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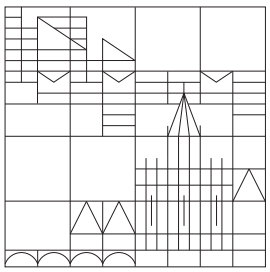

Group theory and symmetries in quantum mechanics
Summer semester 2017 - Exercise sheet 2
Distributed: 12.05.2017, Discussion: 17.05.2017 and 18.05.2017

Problem 5: Direct products and induced transformations.

a) A group $G = H \bigotimes K$ is a direct product of groups H and K. Prove that $G/K \cong H$.

b) Prove that for transformations of functions $O(G_a)\psi(\mathbf{r}) = \psi\big(T(G_a^{-1})\mathbf{r}\big)$ $(G_a \in G)$, induced by an arbitrary group G, the multiplication order is preserved: $G_c{=}G_aG_b \Rightarrow O(G_c){=}O(G_a)O(G_b)$.

Problem 6: Diagonalization of Hermitian matrices.

Prove that any Hermitian matrix H can be diagonalized by a unitary transformation U.

Problem 7: Group D_3 and quadratic polynomials of x and y.

Consider the representation O of the group D_3 in the basis of functions $\psi_1 = x^2$, $\psi_2 = y^2$ and $\psi_3 = xy$.

a) The matrices $O(R_1)$ and $O(R_2)$ were given in the lecture. Determine also the matrices $O(R_3)$, $O(R_4)$, and $O(R_5)$. Check if $\big[O(R_3)\big]^2 = \big[O(R_4)\big]^2 = \big[O(R_5)\big]^2 = O(E)$ is fulfilled.

b) Determine the matrices $O'(R_1)$ and $O'(R_4)$ considering the orthonormal basis of functions $\phi_1 = 2\sqrt{\frac{2}{\pi}}x^2$, $\phi_2 = \frac{1}{\sqrt{\pi}}(3y^2 - x^2)$ and $\phi_3 = 2\sqrt{\frac{6}{\pi}}xy$. Which difference in the properties do you see: between $O(R_1)$ and $O'(R_1)$? And between $O(R_4)$ and $O'(R_4)$?