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Group theory and symmetries in quantum mechanics
Summer semester 2016 - Exercise sheet 1
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Problem 1: Conjugate elements and cosets.

a) Prove that the conjugation is an equivalence relation.

b) Prove that any two left cosets of a subgroup H of a group G either contain exactly the same elements or have no common elements at all.

Problem 2: Multiplication tables for groups of order 4.

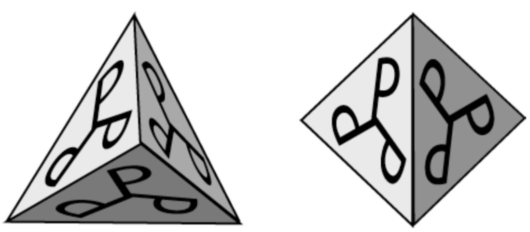
Constructing multiplication tables explicitly, find two distinct (up to relabelling of the elements) structures for groups of order 4. Are these groups Abelian?

Problem 3: Group D_4.

The group D_4 is the group of all possible (in 3 dimensions) rotations (only rotations!) of a square (similar to the considered case of the equilateral triangle). Construct the multiplication table for D_4 and divide the elements into classes.

Problem 4: Converse of the Lagrange's theorem.

Consider a 3-dimensional object having the chiral tetrahedral symmetry T (see Fig. A, or Fig. B for a practical realization).

(A) 

(B) 

(A) Labelled tetrahedron, (B) A beaded bead having the chiral tetrahedral symmetry [G.L. Fisher and B. Mellor, J. Math. Art 1, 85 (2007)].

a) Find all possible symmetry operations for such an object.

b) Convince yourself that the converse of the Lagrange's theorem is not true in this case.