Seminar on the electronic properties of graphene

P-N Junctions in Graphene

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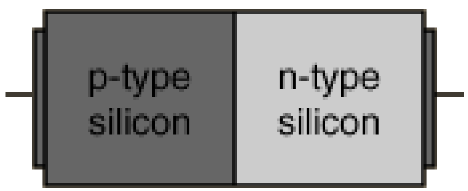
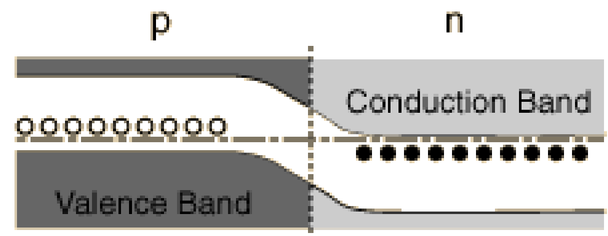
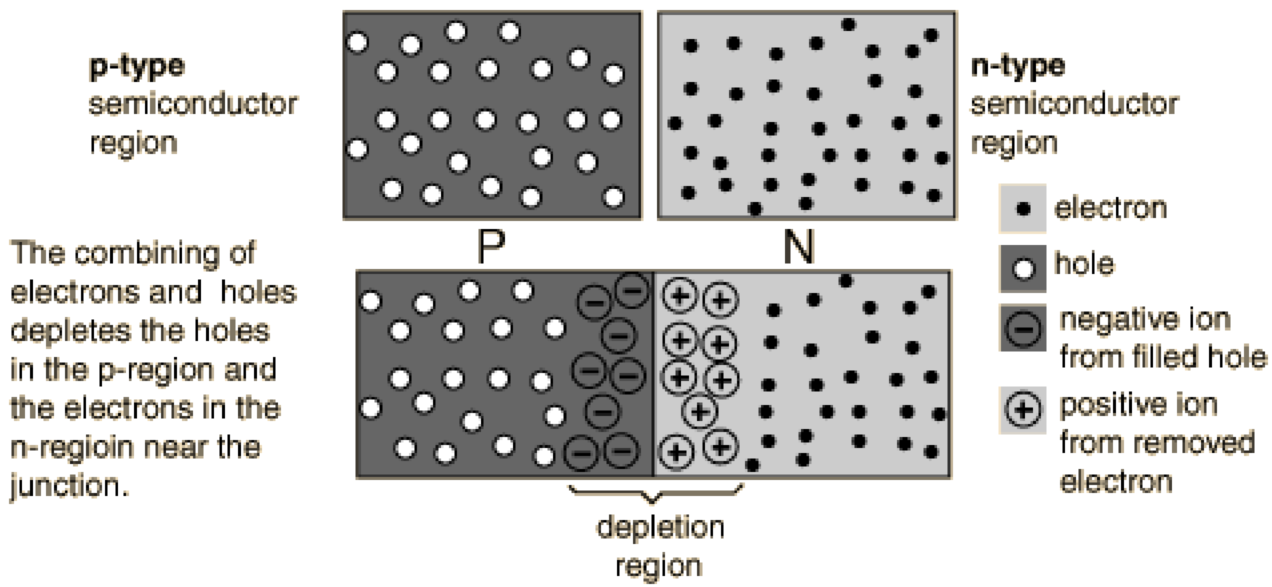
P-N junction in semiconductors



p-n junction

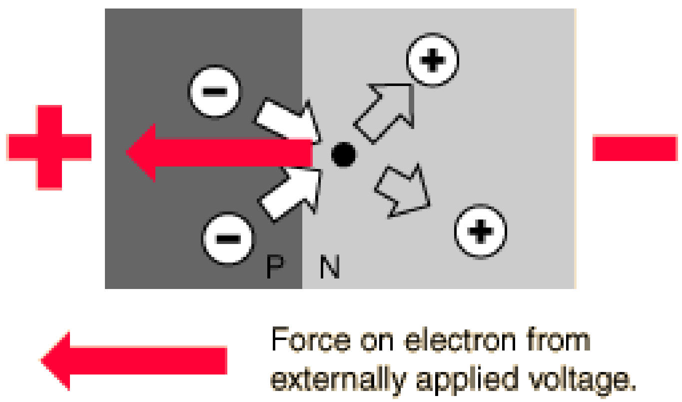


Energy bands at equilibrium



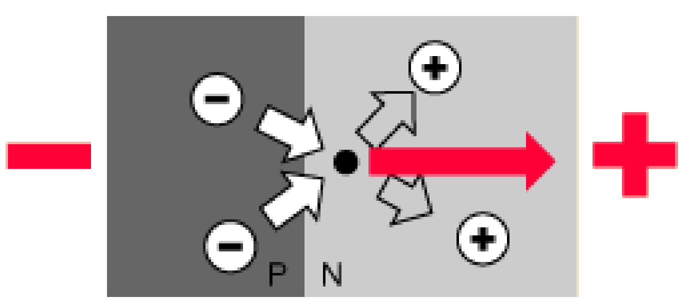
Applying a bias to a p-n junction

forward bias

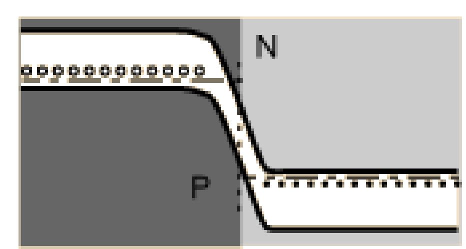


band structure for a forward bias

reverse bias



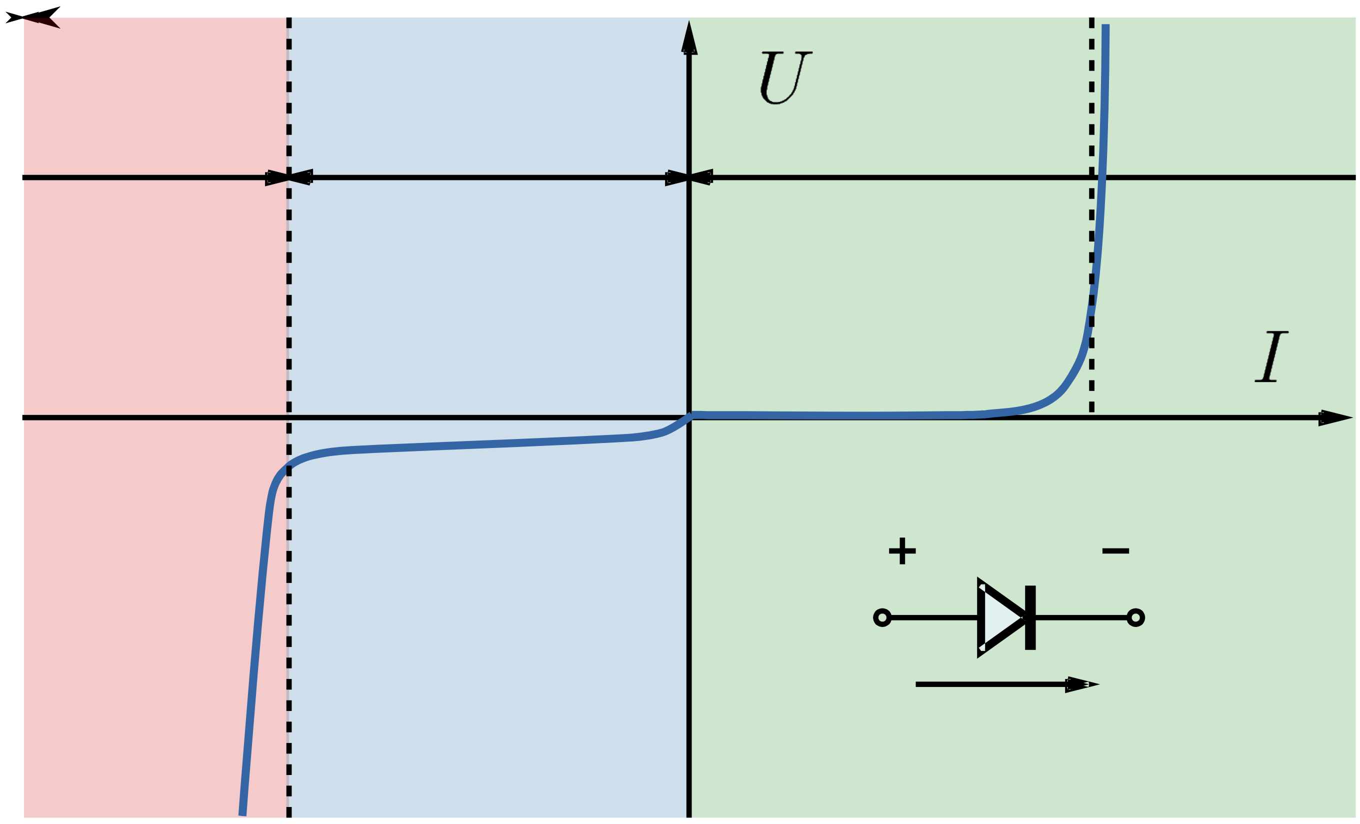
band structure for a forward bias



http://hyperphysics.phy-astr.gsu.edu/hbase/solids/pnjun.html

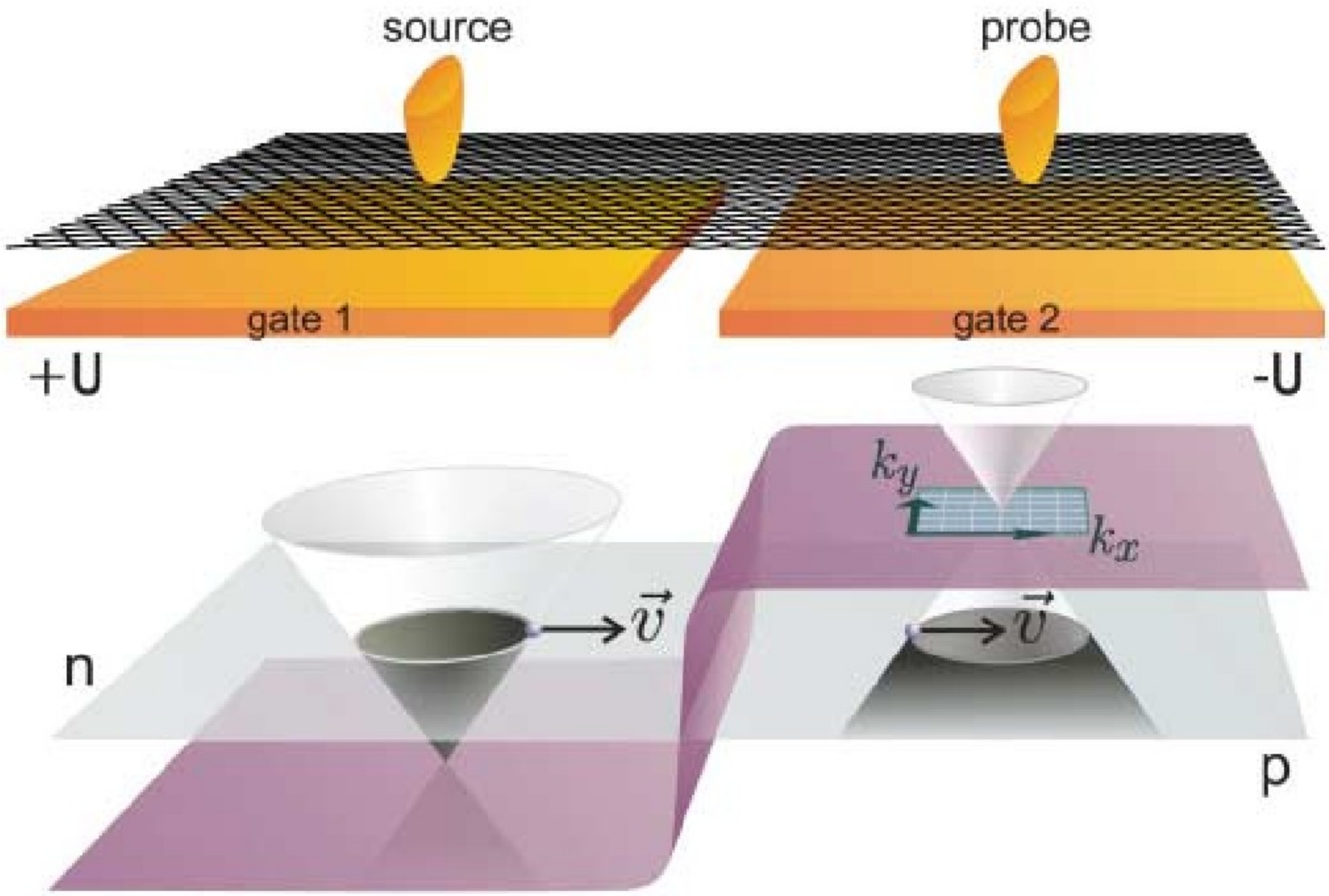
P-N junctions and solid state electronics

- diode



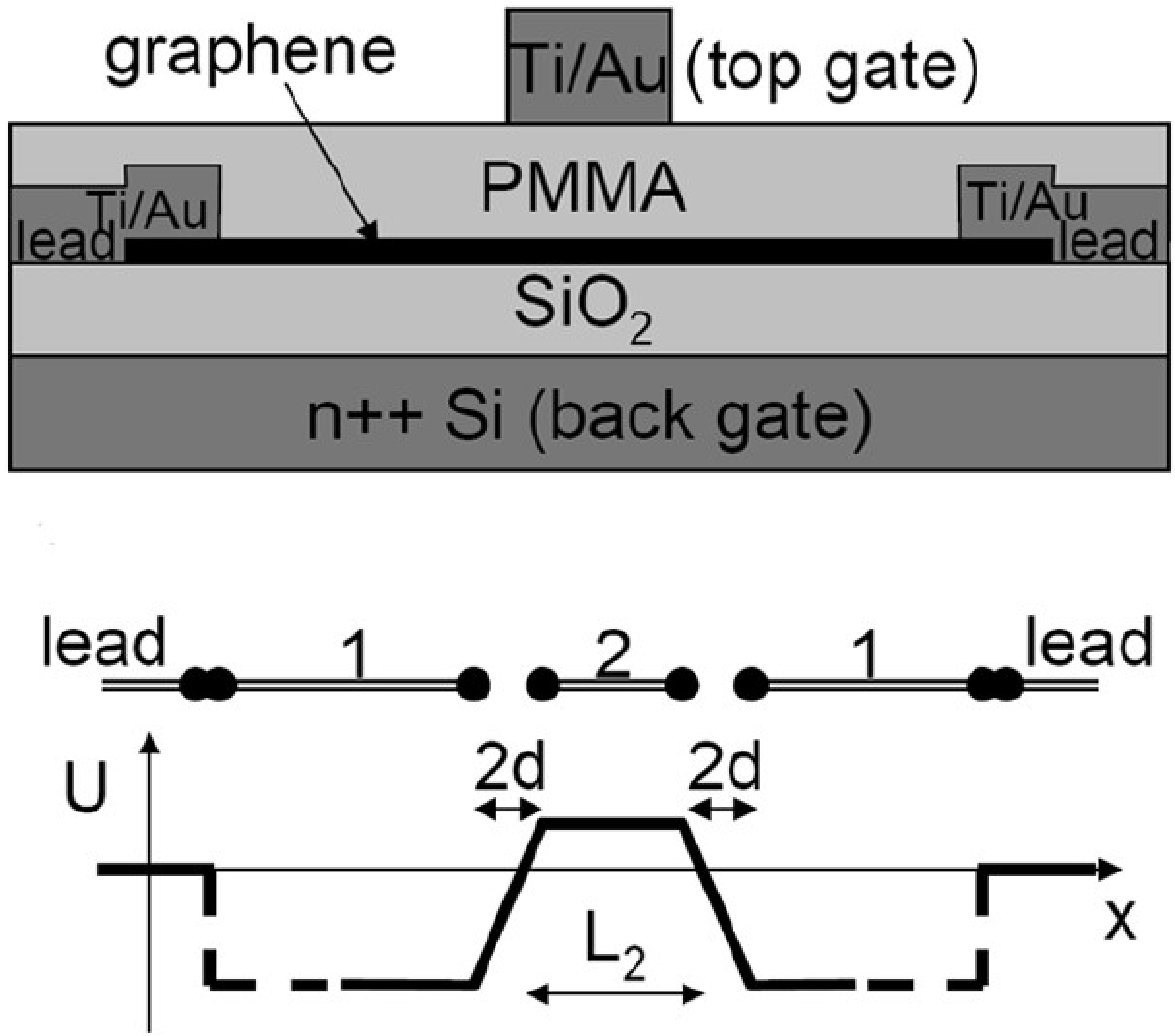
http://en.wikipedia.org/wiki/Diode

Realization of a P-N Junction in graphene



Cheianov *et al.*, *Science 315*, 1252

Realization of a P-N junction in graphene



Huard *et al.*, *PRL 98*, 236803

Transport in a P-N junction for B $= 0$

Shytov *et al.*, arXiv:0708.3081
Cheianov *et al.*, *PRB 74*, 041403(R)

What is the probability T that a Dirac fermion tunnels through the P-N junction ?

- Massless Dirac Hamiltonian with a uniform electric field :

$$\mathcal{H} = \xi \sigma \cdot \mathbf{p} + e\varphi(\mathbf{x})$$

- Electrostatic potential : $\varphi(\mathbf{x}) = -Ey$

- General form of the eigenstates : $\psi(t,\, \mathbf{x}) = \mathrm{e}^{-\mathrm{i}\varepsilon t + \mathrm{i} p_x x} \psi(y)$

Transport in a P-N junction for B $= 0$

- Schrödinger equation in momentum representation :

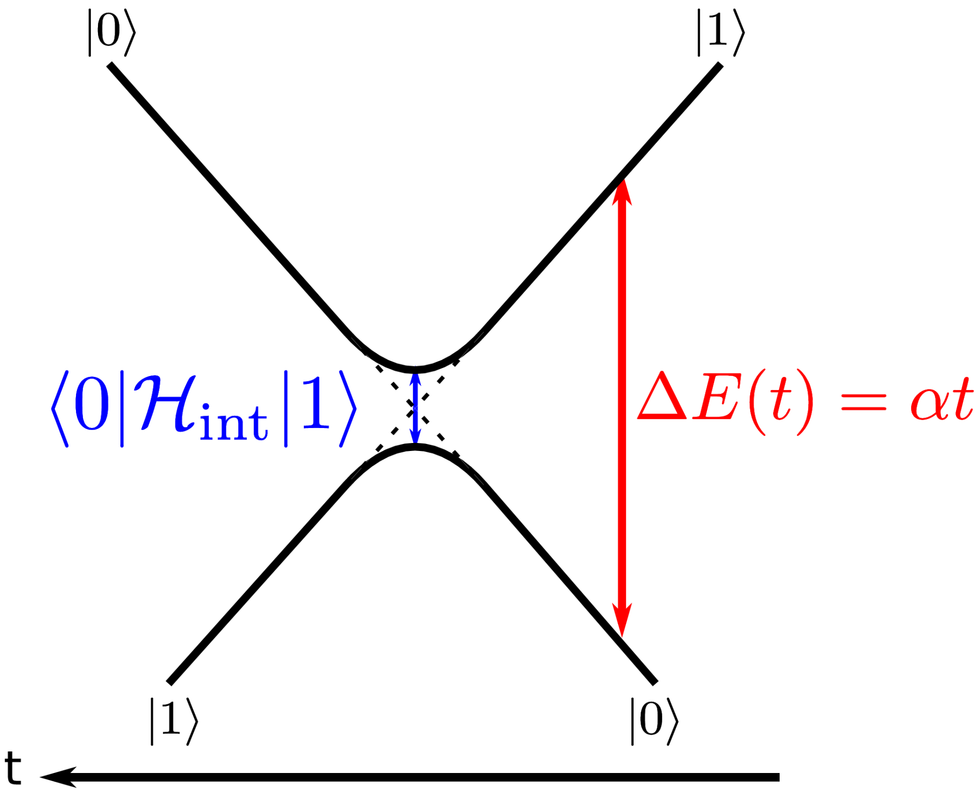
$$-\mathrm{i}eE\frac{\mathrm{d}\psi}{\mathrm{d}p_y} = (\varepsilon - v_\mathrm{F}(\sigma_x p_x + \sigma_y p_y))\psi$$

- Mapping to the Landau-Zener-Stückelberg problem :

$$\begin{cases} eE \rightarrow \hbar \\ p_y \rightarrow t \\ \varepsilon - v_\mathrm{F}(\sigma_x p_x + \sigma_y p_y)) \rightarrow \tilde{\mathcal{H}} \end{cases}$$

The Landau-Zener-Stückelberg problem

- Two level system with Hamiltonian : $\mathcal{H} = \mathcal{H}_0(t) + \mathcal{H}_{\mathrm{int}}$



L. D. Landau, Phys. Z. 2, 46 (1932)
C. Zener, Proc. R. Soc. London A 137 696 (1932)
E. C. G. Stückelberg, Helv. Phys. Acta 5, 370 (1932)

- Result : $P_{0\to 1} = 1 - P_{0\to 0} = 1 - \exp\left(-2\pi \frac{|\langle 0|\mathcal{H}_{\mathrm{int}}|1\rangle|^2}{\alpha\hbar}\right)$

Transport in a P-N junction for B $= 0$

- Mapping of the Landau-Zener-Stückelberg result to the Dirac problem

$$\begin{cases} P_{0\to 1} \to R \\ P_{0\to 0} \to T \end{cases}$$

- The transmission coefficient is

$$T(p_x) = \mathrm{e}^{\frac{-\pi \hbar v_{\mathrm{F}} p_x^2}{|eE|}}$$

Transport in a P-N junction for B $\neq 0$

- We write the Dirac equation in the covariant form

$$\gamma^{\mu}(p_{\mu} - a_{\mu})\psi = 0$$

$$\begin{cases} \gamma^{\mu} = (\sigma_3,\, -\mathrm{i}\sigma_2,\, -\mathrm{i}\sigma_1) \\ x_{\mu} = (v_{\mathrm{F}}t,\, x_1,\, x_2) \\ p_{\mu} = \hbar(\frac{\mathrm{i}}{v_{\mathrm{F}}}\partial_t,\, -\mathrm{i}\partial_x,\, -\mathrm{i}\partial_y) \\ a_{\mu} = (-\frac{e}{v_{\mathrm{F}}}Ey,\, -\frac{e}{c}Bx,\, 0) \end{cases}$$

- The Dirac equation is Lorentz invariant !

Is there a frame where B = 0 ?

A little bit of special relativity

- Lorentz invariants $\begin{cases} F_{\mu\nu}F^{\mu\nu} = 2(c^2B^2 - E^2) \\ \varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} = -4c(\mathbf{E}\cdot\mathbf{B}) \end{cases}$

In our problem $\mathbf{E}\cdot\mathbf{B} = 0 = \mathbf{E}'\cdot\mathbf{B}' \Rightarrow \mathbf{B}' = 0$ is possible !

- Transformation of the electromagnetic field

$$\begin{cases} \mathbf{E}'_{\parallel} = \mathbf{E}_{\parallel} \\ \mathbf{E}'_{\perp} = \gamma(\mathbf{E}_{\perp} + c\beta \wedge \mathbf{B}) \end{cases}$$

$$\begin{cases} \mathbf{B}'_{\parallel} = \mathbf{B}_{\parallel} \\ \mathbf{B}'_{\perp} = \gamma(\mathbf{B}_{\perp} - \frac{1}{c}\beta \wedge \mathbf{E}) \end{cases}$$

In our case :

$$E'_y = \frac{E_y}{\gamma}$$

$$\beta_x = c\frac{B_z}{E}$$

Transport in a P-N junction for B $\neq 0$

- Hum... we are in a solid state device not in the vacuum : $c \longrightarrow \frac{v_{\mathrm{F}}}{c}$
- Is the transformation always possible ?

$$0 \leq \beta_x = \frac{v_{\mathrm{F}}}{c}\frac{B}{E} \leq 1 \quad \Rightarrow \quad B \leq \frac{c}{v_{\mathrm{F}}} E \,(\equiv B^*)$$

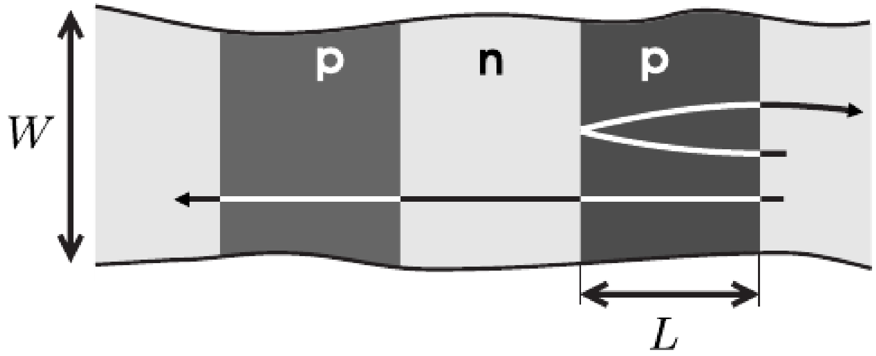
- In the new frame the transmission coefficient is : $T' = \mathrm{e}^{\frac{-\pi \hbar v_{\mathrm{F}} p_1'^2}{|eE'|}}$

- T is a scalar with respect to a Lorentz transformation.

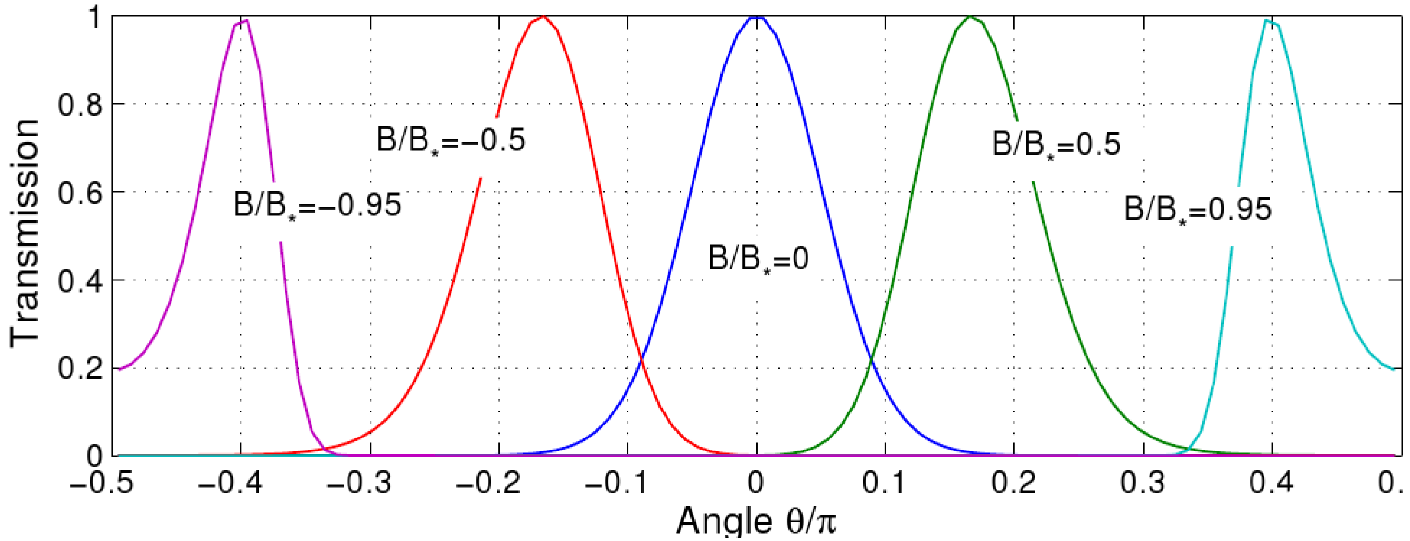
$$T(p_x, B) = \mathrm{e}^{-\pi \frac{1}{\left(1 - \frac{v_{\mathrm{F}}}{c}\frac{B}{E}\right)^{\frac{3}{2}}} \frac{\hbar v_{\mathrm{F}}}{|eE|}\left(p_x + \frac{\varepsilon}{c}\frac{B}{E}\right)^2}$$

Transport in a P-N junction for B$\neq 0$

- Suppression of the tunneling with a B field possible except for $p_x = -\beta\varepsilon$



$$T(\theta) = \mathrm{e}^{-\alpha\gamma^3(\sin(\theta) - \frac{B}{B*})^2}$$



Conductance in a P-N junction

- Conductance is given by Landauer formula

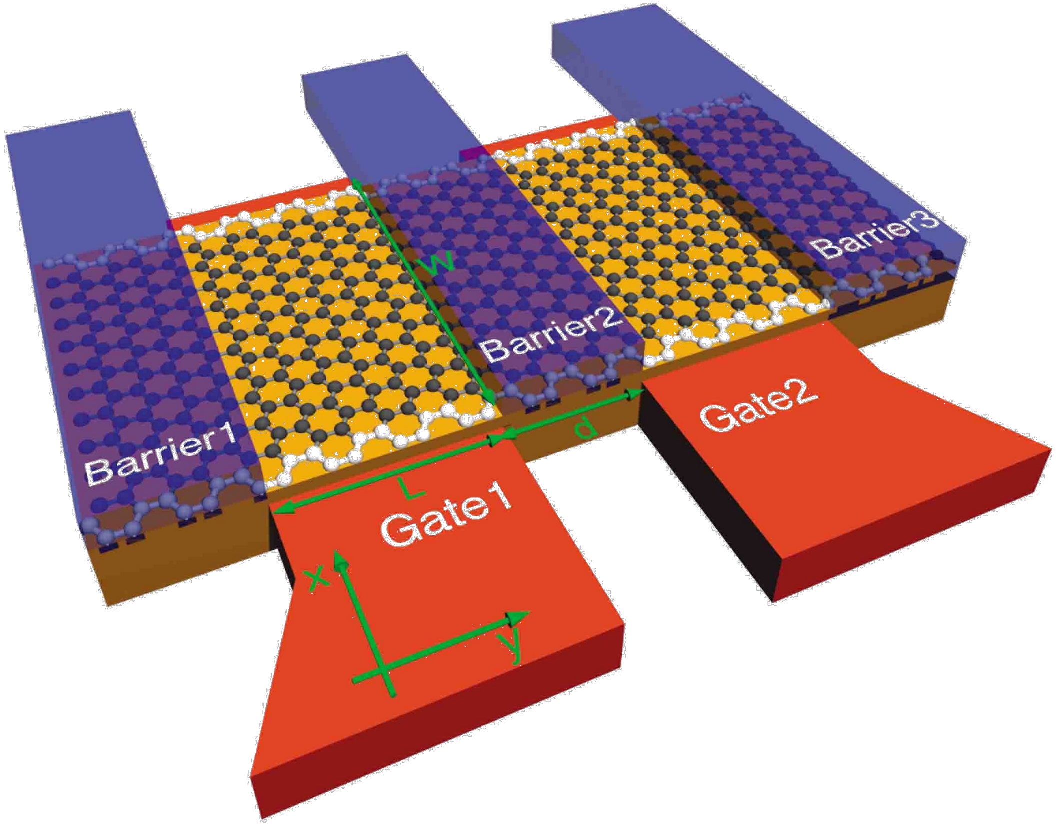
$$G = \frac{e^2}{h} \sum_{-k_\mathrm{F} < p_x < k_\mathrm{F}} T(p_x) = \frac{we^2}{2\pi h} \int_{-k_\mathrm{F}}^{k_\mathrm{F}} \mathrm{d}p_x \, T(p_x)$$

- We can extend the integration range to $\pm\infty$ (T vanishes extremely fast)
- We assume $\frac{\lambda_\mathrm{F}}{2\pi} \ll d \ll w$

$$G(B \leq B^*) = \frac{e^2}{2\pi h} \frac{w}{d} \left(1 - \left(\frac{B}{B^*}\right)^2\right)^{\frac{3}{4}}$$

Graphene P-N junctions in spintronics

- Graphene quantum dots



Trauzettel *et al.*, *Nature Physics 3*, 192