Polaritons

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Wannier Equation

Wannier Equation including center-of-mass motion

$$-\left[\frac{\hbar^2\nabla^2_{\boldsymbol{R}}}{2M}+\frac{\hbar^2\nabla^2_{\boldsymbol{r}}}{2m_r}+V(r)\right]\psi(\boldsymbol{R},\boldsymbol{r})=E_{tot}\psi(\boldsymbol{R},\boldsymbol{r}) \qquad (1)$$

with total Mass $M=m_e+m_h$

$$\psi(\boldsymbol{R},\boldsymbol{r})=\frac{e^{i\boldsymbol{K}\cdot\boldsymbol{R}}}{L^{3/2}}\psi(\boldsymbol{r}) \qquad (2)$$

plane wave for center-of-mass motion
and eigenvalues

$$E_{tot}=E_g+E_n+\frac{\hbar^2K^2}{2M} \qquad (3)$$

Exciton and Photon Dispersion

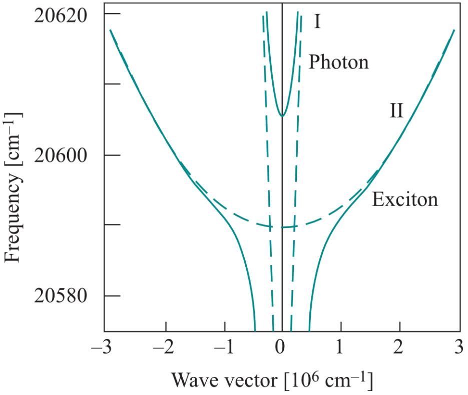

Figure: Dispersion in CdS [Yu]

Photon dispersion

$\omega = \frac{ck}{\sqrt{\epsilon}}$

Exciton dispersion

$\omega \propto k^2$

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Dielectric Theory

$$\begin{aligned} \epsilon(\omega) &= \epsilon_0 \left[1 + 4\pi\chi(\omega)\right] \\ &= \epsilon_0 \left[1 - 8\pi |d_{cv}|^2 \sum_n \frac{|\psi_n(\boldsymbol{r}=0)|^2}{\hbar(\omega + i\delta) - E_g - E_n}\right] \end{aligned} \tag{4}$$

$$\left[-k^2 + \frac{\omega^2}{c^2}\epsilon(\omega)\right] \boldsymbol{E}_\omega e^{i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)} = 0 \tag{5}$$

with $\boldsymbol{E}_\omega \neq 0$

Transverse Eigenmodes

$$c^2 k^2 = \omega^2 \epsilon(\omega) \tag{6}$$

Polaritons without Spatial Dispersion

only 1s-excitons $(n = 0)$

$$\epsilon(\omega) \simeq \epsilon_0 \left[1 - 8\pi|d_{cv}|^2 \frac{|\psi_0(\boldsymbol{r} = 0)|^2}{\hbar(\omega + i\delta) - E_g + E_0}\right] \tag{7}$$

with $\hbar\Delta = 8\pi|dcv|^2|\psi_0(\boldsymbol{r} = 0)|^2$ and $\hbar\omega_0 = E_g - E_0$

$$\epsilon(\omega) = \epsilon_0 \left(1 - \frac{\Delta}{\omega - \omega_0 + i\delta}\right) \tag{8}$$

using the Dirac identity $\lim_{\delta\to 0} \frac{1}{\omega - i\delta} = \mathcal{P}\frac{1}{\omega} + i\pi\delta(\omega)$
into eigenmode equation $c^2k^2 = \omega^2\epsilon(\omega)$

Polaritons without Spatial Dispersion

complex wave number $k = k' + ik''$

$$c^2k^2 = \omega^2\epsilon(\omega) \quad \text{in} \tag{9}$$

$$\text{real part} \quad \frac{\omega^2\epsilon_0}{c^2}\left(1 - \frac{\Delta}{\omega - \omega_0}\right) = k'^2 - k''^2 \tag{10}$$

and

$$\text{imaginary part} \quad \pi\delta(\omega - \omega_0)\frac{\omega^2\epsilon_0}{c^2} = 2k'k'' \tag{11}$$

for $\omega \neq \omega_0$ no absorption

Polariton Dispersion

$$\omega\sqrt{\frac{\omega-\omega_0-\Delta}{\omega-\omega_0}} = \frac{ck'}{\sqrt{\epsilon_0}} \quad (12)$$

with "band gap"
$\omega_0 < \omega < \omega_0 + \Delta$

$$\omega \simeq \frac{ck'}{\sqrt{\epsilon_0(1+\Delta/\omega_0)}} \quad (13)$$

for $\omega \ll \omega_0$

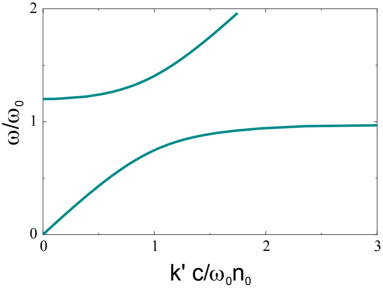


Figure: Polariton dispersion without spatial dispersion for $\Delta/\omega_0 = 0.2$ [Haug]

Polaritons with Spatial Dispersion and Damping

include photon momentum

$$\epsilon(\boldsymbol{K},\omega) = \epsilon_0 \left(1 - \frac{\Delta}{\omega - \omega_0 - \frac{\hbar K^2}{2M} + i\gamma}\right) \tag{14}$$

eigenmode equation

$$\frac{c^2 K^2}{\epsilon_0} = \omega^2 \left(1 - \frac{\Delta}{\omega - \omega_0 - \frac{\hbar K^2}{2M} + i\gamma}\right) \tag{15}$$

Polaritons with Spatial Dispersion and Damping

Dispersion Relation

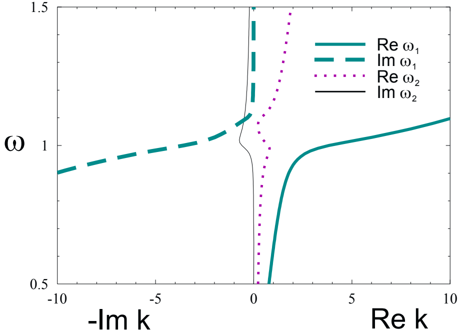


Figure: Polariton dispersion with spatial dispersion and damping for $\Delta/\omega_0 = 0.1$[Haug]

hole operator $\beta^\dagger_{-\boldsymbol{k},-s} \equiv a_{v,\boldsymbol{k},s}$
el. operator $\alpha^\dagger_{\boldsymbol{k},s} \equiv a^\dagger_{c,\boldsymbol{k},s}$
el-hole pair operator
$\alpha^\dagger_{\boldsymbol{k}}\beta^\dagger_{-\boldsymbol{k}} = |\boldsymbol{k}, -\boldsymbol{k}\rangle\langle 0|$
exciton operator $B^\dagger_{\nu,\boldsymbol{K}} = |\nu\boldsymbol{K}\rangle\langle 0|$

$$B^\dagger_{\nu,\boldsymbol{K}} = \sum_{\boldsymbol{k},\boldsymbol{k}'} \langle \boldsymbol{k}, -\boldsymbol{k}'|\nu\boldsymbol{K}\rangle |\boldsymbol{k}, -\boldsymbol{k}'\rangle\langle 0| = \sum_{\boldsymbol{k},\boldsymbol{k}'} \langle \boldsymbol{k}, -\boldsymbol{k}'|\nu\boldsymbol{K}\rangle \alpha^\dagger_{\boldsymbol{k}}\beta^\dagger_{-\boldsymbol{k}'}$$

$$\begin{aligned}\langle \boldsymbol{k}, -\boldsymbol{k}'|\nu\boldsymbol{K}\rangle &= \int \mathrm{d}^3r\mathrm{d}^3r' \langle \boldsymbol{k}, -\boldsymbol{k}'|\boldsymbol{r}, \boldsymbol{r}'\rangle\langle \boldsymbol{r}, \boldsymbol{r}'|\nu\boldsymbol{K}\rangle \\ &= \int \mathrm{d}^3r\mathrm{d}^3r' e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} e^{i\boldsymbol{k}'\cdot\boldsymbol{r}'} e^{i\boldsymbol{K}\cdot(\boldsymbol{r}+\boldsymbol{r}')/2} \psi_\nu(\boldsymbol{r}-\boldsymbol{r}') \\ &= \delta[\boldsymbol{K} - (\boldsymbol{k}-\boldsymbol{k}')]\psi_\nu\left(\frac{\boldsymbol{k}+\boldsymbol{k}'}{2}\right)\end{aligned}$$

Hamiltonian Theory of Polaritons

$$B^{\dagger}_{\nu,\boldsymbol{K}} = \sum_{\boldsymbol{k},\boldsymbol{k}'} \delta[\boldsymbol{K} - (\boldsymbol{k} - \boldsymbol{k}')]\psi_\nu \left(\frac{\boldsymbol{k} + \boldsymbol{k}'}{2}\right) \alpha^{\dagger}_{\boldsymbol{k}}\beta^{\dagger}_{-\boldsymbol{k}} \tag{16}$$

Exciton Creation Operator

$$B^{\dagger}_{\nu,\boldsymbol{K}} = \sum_{\boldsymbol{k}} \psi_\nu(\boldsymbol{k} - \boldsymbol{K}/2)\alpha^{\dagger}_{\boldsymbol{k}}\beta^{\dagger}_{\boldsymbol{K}-\boldsymbol{k}} \tag{17}$$

Hamiltonian Theory of Polaritons
Exciton Commutator

$$[B_{0,0}, B^{\dagger}_{0,0}] = \sum_{\boldsymbol{k},\boldsymbol{k}'} \psi_0(k)\psi^*_0(k') \left[\beta_{-\boldsymbol{k}}\alpha_{\boldsymbol{k}}, \alpha^{\dagger}_{\boldsymbol{k}'}\beta^{\dagger}_{-\boldsymbol{k}'}\right]$$

$$= \sum_{\boldsymbol{k}} |\psi_0(k)|^2 (1 - \alpha^{\dagger}_{\boldsymbol{k}}\alpha_{\boldsymbol{k}} - \beta^{\dagger}_{-\boldsymbol{k}}\beta_{-\boldsymbol{k}}) \tag{18}$$

$$\langle [B_{0,0}, B^{\dagger}_{0,0}] \rangle = 1 - \mathcal{O}(n a_0^3) \tag{19}$$

$\rightarrow$ boson particle at low densities

Free Excitons

$$\mathcal{H}_0 = \sum_{\boldsymbol{k}} \hbar e_{\nu k} B^{\dagger}_{\nu,\boldsymbol{k}} B_{\nu,\boldsymbol{k}} \tag{20}$$

with the wavenumber of the photon q

$$\mathcal{H}_I = -\sum_{\boldsymbol{k},\boldsymbol{q}} d_{cv}\left[\alpha^\dagger_{\frac{1}{2}\boldsymbol{q}+\boldsymbol{k}}\beta^\dagger_{\frac{1}{2}\boldsymbol{q}-\boldsymbol{k}}\mathcal{E}(\boldsymbol{q})e^{-i\omega_q t} + h.c.\right]$$
$$= -\sum_{\boldsymbol{k},\boldsymbol{q},\nu} d_{cv}\left[\psi^*_\nu(k)B^\dagger_{\nu,\boldsymbol{q}}\mathcal{E}(\boldsymbol{q})e^{-i\omega_q t} + h.c.\right] \quad (21)$$

with

$$\sum_\nu \psi^*_\nu(\boldsymbol{k})B^\dagger_{\nu,\boldsymbol{q}} = \sum_{\nu\boldsymbol{k'}} \psi^*_\nu(\boldsymbol{k})\psi_\nu(\boldsymbol{k'} - \boldsymbol{q}/2)\alpha^\dagger_{\boldsymbol{k'}}\beta^\dagger_{\boldsymbol{q}-\boldsymbol{k'}}$$
$$= \sum_{\boldsymbol{k'}} \delta_{\boldsymbol{k}}, \boldsymbol{k'} - \boldsymbol{q}/2\alpha^\dagger_{\boldsymbol{k'}}\beta^\dagger_{\boldsymbol{q}-\boldsymbol{k'}} = \alpha^\dagger_{\frac{1}{2}\boldsymbol{q}+\boldsymbol{k}}\beta^\dagger_{\frac{1}{2}\boldsymbol{q}-\boldsymbol{k}} \quad (22)$$

Field Quantization

$$\hat{\boldsymbol{A}}(\boldsymbol{r},t) = \sum_{\boldsymbol{k},\lambda} \sqrt{\frac{2\pi c^2 \hbar}{\omega_k}} [b_{\lambda \boldsymbol{k}} \boldsymbol{u}_{\lambda \boldsymbol{k}}(\boldsymbol{r}) + b^{\dagger}_{\lambda \boldsymbol{k}} \boldsymbol{u}^{*}_{\lambda \boldsymbol{k}}(\boldsymbol{r})] \tag{23}$$

with field

$$\boldsymbol{u}_{\lambda \boldsymbol{k}} = \frac{e^{i\boldsymbol{k}\cdot\boldsymbol{r}}}{L^{3/2}} \boldsymbol{e}_{\lambda \boldsymbol{k}} \tag{24}$$

in wave equation

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \nabla^2 \right) \hat{\boldsymbol{A}} = 0 \quad \rightarrow \quad \frac{\partial^2}{\partial t^2} b_{\lambda \boldsymbol{k}} = -\omega_k^2 b_{\lambda \boldsymbol{k}} \tag{25}$$

$$\mathcal{E}(\boldsymbol{q},t) = -\frac{1}{c} \frac{\partial A(\boldsymbol{q},t)}{\partial t} = i\sqrt{2\pi\hbar\omega_q}(b_{\boldsymbol{q}} - h.c.) \tag{26}$$

and $\omega_q = \frac{cq}{\sqrt{\epsilon_0}}$

Interaction Hamiltonian

$$\mathcal{H}_I = i\hbar \sum_{\boldsymbol{q}} g_{\nu q}(B_{\boldsymbol{q}} + B^\dagger_{-\boldsymbol{q}})(b_{-\boldsymbol{q}} - b^\dagger_{\boldsymbol{q}}) \tag{27}$$

$$\hbar g_{\nu\boldsymbol{q}} = d_{cv}\psi^*_\nu(\boldsymbol{r} = 0)\sqrt{\pi\hbar\omega_q/2} \tag{28}$$

diagonalized by Hopfield transformation

$$p_{i,\boldsymbol{q}} = u_{i,1,\boldsymbol{q}}B_{\boldsymbol{q}} + u_{i,2,\boldsymbol{q}}b_{\boldsymbol{q}} + u_{i,3,\boldsymbol{q}}B^\dagger_{-\boldsymbol{q}} + u_{i,4,\boldsymbol{q}}b^\dagger_{-\boldsymbol{q}} \tag{29}$$

only resonant terms

$$\mathcal{H}_I = -i\hbar \sum_{\boldsymbol{q},\nu} g_{\nu\boldsymbol{q}}(B^{\dagger}_{\nu\boldsymbol{q}} b_{\boldsymbol{q}} - h.c.) \tag{30}$$

Exciton-Photon Hamiltonian

$$\mathcal{H}_{xp} = \hbar \sum_{\boldsymbol{q}} \left[\sum_{\nu} e_{\upsilon q} B^{\dagger}_{\nu\boldsymbol{q}} B_{\nu\boldsymbol{q}} + \omega_q b^{\dagger}_{\boldsymbol{q}} b_{\boldsymbol{q}} - i \sum_{\nu} g_{\nu\boldsymbol{q}}(B^{\dagger}_{\nu\boldsymbol{q}} b_{\boldsymbol{q}} - h.c.) \right] \tag{31}$$

Bogoliubov transformation

$$p_{\boldsymbol{q}} = u_{\boldsymbol{q}} B_{\boldsymbol{q}} + v_{\boldsymbol{q}} b_{\boldsymbol{q}} \tag{32}$$

$$[p_{\boldsymbol{q}}, p_{\boldsymbol{q}}^{\dagger}] = |u_{\boldsymbol{q}}|^2 + |v_{\boldsymbol{q}}|^2 = 1 \tag{33}$$

Polariton Hamiltonian

$$\mathcal{H}_p = \hbar \sum_{\boldsymbol{q}} \Omega_q p_{\boldsymbol{q}}^{\dagger} p_{\boldsymbol{q}} \tag{34}$$

evaluate the commutator $[p, H_p]/\hbar = [p, H_{xp}]/\hbar$

$$\begin{aligned} \Omega_{\boldsymbol{q}} p_{\boldsymbol{q}} &= \Omega_{\boldsymbol{q}}(u_{\boldsymbol{q}} B_{\boldsymbol{q}} + v_{\boldsymbol{q}} b_{\boldsymbol{q}}) \\ &= u_{\boldsymbol{q}}(e_{\boldsymbol{q}} B_{\boldsymbol{q}} - i g_{\boldsymbol{q}} b_{\boldsymbol{q}}) + v_{\boldsymbol{q}}(\omega\ b_{\boldsymbol{q}} + i g_{\boldsymbol{q}} B_{\boldsymbol{q}}) \end{aligned} \tag{35}$$

$B_{\boldsymbol{q}}$ and $b_{\boldsymbol{q}}$ have to be zero independently

$$0 = (\Omega_{\boldsymbol{q}} - e_{\boldsymbol{q}}) u_{\boldsymbol{q}} + i g_{\boldsymbol{q}} v_{\boldsymbol{q}} \tag{36}$$

$$0 = -i g_{\boldsymbol{q}} u_{\boldsymbol{q}} + (\Omega_{\boldsymbol{q}} - \omega_{\boldsymbol{q}}) v_{\boldsymbol{q}} \tag{37}$$

solution of linear system of equations with $\det = 0$

Polariton Dispersion

$$\Omega_{\boldsymbol{q},1,2} = \frac{1}{2}(e_{\boldsymbol{q}} + \omega_{\boldsymbol{q}}) \pm \frac{1}{2}\sqrt{(e_{\boldsymbol{q}} - \omega_{\boldsymbol{q}})^2 + 4 g_{\boldsymbol{q}}^2} \tag{38}$$

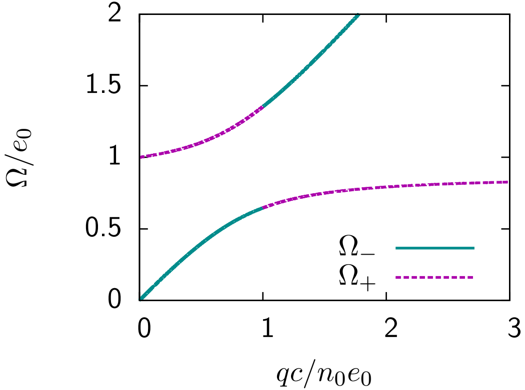


Figure: Polariton dispersion from Hamiltonian theory with $e_{\boldsymbol{q}} = e_0$, $4g_{\boldsymbol{q}}^2 = \Delta \cdot \omega_{\boldsymbol{q}}$ and $\Delta/e_0 = 0.2$

Hamiltonian Theory of Polaritons
Photon and Exciton Contributions

$$p_{\boldsymbol{q}} = u_{\boldsymbol{q}} B_{\boldsymbol{q}} + v_{\boldsymbol{q}} b_{\boldsymbol{q}}$$

$$u_{\boldsymbol{q},1} = \sqrt{\frac{\Omega_{\boldsymbol{q},1} - \omega_{\boldsymbol{q}}}{2\Omega_{\boldsymbol{q},1} - e_{\boldsymbol{q}} - \omega_{\boldsymbol{q}}}}$$

and

$$v_{\boldsymbol{q},1} = i\sqrt{\frac{\Omega_{\boldsymbol{q},1} - e_{\boldsymbol{q}}}{2\Omega_{\boldsymbol{q},1} - e_{\boldsymbol{q}} - \omega_{\boldsymbol{q}}}}$$

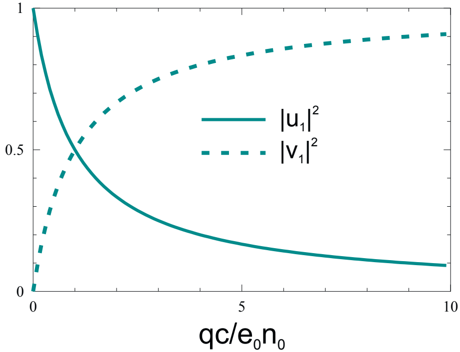


Figure: Dispersion of $|u_i|^2$ and $|v_i|^2$ for the upper polariton branch[Haug]

Microcavity Polaritons

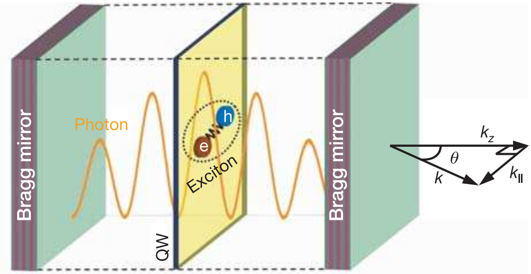


Figure: Microcavity with 2D Quantum well inside [Kasprzak]

with eigenfrequencies

$$\omega_{0,k_{\parallel}} = \frac{c}{n}\sqrt{\frac{\pi^2}{L^2} + k_{\parallel}^2}$$

$$L = \frac{\lambda}{2} \quad \rightarrow \quad \boldsymbol{E}(\boldsymbol{r},\omega) = \boldsymbol{E}_0 \sin\left(\frac{\pi z}{L}\right) e^{i\boldsymbol{k}_{\parallel}\cdot\boldsymbol{r}} \tag{39}$$

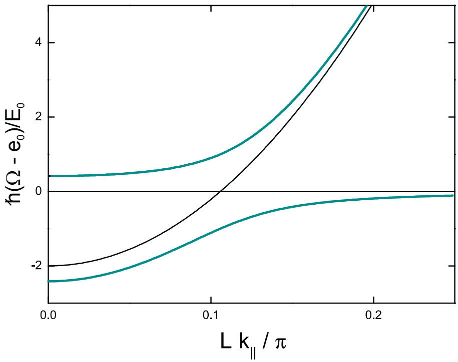
Microcavity Polaritons

$$\mathcal{H} = \hbar \sum_{\boldsymbol{k}_{\parallel}} \Big[e_{0,k_{\parallel}} B^{\dagger}_{0,\boldsymbol{k}_{\parallel}} B_{0,\boldsymbol{k}_{\parallel}} + \omega_{0,k_{\parallel}} b^{\dagger}_{0,\boldsymbol{k}_{\parallel}} b_{0,\boldsymbol{k}_{\parallel}} - i g_{k_{\parallel}} (B^{\dagger}_{0,\boldsymbol{k}_{\parallel}} b_{0,\boldsymbol{k}_{\parallel}} - \text{h.c.}) \Big] \tag{40}$$

$$\hbar g_{k_{\parallel}} = d_{cv} \psi_0^{2D}(r=0) \sqrt{\pi \hbar \omega_{0,k_{\parallel}}} \tag{41}$$

Dispersion Relation inside Quantum Well

$$\Omega_{k_{\parallel},1,2} = \frac{1}{2}(e_{0,k_{\parallel}} + \omega_{0,k_{\parallel}}) \pm \frac{1}{2}\sqrt{(e_{0,k_{\parallel}} - \omega_{0,k_{\parallel}})^2 + 4 g_{k_{\parallel}}^2} \tag{42}$$



tunable coupling

$$g_{k_\parallel} \propto \sqrt{\omega_{0,k_\parallel}}$$

$$\omega_{0,k_\parallel} = \frac{c}{n}\sqrt{\frac{\pi^2}{L^2} + k_\parallel^2}$$

Figure: Microcavity Polariton spectrum [Haug]

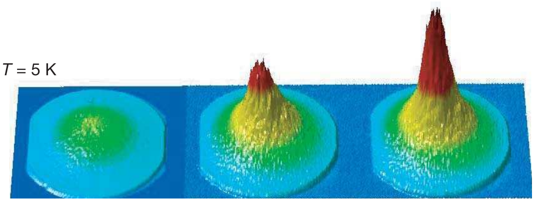


Figure: Far-field polariton emission [Kasprzak]

- ground state emission
- spontaneous polarization
- coherence length

But: interaction, thus no ideal Bose-gas

Application II
Generation of Entangled Photons

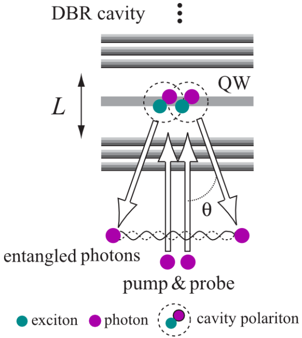


Figure: Entangled-photon generation from cavity bipolariton [Oka]

other applications:

- use long lifetime
- mediation of e.g. spin coupling

Summary

- Interaction between photons and excitons in semiconductor
- Coupling introduces joint dispersion with gab described by quasiparticles
- Dielectric function
- Microscopic theory
  - Properties of quasiparticles
  - Distinguish photon and exciton contributions
  - Microcavity polaritons
  - Left out damping (phonons)
- Applications
  - BEC in microcavities
  - Generation of entangled photons

References

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Longitudinal-transverse Splitting

Table: LT Splitting for 1s exciton-polaritons at $T \leq 4.2$ K

Material	$\hbar\Delta_{LT}$ / meV
GaAs	≈ 0.1
CdTe	≈ 0.4
ZnSe	≈ 1.3
CdS	$\approx 2(A)$
CuCl	$5.4 - 5.7(Z_3)$

$$\Delta_{LT} \propto \frac{|d_{vc}|^2}{a_0^3} \tag{43}$$

[Schäfer and Wegener. *Semiconductor optics and transport phenomena*. Springer 2002]