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Group theory and symmetries in quantum mechanics
Summer semester 2017 - Exercise sheet 11
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Problem 29: One Casimir invariant for the fundamental representation of SU(n).

The so-called "quadratic" Casimir constant c for the fundamental representation of the SU(n) group is determined via $\sum_a (T_a T_a)_{ij} = c\,\delta_{ij}$.

a) Using the completeness relation for the generators T_a of this group show that

$$c = \frac{n^2-1}{2n}\ .$$

b) What is the particular result for the case of SU(2)? How it is related to the spin value?

Problem 30: Representations of SU(2).

a) Prove that the representations 2 and 2^* of SU(2) are equivalent.

Hint : Make use of the matrix $\sigma_2 \equiv \sigma_y$ and the exponential form of the group elements.

b) What is the fundamental weight?

c) What are simple roots?

d) What is the relation between q^1 for an arbitrary representation and the spin (total angular momentum) s?

e) Convince yourself that there is exactly one representation for any positive integer dimension d.

f) Decompose $2 \otimes 2 \otimes 2$ into the direct sum of irreducible representations.

Problem 31: Representations of SU(3).

a) Consider the representations (q^1, q^2) of SU(3). Their dimensions are given by

$$d(q^1, q^2) = \frac{(q^1+1)(q^2+1)(q^1+q^2+2)}{2}\ .$$

Do we have representations with any positive integer dimension $d = 1, 2, 3\ldots$?

b) Consider the representation 3^* having the generators $-T_a^*$. Prove that the weights of 3^* have the opposite sign prefactor with respect to the weights of 3.

c) Explain why $3^* = (0, 1)$.

d) Construct the weight diagram for the representation $(3, 0)$.