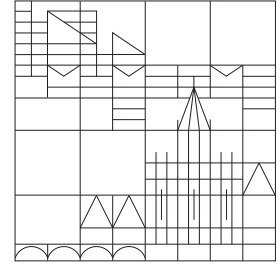


UNIVERSITY OF KONSTANZ
Department of Physics
Dr. Andrey Moskalenko



Group theory and symmetries in quantum mechanics
Summer semester 2017 - Exercise sheet 5
Distributed: 2.06.2017, Discussion: 7.06.2017 and 8.06.2017

Problem 13: Character table for the group C_{6v} using multiplication of classes.

Construct the character table for the group C_{6v} (treated in problem 8) using the common rules and the multiplication of classes (rule 7c of the lecture).

Problem 14: The regular representation.

Prove that the matrices $T^{(R)}(G_a)$, $G_a \in G$ introduced in the lecture indeed form a representation of a group G .

Problem 15: Rotational symmetries of Bravais lattices.

Let us consider a two-dimensional Bravais lattice in the $x - y$ plane and assume that a rotation by an angle φ around the z axis is also a symmetry of the lattice. Due to the translational invariance, the angle φ cannot take on arbitrary values. To show this, assume the perpendicular rotation axis located at a lattice point and choose an in-plane coordinate system such that one of the primitive lattice vectors (say, \mathbf{a}_1) is along the x axis. Assuming that a rotation by $+\varphi$ and $-\varphi$ are symmetries of the Bravais lattice and taking into account the translational invariance, find the allowed values of φ . Can the result be generalized to three-dimensional Bravais lattices?