University of Konstanz

Seminar on the optical properties of semiconductors

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# Magneto Optics

by

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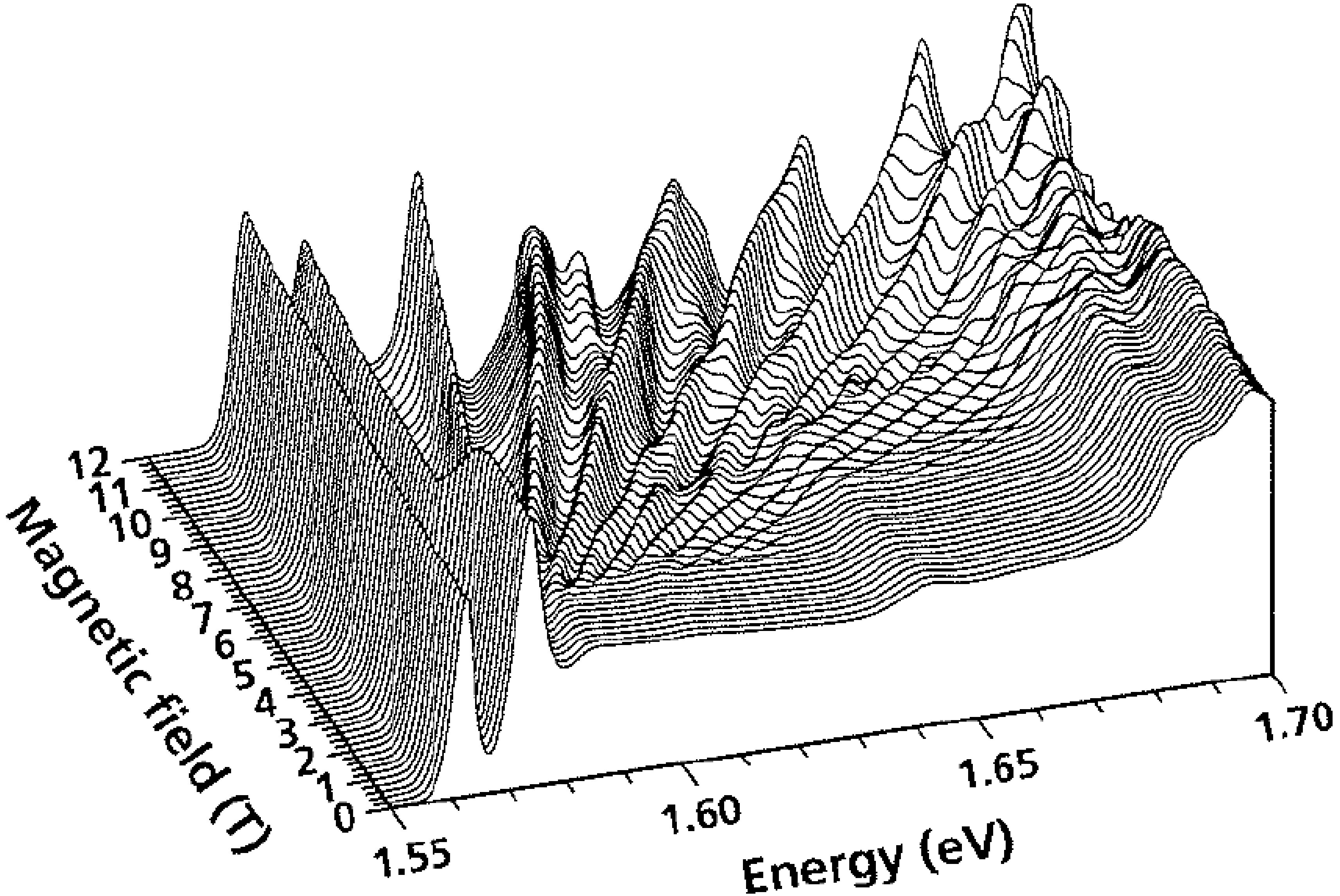


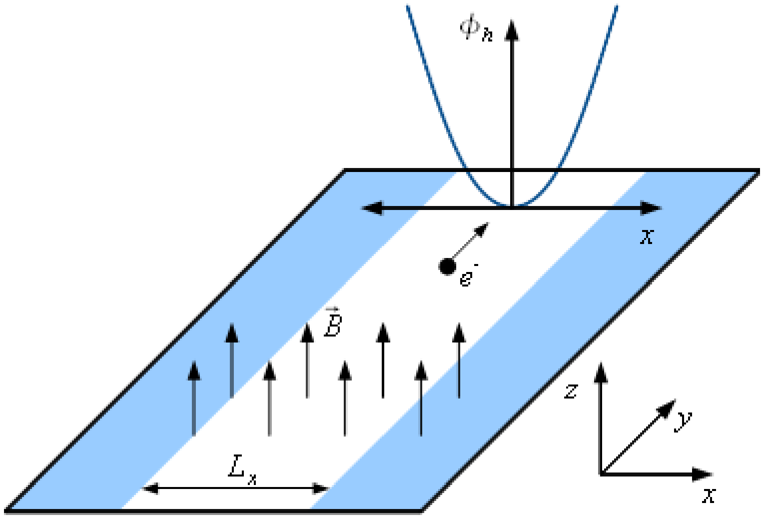
# Magneto Optics

Contents :

# Magneto Optics

## Single electron in a magnetic field

- Quantum well in x-y plane
- Weak harmonic confinement potential in x-direction
- Static Magnetic Field in z-direction
- Disregard all band-mixing effects
- Respect only lowest quantum well electron and heavy-hole bands




- Effective mass approximation

$$\Psi \approx u_\lambda(k=0,r)\,\psi_{k,\nu}(r)$$

Bloch function — Envelope function

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# Magneto Optics

## Single electron in a magnetic field

- Single particle Hamiltonian

$$H_j^0 = \frac{1}{2\mathrm{m}_j}[\vec{p}_j - e_j \vec{A}(\vec{r}_j)]^2 + \frac{1}{2} m_j \Omega_j^2 x_j^2 + \frac{E_g}{2} \qquad j = e, h \qquad \vec{p}_j = -i\hbar \begin{pmatrix} \frac{\partial}{\partial x_j} \\ \frac{\partial}{\partial y_j} \end{pmatrix}$$

$E_g$ *band gap of the quantum well*

$\Omega_j$ *oscillator frequency*

$\vec{A}$ *vector potential*

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# Magneto Optics

## Single electron in a magnetic field

- We need to choose a gauge for the vector potential :

  - Symmetric gauge $\vec{A}(\vec{r})=\frac{B}{2}(-y\,\vec{e}_x+x\,\vec{e}_y)$

  - Asymmetric Landau gauge $\vec{A}(\vec{r})=B\,x\,\vec{e}_y$

- Using the Landau gauge :

$$H_j^0=\frac{1}{2\mathrm{m}_j}[\vec{p}_j-e_j B\,x_j\vec{e}_y]^2+\frac{1}{2}m_j\Omega_j^2x_j^2+\frac{E_g}{2}$$

$$H_j^0=-\frac{\hbar^2}{2\mathrm{m}_j}\frac{\partial^2}{\partial x_j^2}-\frac{\hbar^2}{2\mathrm{m}_j}\frac{\partial^2}{\partial y_j^2}-\hbar\omega_{c,j}\frac{x_j}{i}\frac{\partial}{\partial y_j}+\frac{1}{2}m_j(\Omega_j^2+\omega_{c,j}^2)x_j^2+\frac{E_g}{2}$$

$$\omega_{c,j}=\frac{e_j B}{m_j}$$ cyclotron frequency

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# Magneto Optics

## Single electron in a magnetic field

- Ansatz : $\psi_j(x_j, y_j) = \frac{e^{iky_j}}{\sqrt{L_y}} \phi(x_j)$ $L_y$ wire length

- Inserting this ansatz into Schrödinger´s equation :

$$H_j^0 \psi_j(x_j, y_j) = \frac{e^{iky_j}}{\sqrt{L_y}} \left( -\frac{\hbar^2}{2\mathrm{m}_j} \frac{\partial^2}{\partial x_j^2} + \frac{\hbar^2 k^2}{2\mathrm{m}_j} - \hbar \omega_{c,j} x_j k + \frac{1}{2} m_j (\Omega_j^2 + \omega_{c,j}^2) x_j^2 + \frac{E_g}{2} \right) \phi(x_j)$$

- We obtain the x-dependent Hamiltonian :

$$H^0(x_j) = -\frac{\hbar^2}{2\mathrm{m}_j} \frac{\partial^2}{\partial x_j^2} + \frac{\hbar^2 k^2}{2\mathrm{m}_j} - \hbar \omega_{c,j} x_j k + \frac{1}{2} m_j (\Omega_j^2 + \omega_{c,j}^2) x_j^2 + \frac{E_g}{2}$$

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# Magneto Optics

## Single electron in a magnetic field

- Applying quadratic completion and using :

$$\omega_{c,j}=\frac{e_j B}{m_j}$$ cyclotron frequency

$$\Delta x_j = k\,\delta_j = k\,\frac{\hbar}{m_j}\frac{\omega_{c,j}}{\omega_{eff,j}^2}$$

$$\omega_{eff,j}=\sqrt{\Omega_j^2+\omega_{c,j}^2}$$ effective oscillator frequency

$$m_{eff,j}=m_j\left(\frac{\omega_{eff,j}}{\Omega_j}\right)^2$$ effective mass

$$H^0(x_j)=-\frac{\hbar^2}{2\mathrm{m}_j}\frac{\partial^2}{\partial x_j^2}+\frac{1}{2}m_j\omega_{eff,j}^2\left(x_j-\Delta x_j\right)^2+\frac{\hbar^2 k^2}{2m_{eff,j}}+\frac{E_g}{2}$$

similar to Harmonic Oscillator

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# Magneto Optics

## Single electron in a magnetic field

- Write the harmonic potential as :

$$\frac{1}{2} m_j \omega_{eff,j}^2 \left(x_j - \Delta x_j\right)^2 = \frac{\left(x_j - \Delta x_j\right)^2}{\left(l_{eff,j}\right)^4}$$

- Identify the characteristic length :

$$l_{eff,j} = \sqrt{\frac{\hbar}{m_j \omega_{eff,j}}}$$

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# Magneto Optics

## Single electron in a magnetic field

- Modified Landau ladder :

$$E^{j}_{k,\nu} = \frac{\hbar^2 k^2}{2 m_{eff,j}} + \frac{E_g}{2} + \hbar \omega_{eff,j} (\nu + \frac{1}{2}) \qquad \omega_{eff,j} > \Omega_j$$

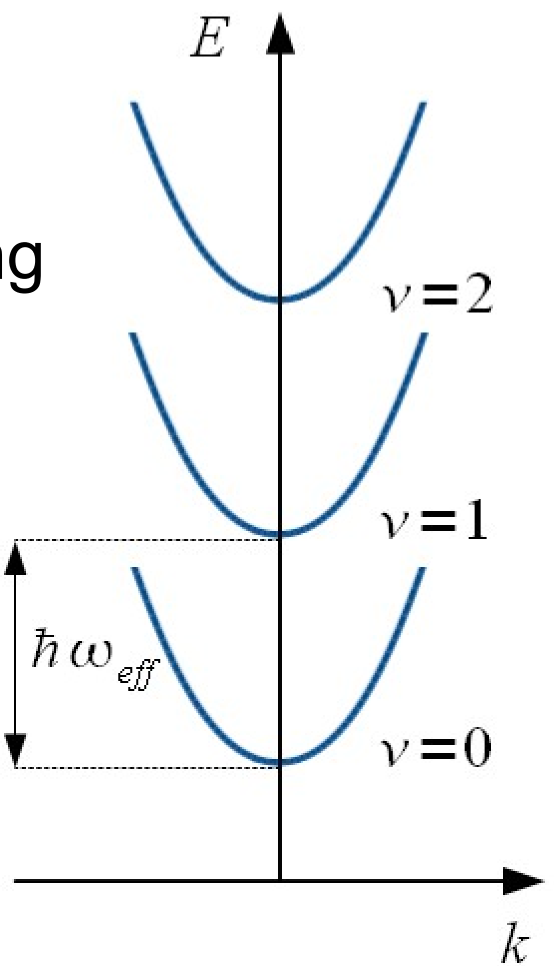
⇨ Increase of the lateral subband spacing

- Modified Landau eigenfunctions :

$$\psi^{j}_{k,\nu}(x_j, y_j) = \frac{e^{iky_j}}{\sqrt{L_y}} \phi_\nu (x_j - \Delta x_j) \qquad \nu = 0,1,2,\ldots$$

⇧ Harmonic Oscillator wavefunctions

$$\Delta x_j = k \delta_j = k \frac{\hbar}{m_j} \frac{\omega_{c,j}}{\omega^2_{eff,j}} = k\, l^2_{eff} \frac{\omega_{cj}}{\omega_{eff,j}}$$

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# Magneto Optics

## Single electron in a magnetic field

- Quantum well without confinement potential
  - Landau ladder

$$E^j_{k,\nu}=\frac{\hbar^2 k^2}{2m_j}+\frac{E_g}{2}+\hbar\omega_{c,j}(\nu+\frac{1}{2})$$

  - Landau eigenfunctions

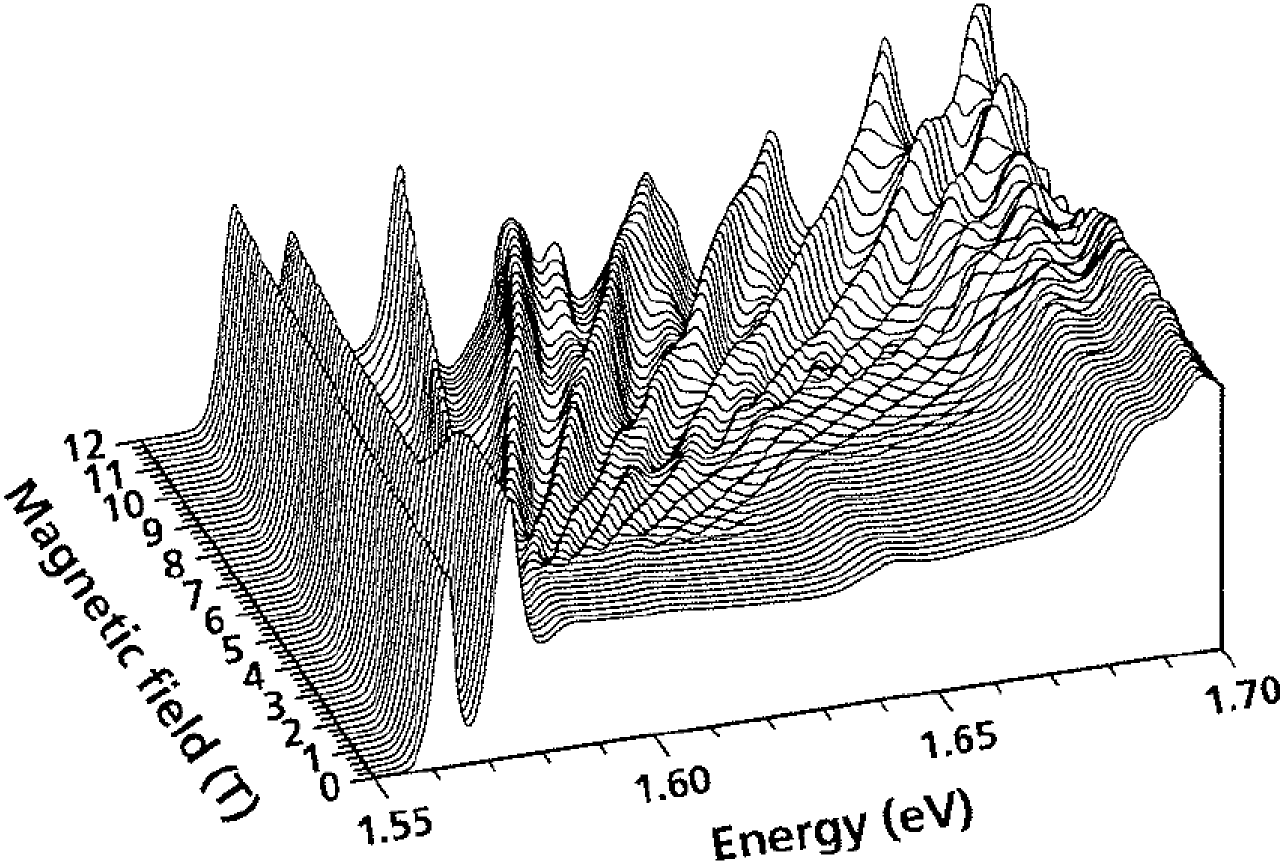
$$\psi^j_{k,\nu}(x_j,y_j)=\frac{e^{iky_j}}{\sqrt{L_y}}\phi_\nu(x_j-\Delta x_j) \quad \nu=0,1,2,\ldots$$

$$\Delta x_j=k\delta_j=k\frac{\hbar}{m_j\omega_{c,j}}$$

# Magneto Optics

## Single electron in a magnetic field

Measured absorption spectra of an 8.5 nm wide GaAs quantum well under the influence of a perpendicular magnetic field.

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# Magneto Optics

## Density of states

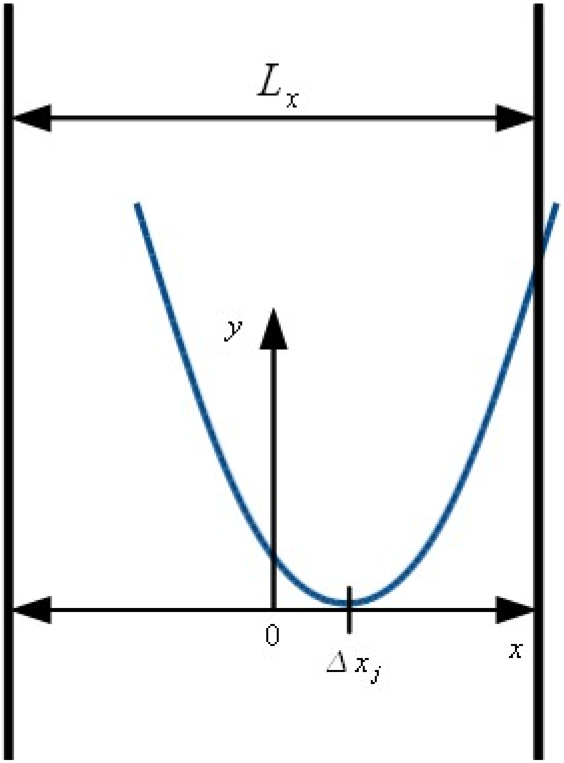
- For simplicity :

$$m_e \Omega_e = m_h \Omega_h = m \Omega \Rightarrow l_{eff,e} = l_{eff,h}$$ local charge neutrality

- In case of applied magnetic field momentum k is restricted

$$\Delta x_j = k \delta_j \leqslant \frac{L_x}{2}$$

$$\Rightarrow \quad k_{max} = \frac{L_x}{2 \delta_j}$$

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# Magneto Optics

## Density of states

- We calculate the density of states :

$$D(E)=\sum_{\nu,k}\delta(E-E_{k,\nu})$$

$$D(E)=\frac{L_y}{2\pi}\int_{-k_m}^{k_m} dk\sum_{\nu}\delta\left(\frac{\hbar^2 k^2}{2m_{eff}}+E_{k=0,\nu}-E\right)$$

$$E_{k=0,\nu}=\frac{E_g}{2}+\hbar\omega_{eff,j}(\nu+\frac{1}{2})$$

$$\tilde{E}_0=\frac{\hbar^2}{2m_{eff}L_x^2}$$

confinement energy

- Using : $\delta(\lambda x)=\frac{1}{|\lambda|}\delta(x)$

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# Magneto Optics

## Density of states

$$D(E)=\frac{L_y}{2\pi L_x \tilde{E}_0}\int_{-\kappa_m}^{\kappa_m} d\kappa \sum_{\nu}\delta\left(\kappa^2-\frac{E-E_{k=0,\nu}}{\tilde{E}_0}\right) \qquad \kappa=L_x k$$

- Using : $\delta(f(x))=\sum_i \frac{1}{|f'(x_i)|}\delta(x-x_i)$ where $f(x_i)=0$

$$D(E)=\frac{L_y}{2\pi L_x \tilde{E}_0}\sum_{\nu}\theta\left(\kappa_m^2-\frac{E-E_{k=0,\nu}}{\tilde{E}_0}\right)\sqrt{\frac{\tilde{E}_0}{E-E_{k=0,\nu}}}$$

- With increasing magnetic field the single particle density of states changes from a $\frac{1}{\sqrt{E}}$ -like behavior (1D) to a $\delta$ -function-like behavior (0D).

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# Magneto Optics

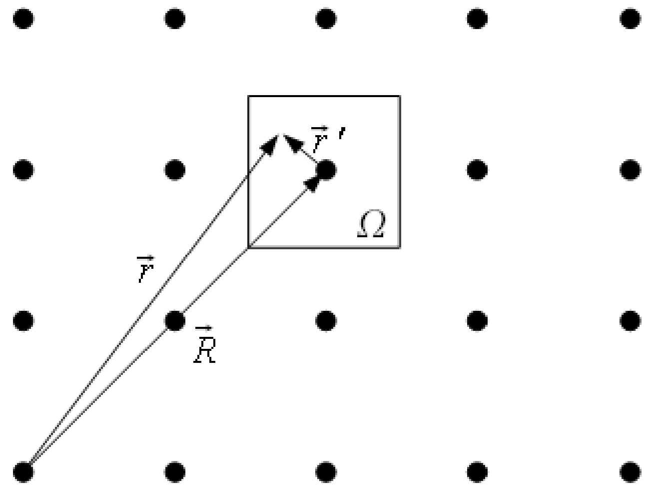
## Optical selection rules

- We consider the optical transition matrix elements :

$$r_{\lambda k \nu, \lambda' k' \nu'} = \langle \lambda k \nu | r | \lambda' k' \nu' \rangle = \int dr\, u_\lambda^*(0,r) \psi_{k\nu}^*(r)\, r\, u_{\lambda'}(0,r) \psi_{k'\nu'}(r)$$

$$r_{\lambda k \nu, \lambda' k' \nu'} = \sum_R \int_\Omega dr'\, u_\lambda^*(0,r') \psi_{k\nu}^*(r'+R)(r'+R) u_{\lambda'}(0,r') \psi_{k'\nu'}(r'+R)$$

$\Omega$ unit cell

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# Magneto Optics

## Optical selection rules

$$r_{\lambda k \nu, \lambda' k' \nu'} = \sum_R \int_\Omega dr' \, u_\lambda^*(0, r') \psi_{k\nu}^*(r'+R) \, r' \, u_{\lambda'}(0, r') \psi_{k'\nu'}(r'+R)$$

$$+ \sum_R \int_\Omega dr' \, u_\lambda^*(0, r') \psi_{k\nu}^*(r'+R) \, R \, u_{\lambda'}(0, r') \psi_{k'\nu'}(r'+R)$$

$$r_{\lambda k \nu, \lambda' k' \nu'} \approx \sum_R \psi_{k\nu}^*(R) \psi_{k'\nu'}(R) \underbrace{\int_\Omega dr' \, u_\lambda^*(0, r') \, r' \, u_{\lambda'}(0, r')}_{r_{\lambda\lambda'}}$$

$$+ \underbrace{\sum_R \psi_{k\nu}^*(R) \, R \, \psi_{k'\nu'}(R)}_{r_{k\nu, k'\nu'}} \int_\Omega dr' \, u_\lambda^*(0, r') u_{\lambda'}(0, r')$$

- dipole matrix element :

$$d_{\lambda k \nu, \lambda' k' \nu'} = -e \, r_{\lambda k \nu, \lambda' k' \nu'} = \delta_{\nu\nu'} \delta_{kk'} d_{\lambda\lambda'} + d_{k\nu, k'\nu'} \delta_{\lambda\lambda'}$$

interband intraband

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# Magneto Optics

## Optical selection rules

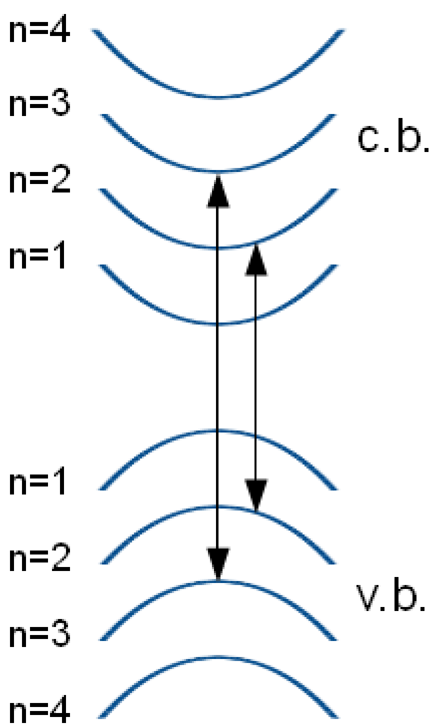
selection rules :

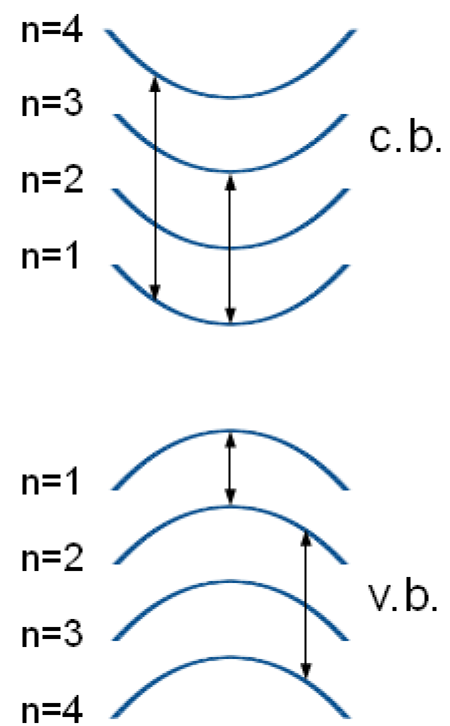
$\nu = \nu'$
$k = k'$

same Landau levels for interband transition between conduction and valence band

$\lambda = \lambda'$

intraband transitions between different Landau levels

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# Magneto Optics

## Semiconductor Bloch Equations

- We use the modified Landau states for the density matrix

$$\rho_{i,\nu;i',\nu}(k,t)=\langle a^{\dagger}_{i,k,\nu}(t)a_{i',k,\nu}(t)\rangle$$

- We only consider interband transitions.

$$\{i,i'\}=\{c,\mathrm{v}\}$$

- The relevant carrier densities are :

$$\rho^{ee}_{\nu,\nu}(k)=\langle\alpha^{\dagger}_{k,\nu}(t)\alpha_{k,\nu}(t)\rangle=n^{e}_{k,\nu}$$ $\alpha,\alpha^{\dagger}$ electron operators

$$\rho^{hh}_{\nu,\nu}(k)=\langle\beta^{\dagger}_{k,\nu}(t)\beta_{k,\nu}(t)\rangle=n^{h}_{k,\nu}$$ $\beta,\beta^{\dagger}$ hole operators

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# Magneto Optics

## Semiconductor Bloch Equations

- Hamiltonian

$$H=\sum_{k\nu} E^{e}_{k\nu}\alpha^{\dagger}_{k\nu}\alpha_{k\nu}+\sum_{k\nu} E^{h}_{k\nu}\beta^{\dagger}_{k\nu}\beta_{k\nu}+H_{C}+H_{L}$$

- Coulomb interaction

$$H_{C}=\frac{1}{2}\sum_{kk'q\neq 0}\Big[V^{ee}_{\mu\mu',\nu\nu'}(q)\,\alpha^{\dagger}_{k+q,\mu}\alpha^{\dagger}_{k'-q,\mu'}\alpha_{k',\nu'}\alpha_{k,\nu}+V^{hh}_{\mu\mu',\nu\nu'}(q)\,\beta^{\dagger}_{k+q,\mu}\beta^{\dagger}_{k'-q,\mu'}\beta_{k',\nu'}\beta_{k,\nu}$$

$$-2V^{eh}_{\mu\mu',\nu\nu'}(q)\,\alpha^{\dagger}_{k+q,\mu}\beta^{\dagger}_{k'-q,\mu'}\beta_{k',\nu'}\alpha_{k,\nu}\Big]$$

- Interaction with light

$$H_{L}=-\sum_{k\nu} E(t)\Big(d_{ck\nu,\nu k\nu}\,\alpha^{\dagger}_{k\nu}\beta^{\dagger}_{-k\nu}+h.c.\Big)$$

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# Magneto Optics

## Semiconductor Bloch Equations

- Matrix elements of the Coulomb potential :

$$\int dx\,dy \int dx'\,dy'\,\psi^*_{k',n'}(x,y)\,\psi^*_{k,\nu}(x',y')\,V(x-x',y-y')\,\psi_{k',n'}(x',y')\,\psi_{k,\nu}(x,y)$$

Use modified Landau eigenfunctions and Fourier transform of 2D Coulomb potential :

$$V(x-x',y-y')=\sum_{q,q_x}\frac{2\pi e^2}{\epsilon_0\sqrt{q^2+q_x^2}}\,e^{iq(y-y')}\,e^{iq_x(x-x')}$$

$$\Longrightarrow \quad V^{j,j'}_{\nu,\nu';\nu',\nu}(q)=\sum_{q_x}\frac{2\pi e_j e_{j'}}{\epsilon_0\sqrt{q^2+q_x^2}}\left|\int dx\,\phi^*_{\nu}(x+\delta q)\,e^{iq_x x}\,\phi_{\nu'}(x)\right|=V^{j,j'}_{\nu,\nu'}$$

$\epsilon_0$ background dielectric function

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# Magneto Optics

## Semiconductor Bloch Equations

- Therefore we find the dynamic equation

$$\left(i\frac{\partial}{\partial t}-e^{e}_{k,\nu}-e^{h}_{k,\nu}\right)P_{k,\nu}=(n^{e}_{k,\nu}+n^{h}_{k,\nu}-1)\omega_{R,k,\nu}(t)+i\frac{\partial P_{k,\nu}}{\partial t}\Big|_{scatt}$$

for the optical interband polarization :

$$\rho^{eh}_{\nu,\nu}(k)=\langle\beta_{-k,\nu}\alpha_{k,\nu}\rangle=P_{k,\nu}$$

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# Magneto Optics

## Semiconductor Bloch Equations

- Carrier energies : renormalized single particle energies

$$\hbar e_{k,\nu}^{j}=E_{k,\nu}^{j}-\sum_{q,\nu',j} V_{\nu,\nu'}^{j,j'}(q)n_{k-q,\nu'}^{eh}(t)$$

$E_{k,\nu}^{j}$ Landau ladder energies

- The exchange renormalized Rabi-frequency is given by :

$$\hbar\omega_{R,k,\nu}(t)=d_{c\nu}E(t)+\sum_{q,\nu'} V_{\nu,\nu'}^{eh}(q)P_{k-q,\nu'}$$

- The equations for the carrier densities are :

$$\frac{\partial}{\partial}n_{k,\nu}^{j}=-2\,\mathrm{Im}[\omega_{R,k,\nu}(t)P_{k,\nu}^{*}]+\frac{\partial n_{k,\nu}^{j}}{\partial t}|_{scatt}$$

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# Magneto Optics

## Magneto-Luminescence of Quantum Wires

- Assume E-field : $E(t)=E_0 e^{-i\omega t}$

- Stationary polarization equation $p_{k-q,\nu}=P_{k,\nu}e^{-i\omega t}$

$$\hbar\left(\omega-e^e_{k,\nu}-e^h_{k,\nu}+i\gamma\right)p_{k,\nu}=\left(f^e_{k,\nu}+f^h_{k,\nu}-1\right)[d_{c\nu}E_0+\sum_{q,\nu'}V^{eh}_{\nu\nu'}(q)\,p_{k-q,\nu}]$$

- Assume for the electron and hole distributions thermal Fermi functions :

$$f^e_{k,\nu}=\frac{1}{e^{\left(\hbar\omega_{eff}(\nu+\frac{1}{2})+\frac{\hbar^2k^2}{2m^e_{eff}-\mu^e}\right)\beta}+1}$$

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# Magneto Optics

## Magneto-Luminescence of Quantum Wires

- Calculate susceptibility :

$$\chi(\omega) = d_{cv} \sum_{k,\nu} P_{k,\nu}(\omega)$$

- We get the luminescence spectrum via the equilibrium relation :

$$I(\omega) \propto \frac{\mathrm{Im}\{\chi(\omega)\}}{e^{\beta(\hbar\omega - \mu)} - 1}$$

$\mu = \mu_e + \mu_h$ Electron-hole chemical potential with respect to the band gap

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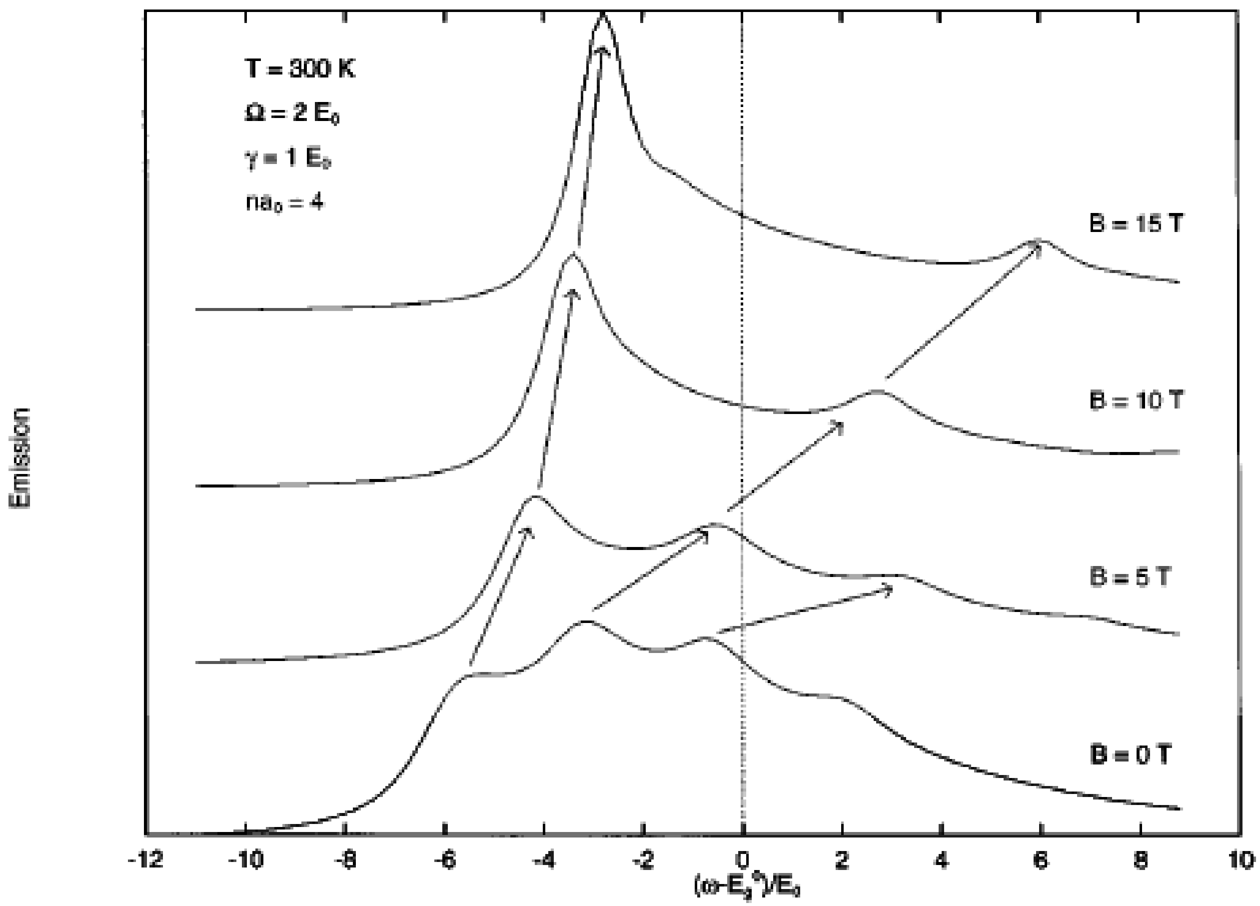
# Magneto Optics

## Magneto-Luminescence of Quantum Wires

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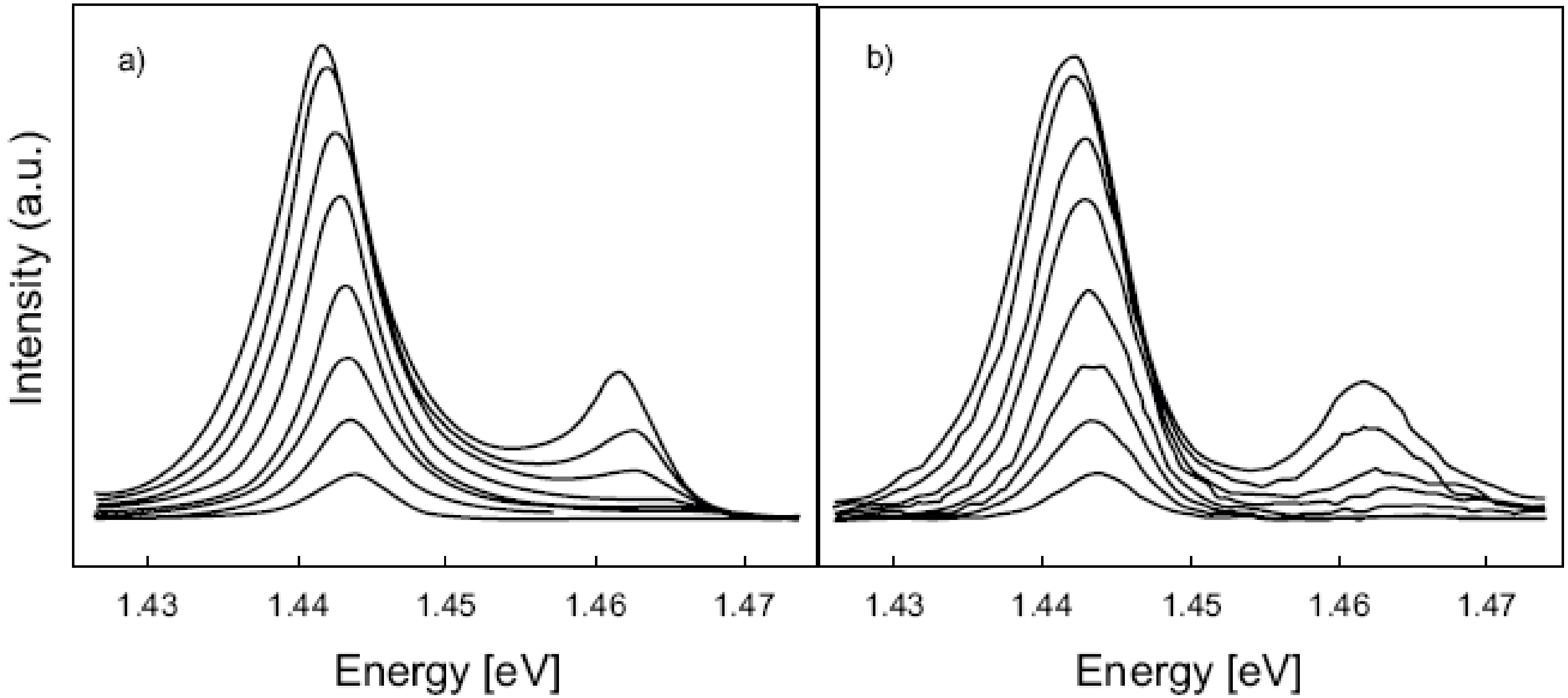
# Magneto Optics

## Magneto-Luminescence of Quantum Wires




Calculated a) and measured b) luminescence spectra

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# Magneto Optics

## Conclusions

- In a magnetic field the single particle states turn into the modified Landau levels :

$$\omega_{eff} = \sqrt{\Omega^2 + \omega_c^2} = \sqrt{\Omega^2 + \left(\frac{e B}{m}\right)^2}$$

- For high magnetic fields 1D- turns into 0D-like behavior

- Optical selection rules :

$$\nu = \nu' \qquad k = k'$$

interband

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# Magneto Optics

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