Entanglement purification with the exchange interaction

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Entanglement purification allows the creation of qubit pairs of arbitrarily high fidelity with respect to a maximally entangled state, starting from a larger number of low-fidelity pairs. Purification requires quantum memory, a role for which electron spins are well suited. However, using existing recurrence protocols involving symmetric local two-qubit operations for spin qubits turns out to be rather unpractical. We present an efficient purification protocol requiring only a single pulsed Heisenberg- or *XY*-type exchange interaction between two qubit pairs. In contrast to known protocols, we allow for asymmetric bilateral two-qubit operations where the two communication parties operate differently on their qubits. In the optimal version of our protocol, the local two-qubit interactions in the case of Heisenberg exchange correspond to the $\sqrt{\text{SWAP}}$ gate and its inverse.

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I. INTRODUCTION

A fundamental resource for the implementation of large-scale quantum communication (QC) networks [1] is the generation of long-distance entanglement between the network nodes. Due to imperfect sources and the inevitable interaction of the entangled particles with their environment, however, the degree of entanglement decreases, and the security or feasibility of many QC protocols like quantum teleportation [2], quantum dense coding [3] and entanglement-based quantum key distribution [4] cannot be guaranteed.

For the practical generation of long-distance entanglement, the concept of quantum repeaters was established [5,6]. The centerpiece of such a device is entanglement purification [7–9], for which a quantum memory [10] is indispensable. The working principle of the quantum repeater is to divide the distance between the network nodes into smaller segments, create entangled states between them, purify these states individually, and, finally, connect them via entanglement swapping [11].

Ideal candidates for the realization of stationary qubits acting as quantum memory are spins in solid-state systems, like electron spins in semiconductor quantum dots (QDs) [12] and nitrogen-vacancy centers in diamond [13]. Due to their long coherence times and their complete controllability by electrical or optical means [13–15] and the possibility of interfacing them with photons [16], considerable potential as quantum memory has been demonstrated. Long-distance entanglement can also be generated using spin chains [17,18].

Pioneering experiments of entanglement purification have been performed using photonic qubits [19–22]. Limitations of these schemes are the destructive measurements of the purified pairs and the requirement for pure input states [23,24], besides the impracticality of using photons as quantum memory. The idea of recurrence protocols for entanglement purification is to use two or more imperfectly entangled qubit pairs to purify one of them with respect to a maximally entangled state. The original CNOT-based protocol of Bennett *et al.* [7] has been implemented with atomic qubits [25], but only locally in the same optical trap.

In this paper, we present a simple purification protocol based solely on the one-time activation of a Heisenberg exchange interaction leading to the $\sqrt{\text{SWAP}}$ gate. The exchange interaction is readily available in many spin-based qubit systems, such as QDs [12]. Following the same approach, we also find a similar result for qubits coupled via an *XY*-type interaction, which happens to be the interaction between superconducting qubits [26] as well as between optically coupled spin qubits [27].

Earlier works have demonstrated purification schemes for spin qubits, e.g., by replacing the CNOT in the bilateral operation by the gate sequence, which uses two-qubit gates directly generated from the interaction Hamiltonian [28], requiring additional single-qubit operations. Other procedures use three input pairs [29–31] or specifically work for two-spin singlet-triplet qubits [32]. Our proposal works with two input pairs of spin-1/2 qubits and only requires a single two-qubit interaction. In comparison with existing protocols, we achieve an advantage by allowing for *different* two-qubit manipulations locally in the bilateral operation. For a single purification step, the derived protocol requires no extra single-spin operations, making it much faster and less susceptible to gate errors. The only needed operation, the $\sqrt{\text{SWAP}}$ gate, has been demonstrated experimentally, with a gate time below 0.2 ns [33] (see also [34]), making the implementation of our proposal within reach of current technology. Furthermore, the required single-shot measurement of an electron spin state has also been successfully performed [35,36].

II. ENTANGLEMENT PURIFICATION

A basis of the two-qubit Hilbert space is given by the maximally entangled Bell states $|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$ and $|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$, where $\{|0\rangle,|1\rangle\}$ is the single-qubit logical basis of the sharing parties, conventionally named Alice and Bob, and we call the overlap of an arbitrary state ρ with the desired entangled state $|\Phi^+\rangle$ its *fidelity* $F \equiv \langle\Phi^+|\rho|\Phi^+\rangle$. Recurrence protocols work on two or more qubit pairs of low fidelity as input to create a single qubit pair with higher fidelity as output using only local unitary operations, measurements,

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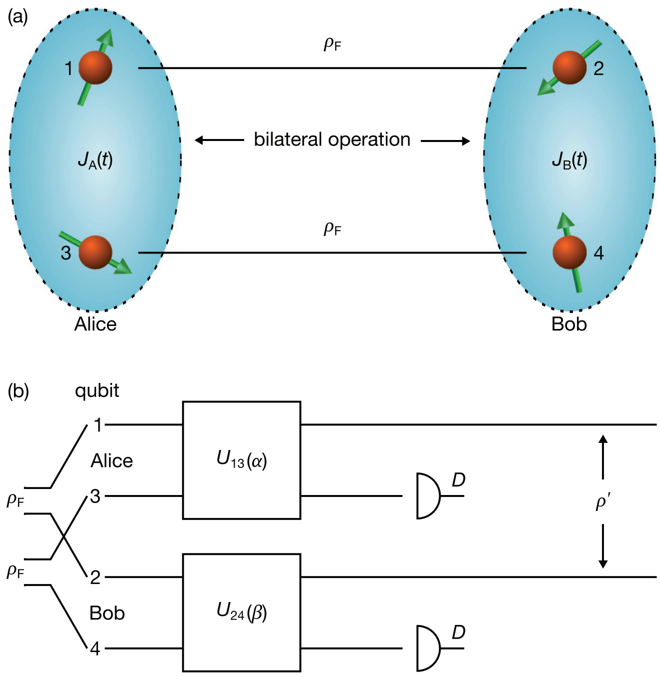

FIG. 1. (Color online) (a) Alice and Bob share two imperfectly entangled qubit pairs ρ_F and locally apply different unitary operations generated by the exchange couplings J_A and J_B. (b) Circuit diagram of the protocol explained in the text, where unitary operations with different pulse areas α and β are applied. After the detection (D) of qubits 3 and 4, Alice and Bob are left with the two-qubit state ρ'.

and two-way communication of the measurement results via a classical channel. Having initially many copies of the low-fidelity pairs and running the purification protocol iteratively on the output pairs of a higher fidelity, one can achieve fidelities arbitrarily close to $F = 1$ and thus obtain a maximally entangled state.

The protocol of Bennett *et al.* [7] (BBPSSW) requires two copies of the state ρ_F (Fig. 1),

$$\rho_F = F|\Phi^+\rangle\langle\Phi^+| + \frac{1-F}{3}(|\Phi^-\rangle\langle\Phi^-| + |\Psi^+\rangle\langle\Psi^+| + |\Psi^-\rangle\langle\Psi^-|), \quad (1)$$

which can be generated from an arbitrary two-qubit state having overlap F with the rotationally invariant singlet state $|\Psi^-\rangle$ by a *twirl* operation [7,37] that retains the singlet component, equalizes the triplet components, and removes all off-diagonal elements, thus creating a so-called Werner state [38], followed by a π rotation about the y axis on the Bloch sphere by one of the sharing parties, hence interchanging the $|\Psi^-\rangle$ and $|\Phi^+\rangle$ components. Next, Alice and Bob each perform a CNOT gate between the two qubits they hold, respectively, where the qubits of the first (second) pair serve as the source (target) bit and the target bit is flipped if the source qubit is in state $|1\rangle$, i.e., $|00\rangle \mapsto |00\rangle$, $|01\rangle \mapsto |01\rangle$, $|10\rangle \mapsto |11\rangle$, and $|11\rangle \mapsto |10\rangle$. After this bilateral CNOT operation, both target qubits (3 and 4) are measured in the logical basis. If the outcomes of Alice's and Bob's measurement are the same, they keep the source pair; otherwise, it is discarded. The fidelity F' of the remaining pair turns out to be larger than the initial fidelity F provided that $1/2 < F < 1$. Another π rotation about the y axis again exchanges $|\Psi^-\rangle$ and $|\Phi^+\rangle$ components. Therefore, iterating the scheme can bring the fidelity arbitrarily close to 1, resulting in a maximally entangled Bell state. Since the state after one purification round is not a Werner state, the last step is necessary as a prerequisite for the twirl in the subsequent round.

The difference in the Deutsch *et al.* [8] protocol (DEJMPS) is that it works generally on Bell-diagonal states and therefore does not need the twirl to come back to Werner form. A purification round begins with Alice performing the single-qubit gate

$$|0\rangle \mapsto \frac{1}{\sqrt{2}}\big(|0\rangle - i|1\rangle\big), \quad |1\rangle \mapsto \frac{1}{\sqrt{2}}\big(|1\rangle - i|0\rangle\big) \quad (2)$$

on both of her qubits, and Bob the inverse operation. As in the BBPSSW protocol, a bilateral CNOT and the measurement of the target pair follow, keeping the source qubits only if the outcomes are equal. If the initial $|\Phi^+\rangle$ component is larger than $1/2$, the states can be purified to a pure $|\Phi^+\rangle$ state, but in a more efficient way than in [7].

III. SPIN QUBITS

The CNOT gate is not directly generated by the typical interaction between spin qubits in gate-defined QDs, where the exchange interaction can be described by a Heisenberg Hamiltonian [12,39]. Two interaction pulses and additional single-qubit operations on both qubits are necessary to construct the CNOT gate [12]. However, since single-spin rotations are much slower than spin-spin exchange interactions, such an implementation is challenging. We therefore construct a purification protocol requiring only a single two-qubit operation each for Alice and Bob, which is directly generated from the Heisenberg Hamiltonian describing the exchange interaction between the spins of two electrons confined to adjacent gate-defined QDs [12,39],

$$H_{ij}(t) = \tfrac{1}{4}J(t)\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j = J(t)\left(\tfrac{1}{4}\mathbb{1} - P_{\Psi^-}\right), \quad (3)$$

where $\boldsymbol{\sigma}_i$ is the Pauli spin operator of the electron in QD i, $P_{\Psi^-} = |\Psi^-\rangle\langle\Psi^-|$, and $J(t)$ is the exchange energy between the two electrons that can be tuned by changing the appropriate gate voltages and, hence, can depend on the time t. The two possible spin orientations of an electron define the logical states $|0\rangle \equiv |\uparrow\rangle$ and $|1\rangle \equiv |\downarrow\rangle$.

The time evolution U generated by (3) is given by $U_{ij}(\alpha) = \exp[-i\int_0^\tau dt\, H_{ij}(t)] = e^{-i\alpha/4}[\mathbb{1} + (e^{i\alpha} - 1)P_{\Psi^-}]$, where we set $\hbar = 1$, assume a pulsed exchange coupling of duration τ, and refer to $\alpha = \int_0^\tau dt\, J(t)$ as the pulse area. Time ordering is not necessary since $[H_{ij}(t), H_{ij}(t')] = 0$ for all t and t'. For each value of α, a specific two-qubit gate is generated, especially the entangling $\sqrt{\text{SWAP}}$ gate for $\alpha = \pi/2$ [12,39], $U_{\sqrt{\text{SWAP}}} = e^{i\pi/8}U_{ij}(\pi/2)$, which, together with arbitrary single-qubit operations, forms a universal set of quantum gates [12,40], or the SWAP operation for $\alpha = \pi$, $U_{\text{SWAP}} = (U_{\sqrt{\text{SWAP}}})^2 = e^{i\pi/4}U_{ij}(\pi)$, whose action is to interchange the states of the qubits [41]. The CNOT can be obtained by the sequence [12] $U_{\text{CNOT}} = e^{-i\pi/2}e^{-i\pi\sigma_j^y/4}e^{i\pi\sigma_i^z/4}e^{-i\pi\sigma_j^z/4}U_{\sqrt{\text{SWAP}}}e^{i\pi\sigma_i^z/2}U_{\sqrt{\text{SWAP}}}e^{i\pi\sigma_j^y/4}$, requiring two $\sqrt{\text{SWAP}}$ and several single-qubit gates.

IV. PURIFICATION WITH THE EXCHANGE INTERACTION

A. General case

We now introduce a purification protocol, which makes use of only a single activation of the Heisenberg interaction, (3). The protocol has the same structure as existing recurrence protocols [7], the crucial difference being that the bilateral operation is *asymmetric*, meaning that one has to apply *different* two-qubit gates. The two parties Alice and Bob each start with a copy of the state ρ_F, with $\rho = \rho_F \otimes \rho_F$, which can be generated as shown above. Then Alice and Bob each activate the exchange interaction between their two qubits with specific pulse areas α and β, respectively (Fig. 1); i.e., they apply the unitary transformation

$$U(\alpha,\beta) = U_{13}(\alpha) \otimes U_{24}(\beta), \tag{4}$$

where Alice holds qubits 1 and 3, and Bob qubits 2 and 4. This transforms the initial state ρ into $U(\alpha,\beta)\rho U(\alpha,\beta)^\dagger$. After this unitary transformation, the two parties continue as in the BBPSSW protocol. The target qubits 3 and 4 are measured in the z basis and the results are communicated via a classical channel. If the spins point in the same direction, Alice and Bob keep qubits 1 and 2 (source pair); otherwise the state is discarded.

We find the fidelity $F' = \langle\Phi^+|\rho'|\Phi^+\rangle$ of the postselected source state ρ' after the described procedure to be

$$F'(F,\alpha,\beta) = \frac{(4F-1)(4F+5)\cos\alpha\cos\beta - (4F-1)(8F+1)\sin\alpha\sin\beta + 8F(4F+1)+5}{6(4F-1)\cos\alpha\cos\beta - 2(4F-1)^2\sin\alpha\sin\beta + 6(4F+5)}, \tag{5}$$

which is the main result of this paper.

To show the feasibility of the protocol, the three fixed points of the map, (5), can be found analytically. Except for the case $\alpha = n\pi$, $\beta = m\pi$, and $\alpha - \beta = 2\pi k$ (n, m, and k integers), where $F' \equiv F$, a constant fixed point is given by $F_c = 1/4$. The values of the two remaining (possibly complex) fixed points $F_{\min}$ and $F_{\max}$ depend on α and β, as illustrated in Fig. 2 within the physically relevant regime $1/2 \leqslant F_{\min} \leqslant F_{\max} \leqslant 1$. While $F_{\max}$ and F_c are attractive, $F_{\min}$ is repulsive. Thus, if the qubit pairs have an initial fidelity $F > F_{\min} \geqslant 1/2$, iterative application of the described scheme can purify them up to a fidelity $F_{\max}$.

However, the output state ρ' is not of the form given in Eq. (1). Hence, for the iteration to work, a twirl operation is needed, requiring a random single-qubit rotation [7,37]. Nevertheless, this operation takes substantially less time than the single-qubit gates that would be required for the realization of the standard CNOT sequence.

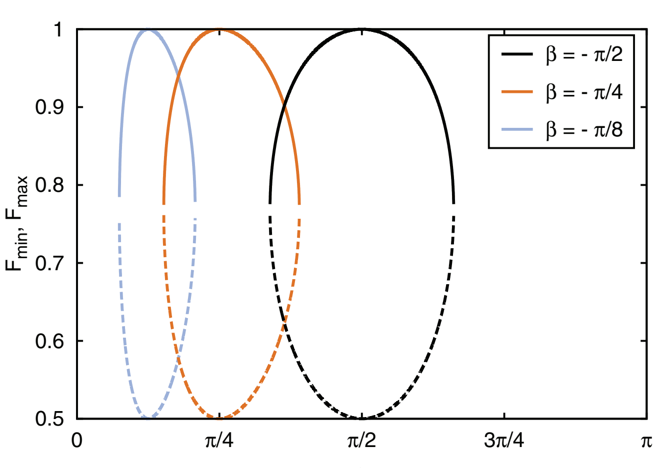

FIG. 2. (Color online) Minimum required initial fidelity $F_{\min}$ (dashed line) for the entanglement purification to work and maximally attainable fidelity $F_{\max}$ (solid line) when iteratively applying the purification protocol generating map (5) as a function of the pulse area α for different pulse areas β.

B. Optimal purification

The minimal and maximal values of the fixed points $F_{\min} = 1/2$ and $F_{\max} = 1$ are obtained in the case $\beta = -\alpha$. This implies that, in principle, maximally entangled states can be created using our protocol if Alice and Bob perform mutually inverse operations. This could be achieved either using ferro- and antiferromagnetic exchange $J_A > 0$, $J_B < 0$, or vice versa, or, in the case of equal coupling types, using the fact that the propagator U_{ij} is 2π periodic (omitting overall phases), with, e.g., $0 < \alpha < \pi$ and $\beta = 2\pi - \alpha > 0$. The fidelity is then given by

$$F'(F,\alpha,-\alpha) = \frac{1}{2} + \frac{3-12F^2}{(F-1)(4F-1)\cos(2\alpha) - F(4F+7) - 7} \tag{6}$$

and has its maximum in the optimal case $\alpha = \pi/2$,

$$F'\left(F,\frac{\pi}{2}+2\pi n,-\frac{\pi}{2}+2\pi m\right) = \frac{16F^2+F+1}{8F^2+2F+8}. \tag{7}$$

The result, Eq. (6), is plotted in Fig. 3(a) for different values of α. The optimal value is therefore achieved if Alice applies a $\sqrt{\text{SWAP}}$ gate to her qubits and Bob performs the inverse $\sqrt{\text{SWAP}}$ gate, $\sqrt{\text{SWAP}}^{-1}$. The square of $\sqrt{\text{SWAP}}^{-1}$ is also the SWAP operation and it can be understood as another root of SWAP.

The described protocol can be slightly improved in terms of resources needed to achieve a specific fidelity. In analogy with the DEJMPS protocol, Alice can begin each purification step with the local operations, (2) (and Bob with the inverse), and the bilateral CNOT is replaced by the asymmetric operation described above, to increase the gain in fidelity [Fig. 3(b)] and therefore reduce the number of purification steps [Fig. 3(c)].

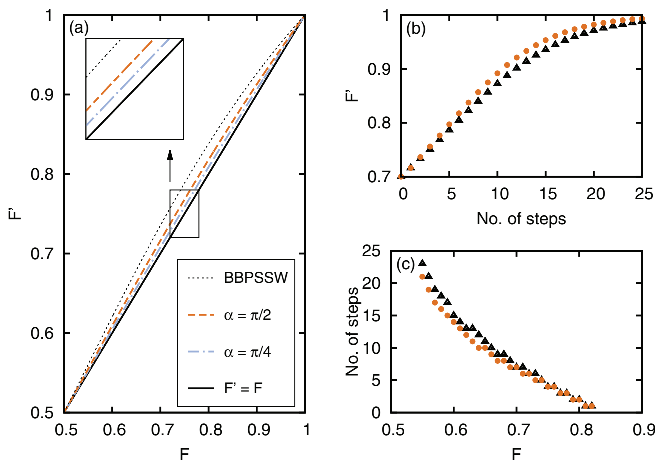

FIG. 3. (Color online) (a) Fidelity F' as a function of the initial fidelity F, shown for different pulse areas α and compared to the BBPSSW protocol. (b) Stepwise fidelity increase for an initial fidelity $F = 0.7$ and (c) number of steps needed to achieve a fidelity $F' > 0.82$ (above the threshold fidelity of the hashing method introduced in [37]) starting from qubit pairs with a fidelity F; shown for the described protocol (triangles) in the optimal case, (7), and in the more efficient manner in analogy with the DEJMPS protocol (circles).

Furthermore, one can see from Fig. 2 that the robustness of our scheme is greatest in the case $\alpha = -\beta = \pi/2$, since deviations from perfectly applied operations have the least effect on the values of $F_{\rm min}$ and $F_{\rm max}$. If Alice applies a rectangular pulse with an exchange interaction of $J = 1\ \mu$eV [33], the deviation $\Delta\tau$ from the optimal case must not exceed 100 ps to still achieve a maximum fidelity of 0.99, assuming that Bob generates a perfect pulse. Such accuracies can be obtained experimentally [33].

V. PURIFICATION WITH THE *XY* INTERACTION

We briefly discuss our approach for entanglement purification and application of an asymmetric bilateral operation for the case of anisotropic *XY*-type qubit interactions, $H_{XY}(t) = \frac{1}{4}J(t)(\sigma_i^x\sigma_j^x + \sigma_i^y\sigma_j^y)$. This kind of interaction appears, e.g., in all-optical cavity-coupled QD electron spins [27] or superconducting qubits [26]. The Hamiltonian $H_{XY}(t)$ generates the iSWAP gate, $|\uparrow\uparrow\rangle \mapsto |\uparrow\uparrow\rangle$, $|\uparrow\downarrow\rangle \mapsto i|\downarrow\uparrow\rangle$, $|\downarrow\uparrow\rangle \mapsto i|\uparrow\downarrow\rangle$, $|\downarrow\downarrow\rangle \mapsto |\downarrow\downarrow\rangle$, for a pulse area $\int_0^\tau {\rm d}t\, J(t) = -\pi$. Following the above scheme, applying interactions with different pulse areas α and β on Alice's and Bob's qubits, the fidelity F' after the protocol is

$$F'(F,\alpha,\beta) = \frac{(12F-3)\cos\alpha\cos\beta - (4F-1)^2\sin\alpha\sin\beta + 4(8F^2+2F-1)\cos\frac{\alpha+\beta}{2} + 4F(4F+1) + 7}{6(4F-1)\cos\alpha\cos\beta - 2(4F-1)^2\sin\alpha\sin\beta + 6(4F+5)}. \quad (8)$$

In the case $\alpha = -\beta$, the result coincides with (6) and therefore is maximal for $\alpha = \pi/2$. The different qubit interactions in the case of the *XY* Hamiltonian correspond in the optimal case to gates whose double-application results in the iSWAP gate.

VI. CONCLUSIONS

We have presented an entanglement purification scheme in which the bilateral two-qubit operation is directly generated from the one-time activation of a Heisenberg-type spin-spin interaction. In general we have shown that an asymmetric unitary evolution of Alice's and Bob's qubits, respectively, can lead to an increased fidelity of one of the shared qubit pairs with respect to $|\Phi^+\rangle$ if the initial fidelity was larger than a given minimal value. In the special case where Alice and Bob apply inverse operations, the maximally obtainable fidelity by iterative application of our protocol is $F = 1$; i.e., in principle maximally entangled states can be generated. We found that the optimal case is when the two communicating parties apply the $\sqrt{\rm SWAP}$ and the $\sqrt{\rm SWAP}^{-1}$ gates locally on their qubits.

Since the coupling of electron spins in gate-controlled QDs is well described by an exchange interaction of the Heisenberg type, the protocol is particularly suitable for spin qubits. In terms of operation times, the presented protocol is much faster than protocols based on CNOT applied to spin qubits. The reason is that the single-qubit gates needed in constructing the CNOT require operation times of the order of 100 ns [42], whereas the $\sqrt{\rm SWAP}$ can be generated two orders of magnitude more rapidly, in about 0.2 ns [33]. Therefore, besides the smaller error proneness due to the smaller number of gate operations, much faster iteration of the protocol is possible. At this stage, we point out that fast single-qubit gates on the picosecond scale have been demonstrated for charge qubits in gate-controlled QDs [43] and spin qubits in self-assembled QDs [15,44]. However, two-qubit gates have not been demonstrated yet for such qubits, and moreover, charge qubits have coherence times in the range of nanoseconds and are therefore not suitable as quantum memory.

In addition, we have shown that our purification method of applying an asymmetric bilateral operation works as well for qubits coupled via an *XY*-type interaction and is therefore suitable for cavity-coupled spin qubits and superconducting qubits.

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