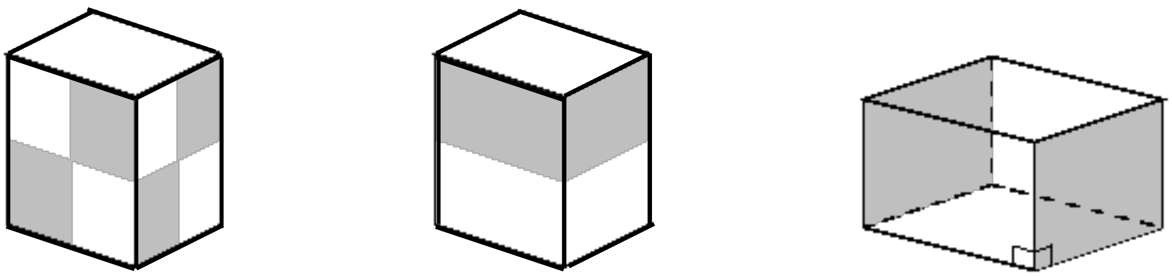
UNIVERSITY OF KONSTANZ
Department of Physics
Prof. Dr. Guido Burkard, Dr. Andrey Moskalenko, Dr. Andor Kormányos

Group theory and symmetries in quantum mechanics
Summer semester 2015 - Exercise sheet 10
Distributed: 18.06.2015, Discussion: 23.06.2015

Problem 28: The double groups $\bar{D}_2$, $\bar{C}_{2v}$ and $\bar{C}_{2h}$



Objects with D_2, C_{2v} and C_{2h} symmetries are shown in the left, middle and right figures, respectively. The single group character table for D_2 reads:

			E	C_2	C_2'	C_2''
x^2, y^2, z^2		A_1	1	1	1	1
xy	R_z, z	B_1	1	1	-1	-1
xz	R_y, y	B_2	1	-1	1	-1
yz	R_x, x	B_3	1	-1	-1	1

while for C_{2v} it is given by

			E	C_2	σ_v	σ_v'
x^2, y^2, z^2	z	A_1	1	1	1	1
xy	R_z	A_2	1	1	-1	-1
xz	R_y, x	B_1	1	-1	1	-1
yz	R_x, y	B_3	1	-1	-1	1

Here σ_v and σ_v' denote mirror reflections.
(a) Consider the character table of D_2 and C_{2v}. Using the Frobenius-Schur criterion find out whether individual irreps are of a), b) or c) type, as discussed in the lecture !

(b) Try to construct the double group character table for $\bar{D}_2$ and $\bar{C}_{2v}$! How many new classes and representations are there in these double groups?

(c) Now consider the group C_{2h}. The character table is given below. How many new classes and representations are there in $\bar{C}_{2h}$? What are the dimensions of the new representations?

			E	C_2	σ_h	i
x^2, y^2, z^2	R_z	A_g	1	1	1	1
	z	A_u	1	1	-1	-1
xz, yz	R_y, x	B_g	1	-1	-1	1
	x, y	B_u	1	-1	1	-1

Table 1: Character table of C_{2h}. i denotes inversion.

Problem 29: Energy eigenstates and time-reversal

a) Consider a spinless Hamiltonian H. Show that if H is invariant under the time reversal and the energy eigenvalue E_n is non-degenerate, then the corresponding eigenstate Ψ_n is real (apart from an arbitrary complex phase factor)!

b) The wave function of a (one-dimenasional) plane wave is $\Psi(x) = e^{ikx}$, where k is the wavenumber. $\Psi(x)$ is the eigenfunction of the Hamiltonian $H = \frac{-\hbar^2 \nabla_x^2}{2m}$. Does this contradict to what we have found in a) ?

c) Suppose a spinless particle is bound in the potential $V(\mathbf{r})$ and that no bound state energy level is degenerate. Show that the expectation value of the angular momentum $\hat{\mathbf{L}}$ is zero for any bound eigenstate!