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Group theory and symmetries in quantum mechanics
Summer semester 2016 - Exercise sheet 3
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Problem 8: Symmetry group C_{6v}.

a) Consider the symmetry group of a regular hexagon. Find all 12 elements of this group. Denote rotations by angle $\frac{m}{n}2\pi$ as C_n^m and reflections in respect to mirror planes which pass through opposite faces (vertices) of the hexagon as $\sigma_{v,i}$ ($\sigma_{d,i}$), where the index i just numbers different reflections of the corresponding type.

b) Identify the six classes of this group.

c) How many inequivalent irreducible representations do exist for this group and what are their dimensions?

Problem 9: Irreducible representations of cyclic groups.

Consider at first the three-element group $G = \{e, a, b\}$.

a) Show that this group is cyclic.

b) Check if the following representation

$$e = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad a = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}, \quad b = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$

satisfies the Great Orthogonality Theorem.

c) Consider a 1-dimensional representation based on choosing $a = z$, where z is a complex number. Show that in order to produce a representation of G, we must require that $z^3 = 1$.

d) Use the result of (c) to obtain three irreducible representations of G. Are there any other irreducible representations of G? Explain your answer.

e) Generalize the result of (d) to any cyclic group of order n.

Problem 10: Great Orthogonality Theorem for abelian groups.

a) Simplify the formulation of the Great Orthogonality Theorem for the case of abelian groups. Viewing irreducible representations of such a group as vectors in a $|G|$-dimensional space (with components corresponding to different group elements), what does the Great Orthogonality Theorem mean for these vectors? Which bound does this put on the number of irreducible representations of an abelian group?

b) Verify that the Great Orthogonality Theorem in the simplified form, obtained in (a), is satisfied for the representations obtained in problem 9(d).