UNIVERSITY OF KONSTANZ
Department of Physics
Dr. Andrey Moskalenko

Group theory and symmetries in quantum mechanics
Summer semester 2017 - Exercise sheet 7
Distributed: 16.06.2017, Discussion: 21.06.2017 and 22.06.2017

Problem 19: Selection rules for a system having the C_{6v} symmetry.

Consider a quantum-mechanical system which has the symmetry of a regular hexagon (cf. problems 8 and 13). Suppose there is a "vector" perturbation of this system, i.e. a perturbation that transforms as $\{x, y, z\}$. Determine the selection rules for transitions induced by this perturbation from the initial state that transforms according to the "parity" representation (i.e. the 1-dimensional representation with a sign change for the inversion, reflections and any improper rotations in general, contained in the symmetry group).

Problem 20: Clebsch-Gordan coefficients for a direct product of representations.

The coordinates x, y of a particle transform according to the 2-dimensional irreducible representation E of the group D_4 (cf. problem 11), so that the products of coordinates x_1x_2, x_1y_2, y_1x_2 and y_1y_2 of two such particles transform according to the product representation $E \otimes E$. Find the four combinations of these product functions which transform irreducibly under D_4 (*Hint: look at the character table of D_4 and the result of problem 11d*). Derive the Clebsch-Gordan coefficients from them.

Problem 21: Projection operators, D_3 and quadratic polynomials of x and y.

Consider the group D_3 and the function x^2. The operations of D_3 generate an invariant subspace L from this function. A representation O of D_3 in this subspace was considered in problem 7, with functions x^2, y^2 and xy as a basis.

a) Using the character table of D_3, construct the projection operators $P^{(1)}$, $P^{(2)}$ and $P^{(3)}$ (corresponding to the irreducible representations $T^{(1)}$, $T^{(2)}$ and $T^{(3)}$ of D_3) and apply them to the function x^2. What is the dimension of the space spanned by $P^{(1)}x^2$, $P^{(2)}x^2$ and $P^{(3)}x^2$?

b) Using the properties of the representation O and the matrices of the irreducible representations of D_3 (given in the lecture), project the function x^2 onto the space $L_{3,i}$ of functions transforming according to the i-th row of the representation $T^{(3)}$, i.e. calculate $P_i^{(3)}x^2$ $(i = 1, 2)$.

c) Solving (b), you can see that with help of $P_i^{(3)}$ you can not generate from x^2 a complete basis of the space L_3 belonging to the representation $T^{(3)}$. Make use of the transfer operators $P_{ij}^{(3)}$ in order to obtain a complete basis in L_3 from x^2. Are the corresponding functions orthogonal or not? Explain the reason.