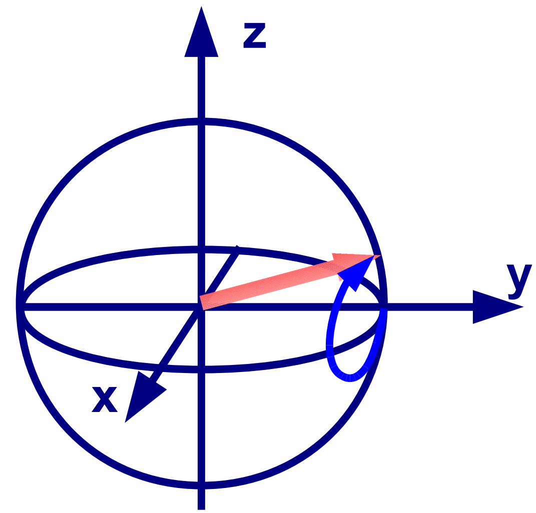
$$t \gg \frac{1}{\Delta\omega}$$

$$\mathrm{Re}\left[x_t\right] \sim \frac{C \cos\left(\Delta\omega t + \phi\right)}{t^2}$$

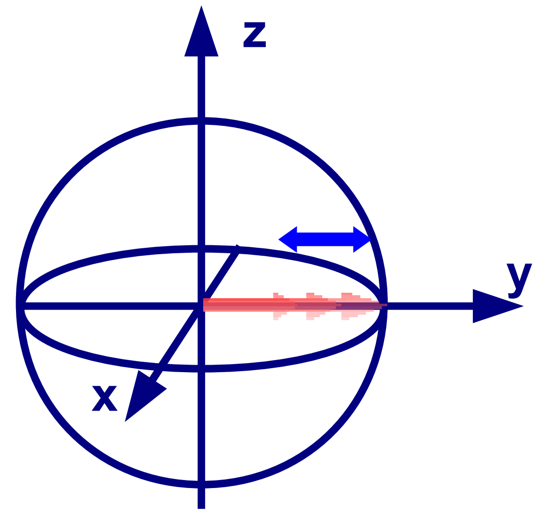
Higher-order corrections needed

Envelope modulations:
A collective quantum effect

Conventional envelope modulations (semiclassical); the spin remains on the surface of the Bloch sphere.



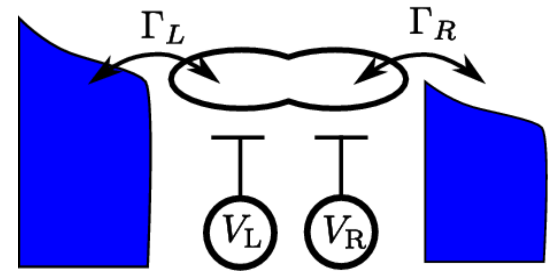
New quantum modulations due to collective excitations of the nuclear spin bath.

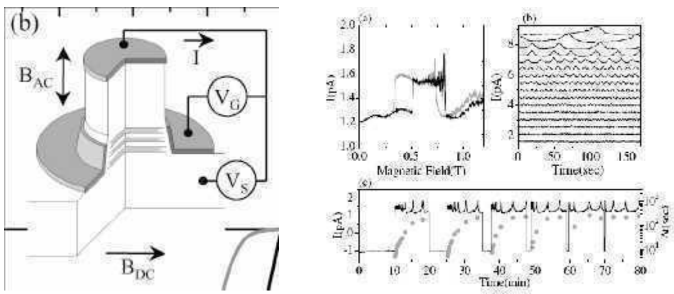


Initialization and readout

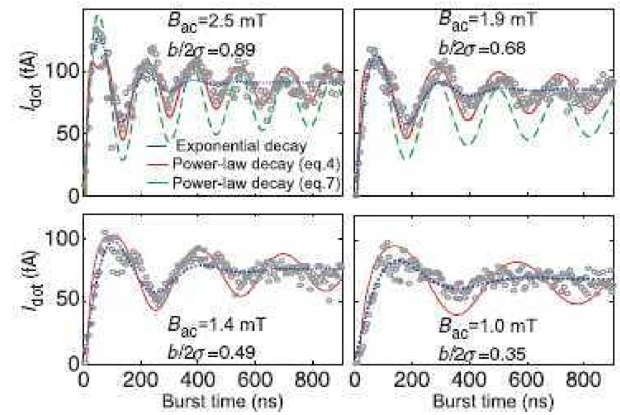
Spin → Charge

Spin dynamics in transport

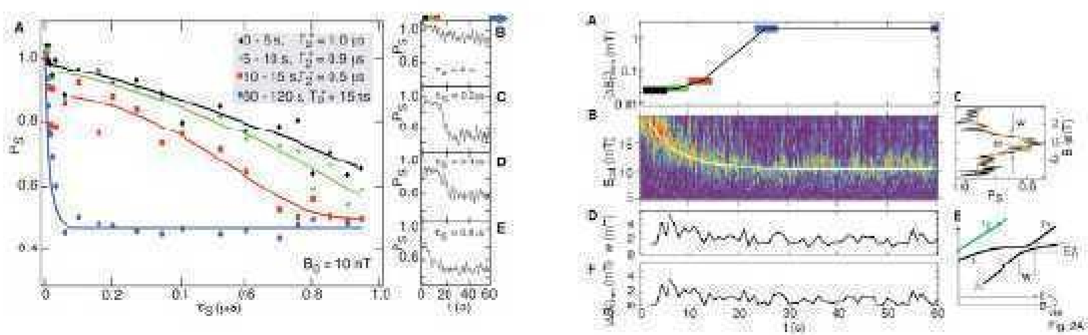




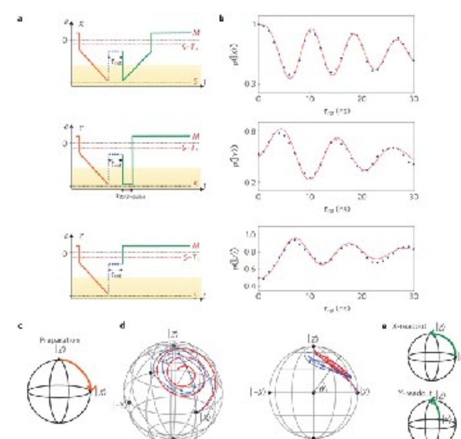
Ono and Tarucha, PRL (2004)



Koppens et al., PRL (2007)



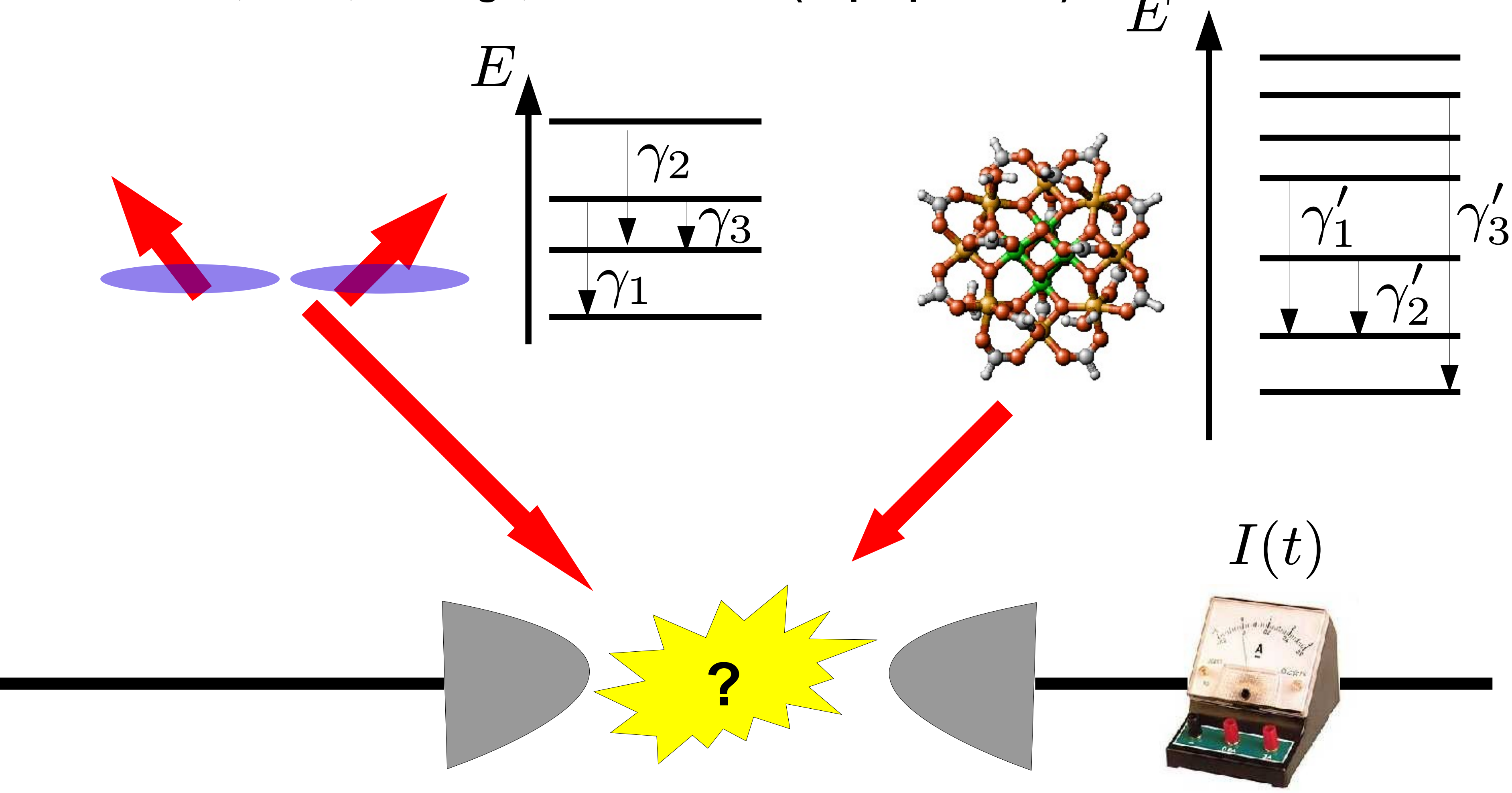
Reilly et al., Science (2008)



Foletti et al., Nature Phys. (2009)

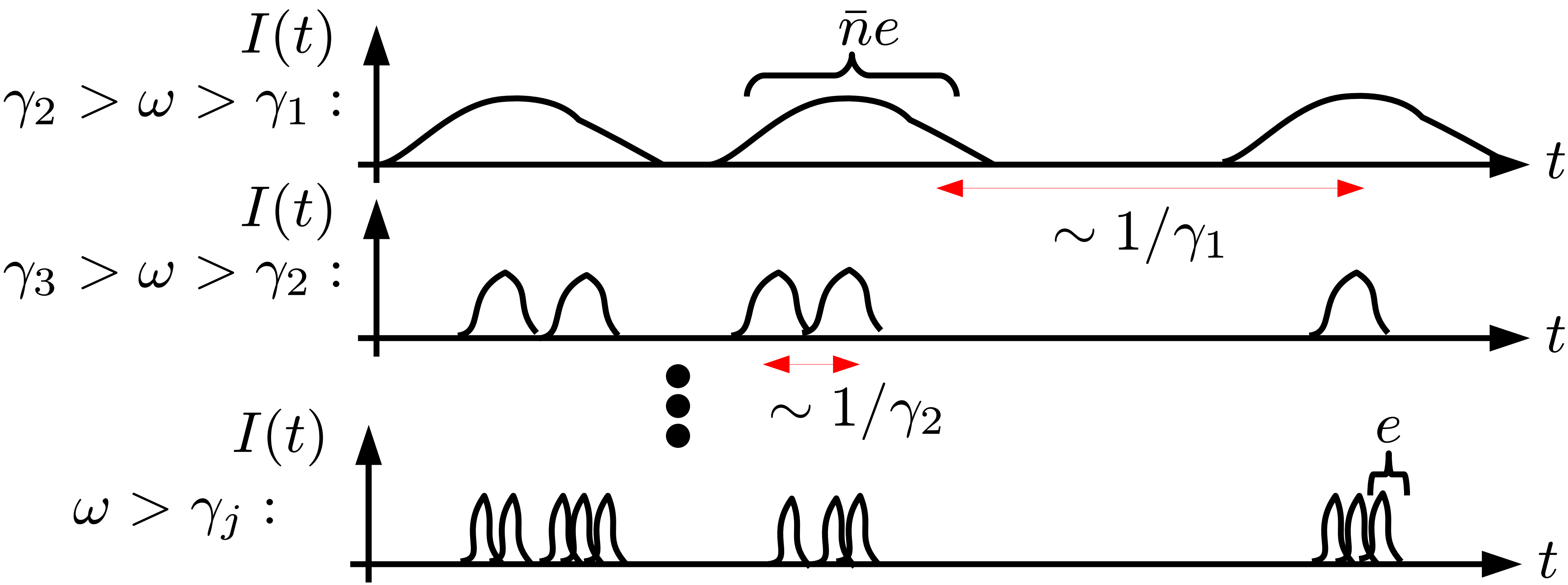
The problem: Identify multiple relaxation rates

F. Qassemi, WAC, J. Bergli, F. K. Wilhelm (in preparation)



Solution?: Frequency-dependent current noise

$$S_{II}(\omega) = \frac{1}{2}\int_{-\infty}^{\infty} dte^{i\omega t}\left\langle \{\delta I(t), \delta I(0)\}\right\rangle \quad \delta I(t) = I(t) - \langle I \rangle$$



Dynamical Channel Blockade

W. Belzig, PRB (2005)

F. Qassemi, WAC, J. Bergli, F. K. Wilhelm (in preparation)

$$\bar{n} = \frac{1}{P_B} \qquad F(0) = 2\bar{n} - 1 \qquad F(\omega) = \frac{S_{II}(\omega)}{e\,\langle I \rangle}$$

Probability to be blocked

Dynamical Channel Blockade

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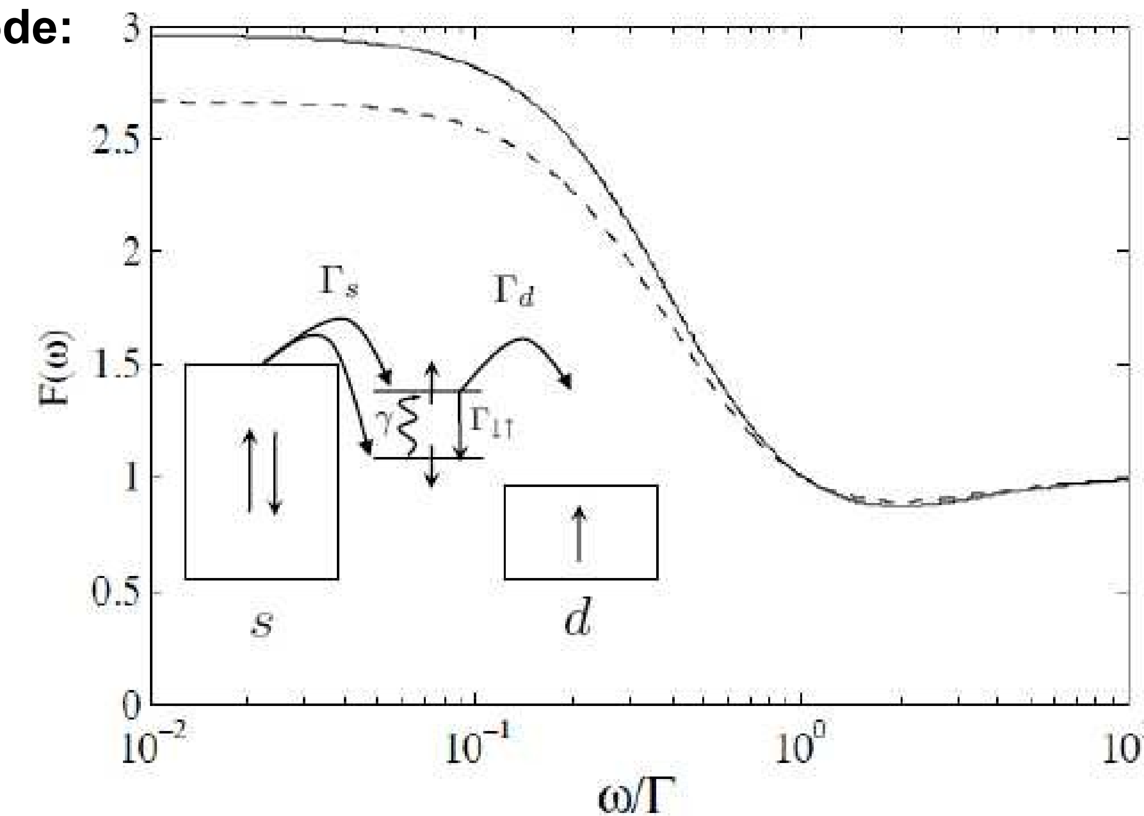
$$\bar{n} = \frac{1}{P_B} \qquad F(0) = 2\bar{n} - 1 \qquad F(\omega) = \frac{S_{II}(\omega)}{e\,\langle I \rangle}$$

Probability to be blocked

Spin diode:



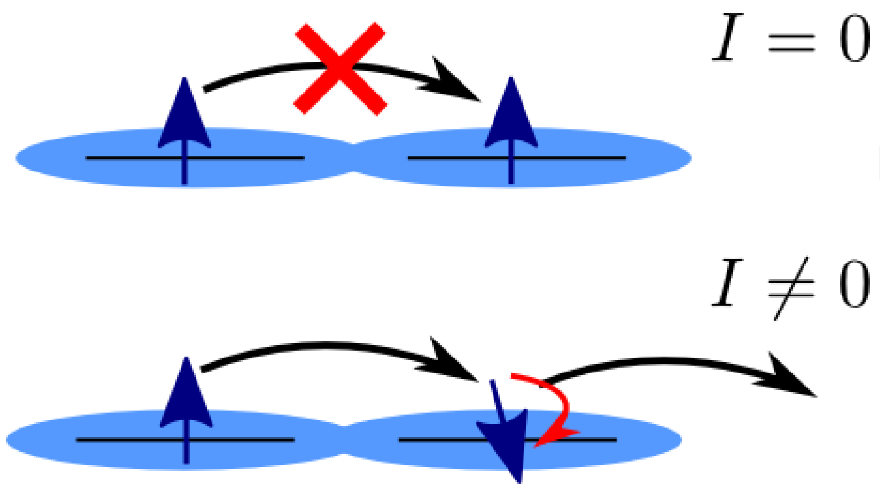
$$\dot{\rho} = M\rho$$

$$F(\omega) = 1 + \sum_j \frac{\lambda_j^2 \Delta F_j}{\omega^2 + \lambda_j^2}$$

λ_j : Eigenvalues of M

ΔF_j : From right/left eigenvectors

Pauli spin blockade



Blocked

$$|T_+\rangle = |\uparrow\uparrow\rangle$$

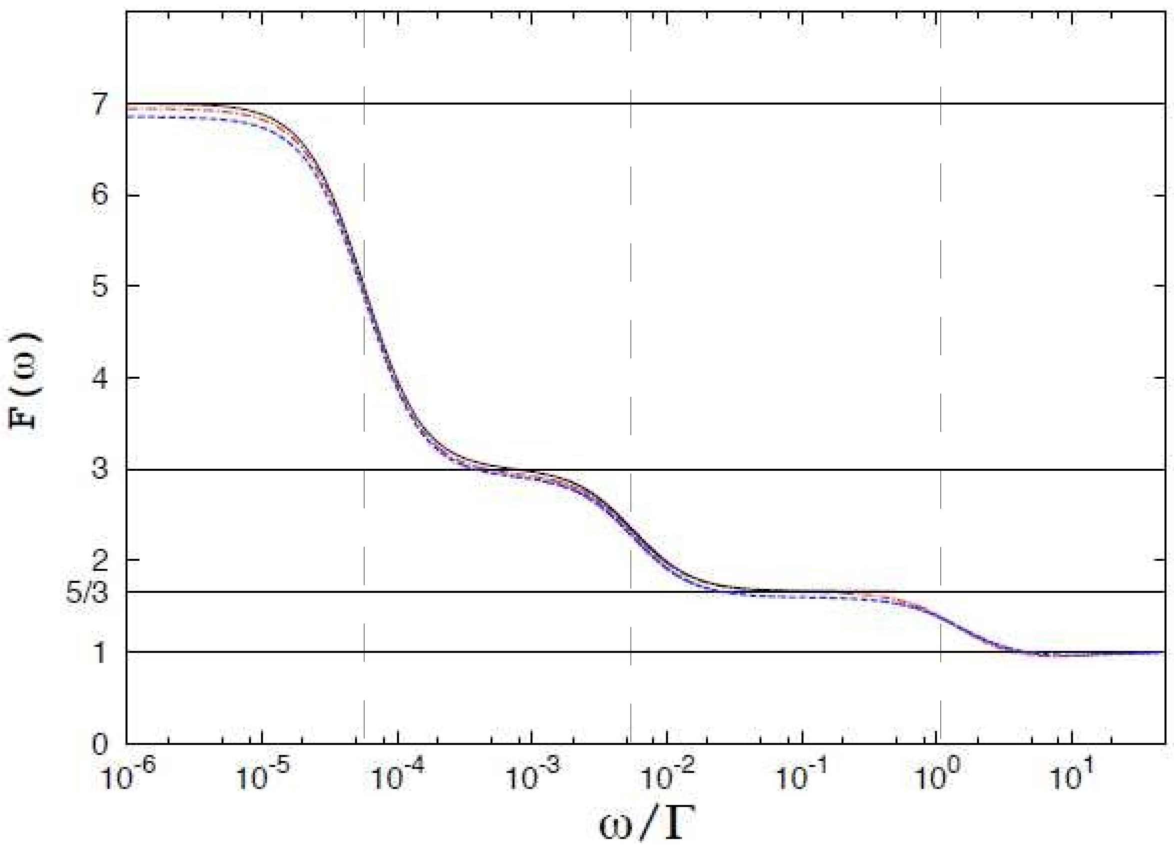
$$|T_0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$$

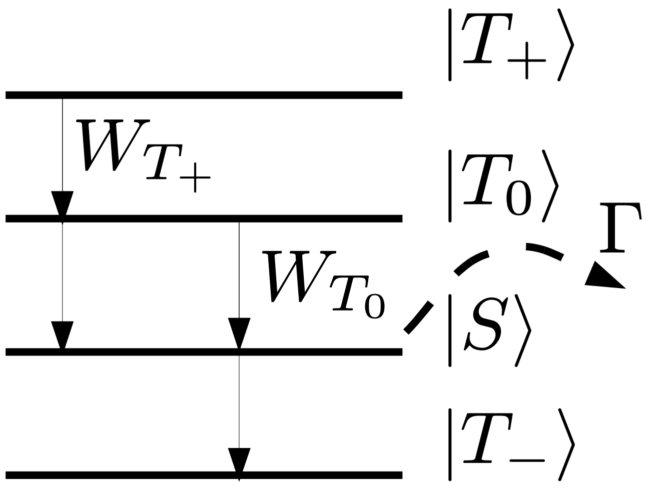
$$|T_-\rangle = |\downarrow\downarrow\rangle$$

$$|S\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

Fano factor

W_{T_+} W_{T_0} Γ





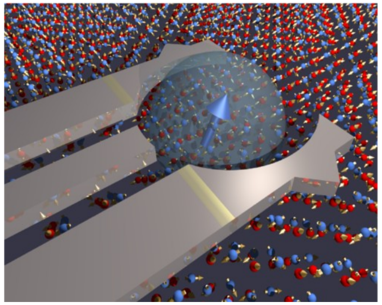
$$F = 2\bar{n} - 1$$

$$\bar{n} = \frac{1}{P_B}$$

$$F(\omega) \simeq \frac{5}{3} + \frac{4W_{T_+}^2}{4\omega^2 + W_{T_+}^2} + \frac{16W_{T_0}^2}{3(9\omega^2 + 4W_{T_0}^2)} \qquad \omega << \Gamma$$

Conclusions

• Electron spin dynamics depend on the nuclear environment (new quantum effects); Gate errors.



• Frequency-dependent noise as a probe of spin relaxation; Initialization and readout.