Tutorial: Quantum Noise (I)

Tobias Brandes (Institut für Theoretische Physik, TU Berlin)

- Introduction.
- Analogy with Quantum Optics.
- Master Equations. Counting Variables.
- Discussion.

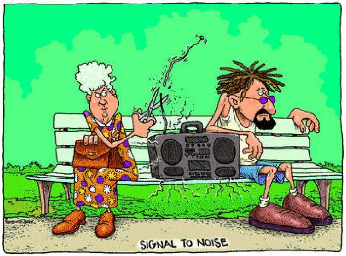
Condensed-matter physics

The noise is the signal

Rolf Landauer

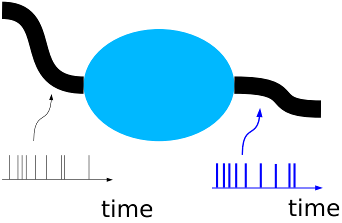
Noise is not only a hindrance to signal detection. Advances in measurement techniques mean that it can now be regulate and reduce the fluctuations.

The new investigations[1–4] were prompted by precise measurements of noise at quan-



'Whether noise is a nuisance or a signal may depend on whom you ask' C. Beenakker, C. Schönenberger, Physics Today (2003).

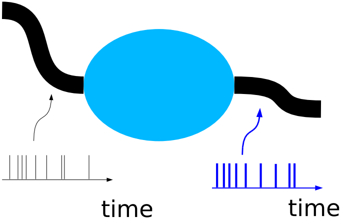
Basic Idea



(Electronic) Transport

- Current comes in individual charges, $I = -e \sum_{i=1}^{\infty} \delta(t - t_i)$.
- Measure times t_i in leads $\rightsquigarrow$ system properties.

Basic Idea



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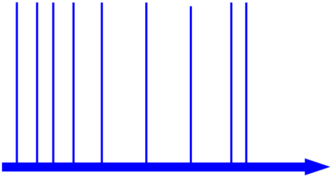

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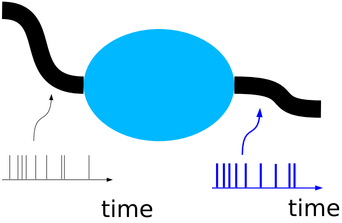

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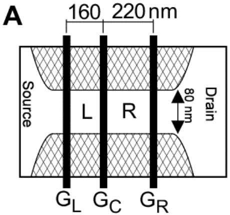
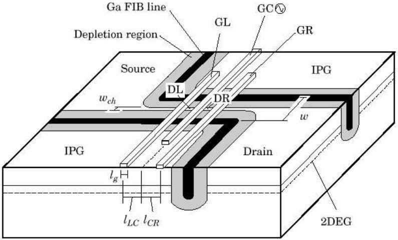
'...daß die Gesamtheit der Schwingungsfrequenzen und der die Intensität der Linien bestimmenden Größen ... als ein vollwertiger Ersatz der Bahnen gelten könnte.'

W. Heisenberg, 'Der Teil und das Ganze'

Some Real Systems





Double Quantum Dots, T. Fujisawa *et al.*

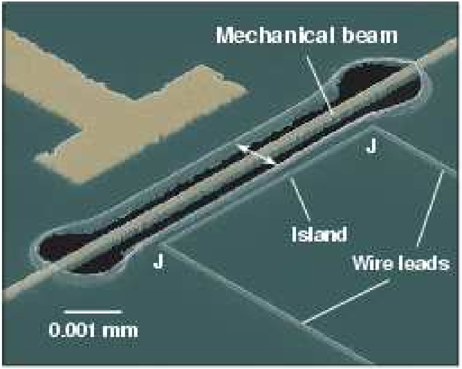
Some Real Systems

PERSPECTIVES

PHYSICS

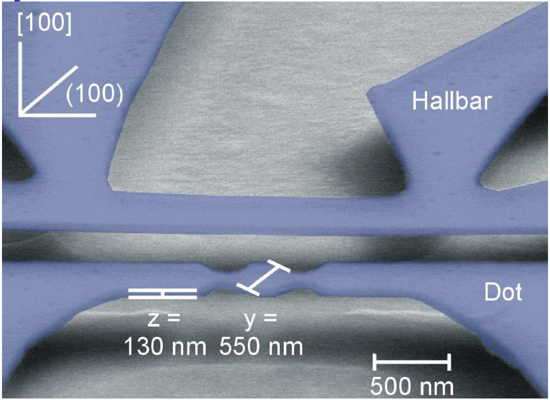
Nanomechanical Quantum Limits

Miles Blencowe



M. D. LaHaye, O. Buu, B. Camarota, K.Schwab, Science 304, 74(2004)

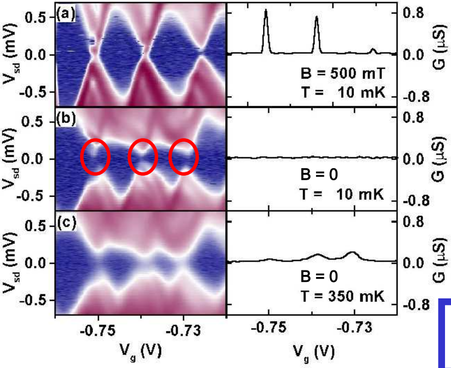
Some Real Systems

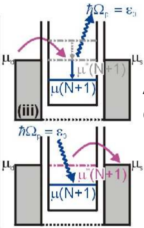


E. Weig *et al.*

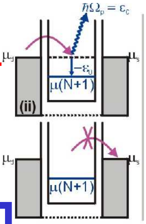
Some Real Systems

Quanten-Dot inPhonon -Kavität





Aufhebung der Phonon-Blockade



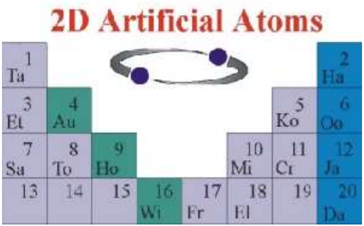
\`Phonon-Blockade'

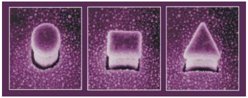
Phononen-Energie ~100µeV

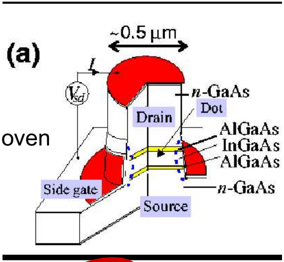
Vgl. Transport durch C_{60} (Park *etal* 00).

Some Real Systems

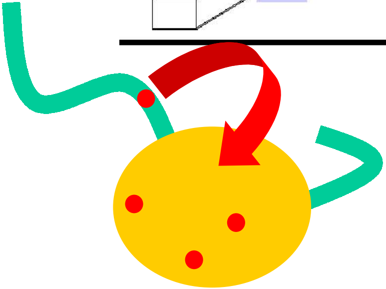
Vertikale Quantenpunkte







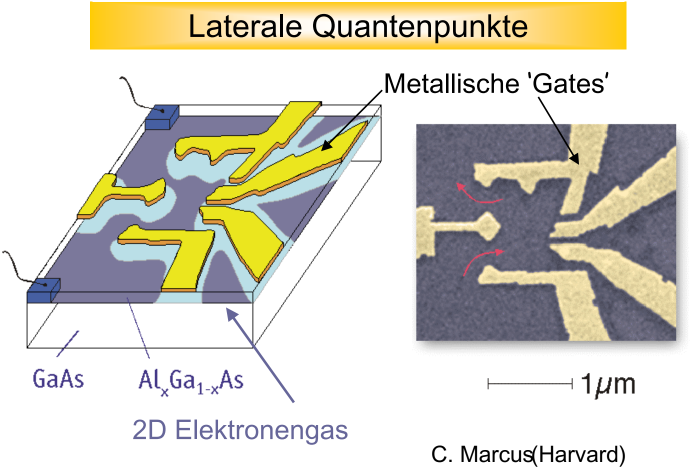
L. Kouwenhoven (Delft)



Ladungsenergie $U=e^2/2C$

→ Coulomb-Blockade

Some Real Systems



Some Real Systems

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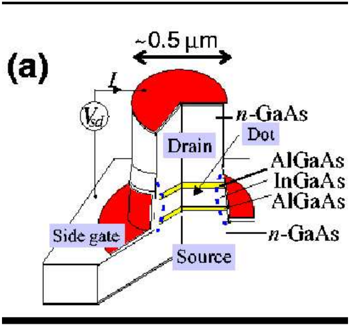
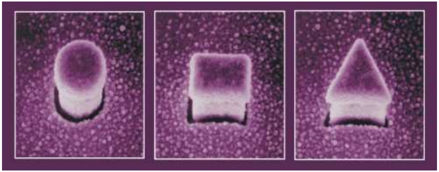


Vertikale Säulen



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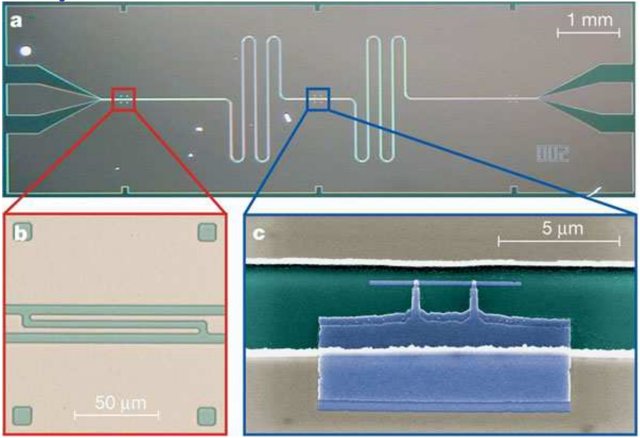
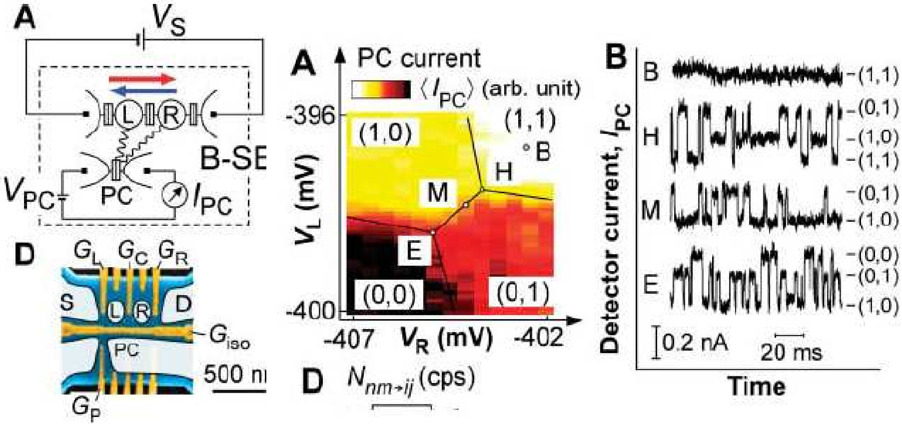


Yale group.

Counting of Single Electrons

Transport Through Double Quantum Dots (T. Fujisawa, T. Hayashi, R. Tomita, and Y. Hirayama, Science **312**, 1634 (2006))



Quantum Optics

M. B. Plenio and P. L. Knight: Quantum-jur

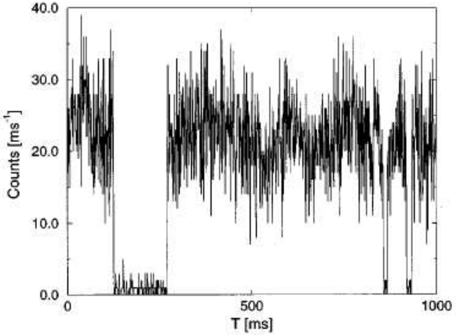


FIG. 2. Recorded resonance fluorescence signal exhibiting quantum jumps from a laser-excited $^{24}Mg^{+}$ ion (Thompson,

Quantum Optics

Counting photons

- Counting photons, but...
- ...'the eternal question: what is a photon'.
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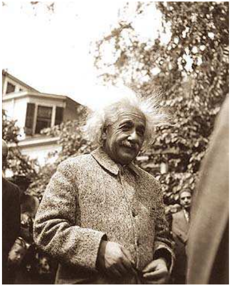


Einstein 1951: '...these days every fool pretends to know what a *photon* is. I have been thinking about this for the whole of my life, and I haven't found the answer'.

Quantum Optics

Photon counting: some issues

- Count photo-electrons instead of photons.
- Counting statistics: correct theory for

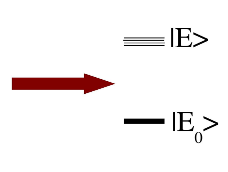
$$p_n(t, t+T) \quad \text{probability for } n \text{ photo-electrons in } [t, t+T).$$

- Detector back-action. System-bath problem 'with two baths'.
- ... no entirely trivial!

Semiclassical theory for $p_n(t, t+T)$: Mandel formula

Photodetector model: ionize single atom

- Classical electromagnetic field, vector potential $\mathbf{A}(\mathbf{r})e^{-i\omega t} + \mathbf{A}^*(\mathbf{r})e^{i\omega t}$.



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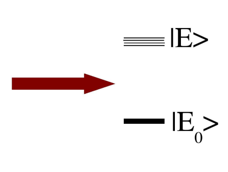

Probability $p_1(t, t+\Delta t)$ of one count: Fermi's Golden Rule

$$p_1(t, t+\Delta t) = \int_0^\infty dE \nu(E) \left| \langle E | \frac{e}{m} \mathbf{p}\mathbf{A}(\mathbf{r}) | E_0 \rangle \right|^2 D_{\Delta t}(E - E_0 - \omega)$$
$$= \eta I(\mathbf{r})\Delta t, \quad I(\mathbf{r}) = |A(\mathbf{r})|^2 (\text{intensity}).$$

- $D_{\Delta t}(\varepsilon) \equiv \left([\sin \frac{1}{2}\varepsilon \Delta t]/[\frac{1}{2}\varepsilon]\right)^2$, $\Delta t \to 0$. Polarisation $\mathbf{A}(\mathbf{r}) = \vec{\varepsilon} A(\mathbf{r})$.

Mandel formula: many counts

How to obtain probability of n transitions $p_n(t, t+T)$

- Short-time probability $p_1(t, t+\Delta t) = \eta I(\mathbf{r})\Delta t$ for *single* electron transition ($\eta I(\mathbf{r})$ transition rate).
- Long-time probability of n transitions $p_n(t, t+T) \leftrightarrow n$ electrons.

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- Individual transitions are statistically independent...
- $\rightsquigarrow$ Poisson distribution.
- Characterized by average $\bar{n}$ only $\rightsquigarrow$

$$p_n(t, t+T) = \frac{\bar{n}^n}{n!} e^{-\bar{n}}, \quad \bar{n} = \eta I(\mathbf{r}) T.$$

Mandel formula: many counts

- Markovian master equation for probabilities.

$$\begin{aligned} p_n(t+dt) &= p_n(t) \times [1 - \eta I(\mathbf{r})dt] + p_{n-1}(t) \times \eta I(\mathbf{r})dt \\ \rightsquigarrow \frac{d}{dt} p_n(t) &= \eta I(\mathbf{r})[p_{n-1}(t) - p_n(t)]. \end{aligned}$$

- Generating function

$$G(\chi, t) \equiv \sum_{n=0}^{\infty} e^{i\chi n} p_n(t) \rightsquigarrow \partial_t G(\chi, t) = \eta I(\mathbf{r})(e^{i\chi} - 1) G(\chi, t).$$

- Solve with $p_0(0) = 1,\ p_n(0) = 0,\ n > 0,\ G(\chi, 0) = 1$.

$$G(\chi, t) = \exp[\eta I(\mathbf{r}) t (e^{i\chi} - 1)] = \sum_{n=0}^{\infty} e^{i\chi n} \frac{\bar{n}^n}{n!} e^{-\bar{n}}, \quad \bar{n} = \eta I(\mathbf{r}) t.$$

Mandel formula: many counts

SUMMARY so far:

- Classical photo-electron counting formula (Mandel formula)

$$p_n(t, t+T) = \frac{\bar{n}^n}{n!} e^{-\bar{n}}, \quad \bar{n} = \eta I(\mathbf{r}) T.$$

- Poisson process.
- Generating function $G(s,t) \equiv \sum_{n=0}^{\infty} s^n p_n(t) = \exp[\eta I(\mathbf{r}) t (s-1)]$.

Scully-Lamb photodetector

M. Scully, W. Lamb Jr., Phys. Rev. **179**, 368 (1969)

- 'Photon statistics' means (reduced) density operator $\rho(t)$ of a light field (more generally: boson field).

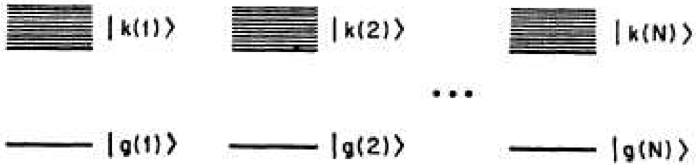
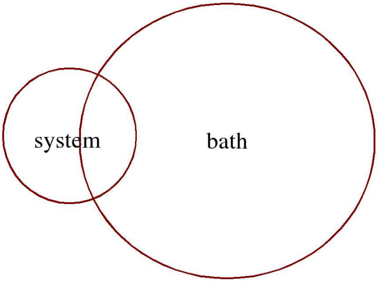


FIG. 1. Pictorial representation of photodetector consisting of N-independent atoms. Each atom in detector has a ground state $|g\rangle$ and continuum of excited states $|k\rangle$. Atoms are labeled by indexing atomic state with particle number, e.g., $|k(m)\rangle$.

System-bath theory

Divide 'total universe' into system S and bath B,



$$\begin{aligned}\mathcal{H} &= \mathcal{H}_{\mathrm{S}} + \mathcal{H}_{\mathrm{B}} + \mathcal{H}_{\mathrm{SB}} \\ &\equiv \mathcal{H}_0 + V, \quad V \equiv \mathcal{H}_{\mathrm{SB}}.\end{aligned}$$

Total density matrix $\chi(t)$ obeys the Liouville-von-Neumann equation

$$\frac{d}{dt}\chi(t) = -i[\mathcal{H}, \chi(t)].$$

Master equation

- Effective density matrix of the system $\rho(t) \equiv \mathrm{Tr}_B[\chi(t)]$.
- Interaction picture with respect to H_0,

$$\frac{d}{dt}\tilde{\rho}(t) = -i\mathrm{Tr}_B[\tilde{V}(t), \chi(t=0)] - \int_0^t dt' \mathrm{Tr}_B[\tilde{V}(t), [\tilde{V}(t'), \tilde{\chi}(t')]].$$

- Born approximation, $\tilde{\chi}(t') \approx R_0 \otimes \tilde{\rho}(t')$, R_0 bath density matrix.
- System-bath interaction as $V = \sum_k S_k \otimes B_k$,
- Bath correlation functions $C_{kl}(t, t') \equiv \mathrm{Tr}_B \left[\tilde{B}_k(t)\tilde{B}_l(t')R_0\right]$,
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$$\begin{aligned}\frac{d}{dt}\tilde{\rho}(t) &= -\int_0^t dt' \sum_{kl} \Big[C_{kl}(t-t')\left\{\tilde{S}_k(t)\tilde{S}_l(t')\tilde{\rho}(t') - \tilde{S}_l(t')\tilde{\rho}(t')\tilde{S}_k(t)\right\} \\ &+ C_{lk}(t'-t)\left\{\tilde{\rho}(t')\tilde{S}_l(t')\tilde{S}_k(t) - \tilde{S}_k(t)\tilde{\rho}(t')\tilde{S}_l(t')\right\}\Big].\end{aligned}$$

Master equation

Statistische Physik I, Skript unter

http://www.itp.tu-berlin.de/brandes/ag_brandes/lehre/

Scully-Lamb Photodetector

Detector model

- *System*: single photon mode a and N detector single level 'quantum dots' j with one ($|1\rangle_j$) or zero ($|0\rangle_j$) electrons.
- Photon absorption empties dots into *bath*: leads j, $c^{\dagger}_{\alpha j}|vac\rangle$.

$$\mathcal{H}_{\rm SB} = \sum_{\alpha j} \left(V^j_\alpha c^\dagger_{\alpha j} |0\rangle_j \langle 1| a + \bar{V}^j_\alpha c_{\alpha j} |1\rangle_j \langle 0| a^\dagger \right) \equiv \sum_k S_k \otimes B_k. \quad (1.1)$$

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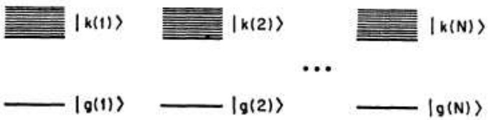

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Master equation: trace out the leads

- Terms $C_{kl}(t-t')\tilde{S}_k(t)\tilde{S}_l(t')\tilde{\rho}(t')$; $C_{kl}(t-t') = \langle \tilde{B}_k(t)\tilde{B}_l(t')\rangle$.
- 'Broadband detection' at all energies, $\sum_\alpha |V^j_\alpha|^2 \delta(\varepsilon - \varepsilon_{\alpha j}) = \nu$.

$$\frac{d}{dt}\tilde{\rho}_t = -\pi\nu \sum_j \left\{ |1\rangle_j\langle 1| a^\dagger a \tilde{\rho}_t + \tilde{\rho}_t a^\dagger a |1\rangle_j \langle 1| - 2|0\rangle_j\langle 1| a \tilde{\rho}_t a^\dagger |1\rangle_j\langle 0| \right\}.$$

Scully-Lamb Photodetector

- Detector states with m excitations,
  $|m;\lambda\rangle \equiv \hat{\Pi}_\lambda |0\rangle_1 ... |0\rangle_m |1\rangle_{m+1} ... |1\rangle_N$ (permutations).
- m-resolved field 'pseudo' density matrix $\tilde{\rho}_t^{(m)} \equiv \sum_\lambda \langle m;\lambda|\tilde{\rho}_t|m;\lambda\rangle$.

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$N \gg m$, $\gamma \equiv 2\pi N\nu \rightsquigarrow$ Master equation

$$\frac{d}{dt}\rho_t^{(m)} = -i[\mathcal{H}_{\rm F}, \rho_t^{(m)}] - \frac{\gamma}{2}\left(a^\dagger a \rho_t^{(m)} + \rho_t^{(m)} a^\dagger a - 2a\rho_t^{(m-1)} a^\dagger\right).$$

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Now counting statistics as $p_m(t) \equiv \mathrm{Tr}\rho_t^{(m)}$!

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Jump super-operator J, $J\rho \equiv \gamma a\rho a^\dagger$, time evolution generator $\mathcal{L}_0$

- Define $\mathcal{L}_0\rho \equiv Y\rho + \rho Y^\dagger$ with $Y \equiv -i\mathcal{H}_{\mathrm{F}} - \frac{\gamma}{2}a^\dagger a$.

$$\boxed{\dot{\rho}_t^{(m)} = \mathcal{L}_0\rho_t^{(m)} + J\rho_t^{(m-1)}.}$$

Summary: counting statistics in Scully-Lamb detector model

m-resolved field density matrix

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Generating operator $\hat{G}(\chi, t)$

- Define $\hat{G}(\chi, t) \equiv \sum_{n=0}^{\infty} e^{i\chi n}\rho_t^{(n)}$, χ: *counting variable.*
- Infinite set of master equations now becomes a single equation,

$$\boxed{\frac{\partial}{\partial t}\hat{G}(\chi, t) = (\mathcal{L}_0 + e^{i\chi}J)\hat{G}(\chi, t).}$$

Results: Photon Distributions

- Coherent state $\rho(0) = |z_0\rangle\langle z_0| \rightsquigarrow$

$$p_m(t) = \frac{(\langle n \rangle \eta_t)^m}{m!} e^{-\langle n \rangle \eta_t}, \quad \eta_t \equiv 1 - e^{-\gamma t}.$$

  - Poisson-distribution.
  - Average $\langle n \rangle \equiv \langle a^\dagger a \rangle = |z_0|^2$.
  - Coincides with semiclassical Mandel formula for $\gamma t \ll 1$.

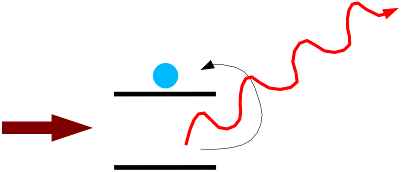
- Fock-state $\rho(0) = |n\rangle\langle n| \rightsquigarrow$

$$p_m(t) = \left(\begin{array}{c} n \\ m \end{array} \right) \eta_t^m (1 - \eta_t)^{n-m}, \quad n \geq m.$$

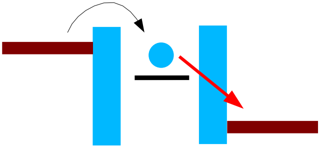
  - Bernoulli-distribution.
  - m successful events (counts), $n - m$ failures (no counts) regardless of order.

Tools: 'n-resolved density matrix'

EXAMPLE: resonance fluorescence - analogy with single electron tunneling



Resonance fluorescence



CB dot, tunneling

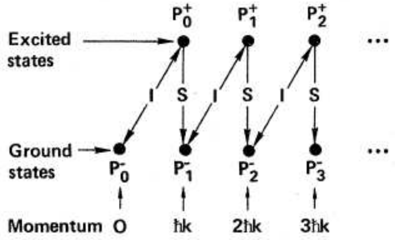
Tools: 'n-resolved density matrix'

Cook's 'counting at the source',R. J. Cook, PRA **23**, 1243 (1981)

n-resolved Master equation for resonance fluorescence of driven TLS

$$\dot{\rho}_t^{(n)} = i\frac{\Omega}{2}[\sigma_+ + \sigma_-, \rho_t^{(n)}] - \beta\left(\sigma_+\sigma_-\rho_t^{(n)} + \rho_t^{(n)}\sigma_+\sigma_- - 2\sigma_-\rho_t^{(n-1)}\sigma_+\right)$$

- Splitting up $\rho_t = \sum_{n=0}^{\infty}\rho_t^{(n)}$, n photon emissions.



- Photodetector theory: count number of spontaneous emission events $\rightsquigarrow$ *n*-resolved Master equation.
- Cook's original idea: momentum transfers between atom and driving field. Count number of discrete displacements $n\hbar k$.

Tools: 'n-resolved density matrix'

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n-resolved Master equation for resonance fluorescence of driven TLS

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- Splitting up $\rho_t = \sum_{n=0}^{\infty} \rho_t^{(n)}$, n photon emissions.

- Jump super-operator J with $J\rho = 2\beta\sigma_-\rho\sigma_+ = 2\beta|-\rangle\langle+|\rho|+\rangle\langle-|$.
- Generating operator, $G(s,t) \equiv \sum_{n=0}^{\infty} s^n \rho_t^{(n)}$; *counting variable s* $(= e^{i\chi})$.

$$\dot{\rho}_t^{(n)} = \mathcal{L}_0\rho_t^{(n)} + J\rho_t^{(n-1)} \rightsquigarrow \boxed{\partial_t G(s,t) = (\mathcal{L}_0 + sJ)G(s,t).}$$

- Counting statistics as $p_n(0,t) = \mathrm{Tr}\rho_t^{(n)}$.

Tools: 'n-resolved density matrix'

Cook's 'counting at the source',R. J. Cook, PRA **23**, 1243 (1981)

n-resolved Master equation for resonance fluorescence of driven TLS

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- Splitting up $\rho_t = \sum_{n=0}^{\infty} \rho_t^{(n)}$, n photon emissions.

- $\hat{G}$ as vector, resolvent matrix

$$[z - \mathcal{L}_0 - sJ] = \begin{pmatrix} z+2\beta & 0 & 0 & -\Omega \\ -2\beta s & z & 0 & \Omega \\ 0 & 0 & z+\beta & 0 \\ \frac{\Omega}{2} & -\frac{\Omega}{2} & 0 & z+\beta \end{pmatrix}.$$

$$\mathrm{Tr}\hat{G}(s,z) = \frac{(z+\beta)(z+2\beta) + \Omega^2 + (s-1)2\beta\left[(z+\beta)\rho_0^{++} + \Omega \mathrm{Im}\rho_0^{+-}\right]}{z(z+\beta)(z+2\beta) + \Omega^2[z+\beta(1-s)]}.$$

Tools: 'n-resolved density matrix'

Resonance fluorescence: sub-Poissonian counting statistics

Information contained in

$$\mathrm{Tr}\hat{G}(s,z) = \frac{f(z)}{zf(z)+\beta\Omega^2(1-s)}, \quad f(z) \equiv (z+\beta)(z+2\beta)+\Omega^2.$$

Tools: 'n-resolved density matrix'

Resonance fluorescence: sub-Poissonian counting statistics

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- Need to transform back into time-domain.

$$\begin{aligned} p_n(0,t) &= \frac{\partial^n}{\partial s^n}\mathrm{Tr}G(s,t)\Big|_{s=0}. \\ \langle n\rangle_t &= \frac{\partial}{\partial s}\mathrm{Tr}G(s,t)\Big|_{s=1} \quad \text{1st moment.} \\ \langle n(n-1)_t\rangle &= \frac{\partial^2}{\partial s^2}\mathrm{Tr}G(s,t)\Big|_{s=1} \quad \text{2nd factorial moment.} \end{aligned}$$

Tools: 'n-resolved density matrix'

Resonance fluorescence: sub-Poissonian counting statistics

Information contained in

$$\mathrm{Tr}\hat{G}(s,z) = \frac{f(z)}{zf(z)+\beta\Omega^2(1-s)}, \quad f(z) \equiv (z+\beta)(z+2\beta)+\Omega^2.$$

- Large t: pole z_0 closest to $z=0$.
- Expand $z_0 = \sum_{m=1}^{\infty} c_m (s-1)^m$

$$\rightsquigarrow \langle n \rangle_{t\to\infty} = \frac{\beta\Omega^2}{2\beta^2+\Omega^2} t$$

$$\rightsquigarrow \sigma_t^2 \equiv \langle \Delta n^2 \rangle_{t\to\infty} = \langle n \rangle_{t\to\infty} \left[1 - \frac{6\beta^2\Omega^2}{(2\beta^2+\Omega^2)^2} \right].$$

- Negative Mandel Q-parameter $Q \equiv F-1$, Fano factor $F \equiv \langle \Delta n^2 \rangle / \langle n \rangle < 1$.

Master Equations For Electronic Transport

Main Ingredients

- Simple microscopic models.
- (Quantum) system $\mathcal{H}_S$ + bath(s) $\mathcal{H}_B$ (electronic leads, phonons).
- Basis of (many body) system states $|\alpha\rangle$.
- System density operator $\hat{\rho}(t)$.

(Dis)Advantages

- Good for exploring new models, mechanisms, concepts.
- Perturbative in coupling to the leads: some/much interesting physics lost.

Master Equations

Main Ingredients

- Markovian generalized Master equation for the reduced system density operator $\rho(t)$,

$$\dot{\rho}_t = \mathcal{L}\rho_t, \quad \mathcal{L} = \mathcal{L}_0 + \mathcal{J}$$

- Single electron jump operator $\mathcal{J}$.

Master Equations

Main Ingredients

- Markovian generalized Master equation for the reduced system density operator $\rho(t)$,

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- Single electron jump operator $\mathcal{J}$.

- Example: Single level quantum dot.
- In the basis where $\rho = (p_0, p_1)$,

$$\mathcal{L}_0 + e^{i\chi}\mathcal{J} = \begin{pmatrix} -\Gamma_L & e^{i\chi}\Gamma_R \\ \Gamma_L & -\Gamma_R \end{pmatrix}$$

Master Equations

Generating function

$$G(\chi, t) = \sum_{n=0}^{\infty} e^{in\chi} p_n(t) = \mathrm{Tr} e^{(\mathcal{L}_0 + e^{i\chi} J)t} \rho_0$$

- k-th moment of the counting statistics

$$\langle n^k \rangle_t \equiv \sum_{n=0} n^k p_n(t) = \left. \frac{\partial^k}{\partial (i\chi)^k} G(\chi, t) \right|_{\chi=0} .$$

- k-th *cumulant* of the counting statistics

$$\langle\langle n^k \rangle\rangle_t = \left. \frac{\partial^k}{\partial (i\chi)^k} \log G(\chi, t) \right|_{\chi=0} .$$

Master Equations

Example: Single level quantum dot.

- For large times t,

$$\langle n \rangle_t = \frac{\Gamma_L \Gamma_R}{\Gamma_L + \Gamma_R} t$$

- Fano factor

$$F \equiv \frac{\langle (n - \langle n \rangle_t)^2 \rangle_t}{\langle n \rangle_t} = 1 - \frac{2\Gamma_L \Gamma_R}{(\Gamma_L + \Gamma_R)^2} < 1$$

- Noise is sub-Poissonian ($F < 1$) as in resonance fluorescence!

Summary/Time-Line

Quantum Optics

- 1905 Photoelectric effect (Einstein).
- 1960s Quantum optics (Glauber). Photocounting formulas (Mandel). Photodetector models (Scully, Lamb)
- 1970s/80s Single atom sources. Quantum Trajectories (Carmichael, Hegerfeldt, ...).

Summary/Time-Line

Quantum Optics

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Solid State Mesoscopics

- No interactions Lesovik (1989); Büttiker (1990); Levitov, Lesovik (1993);...
- Master equations, counting field $s = e^{i\chi}$ Bagrets, Nazarov (2003).
- Many applications and new developments.

Further Topics

- Counting statistics and noise in more complicated systems.
- Higher order cumulants.
- Other noise-related quantities: frequency-dependent noise spectra, waiting times.
- ...

Sessions TT 3 (Mon 9.30h, H 2053) and other sessions at the conference!